

Quantum BCH and Reed-Solomon Entanglement-Assisted Codes*

Francisco R. F. Pereira Ruud Pellikaan

Eindhoven University of Technology

Department of Mathematics and Computing Science

Eindhoven, the Netherlands

f.r.fernandes.pereira@tue.nl g.r.pellikaan@tue.nl

Abstract

Quantum error correcting codes play the role of suppressing noise and decoherence in quantum systems by introducing redundancy. Some resources can be used to improve the parameters of these codes, e.g., entanglement. Such codes are called entanglement-assisted quantum (QUENTA) codes. In this paper, a general method to construct QUENTA codes via cyclic codes is shown. Afterwards, the method is applied to BCH and Reed-Solomon codes, resulting in new families of QUENTA codes, where one of them has maximal entanglement and maximal distance separability.

It is generally accepted that the prospect of practical large-scale quantum computers and the use of quantum communication are only possible with the implementation of quantum error correcting codes. Quantum error correcting codes play the role of suppressing noise and decoherence by introducing redundancy. The capability of correcting errors of such codes can be improved if it is possible to have pre-shared entanglement states. This class of codes is known as Entanglement-Assisted Quantum (QUENTA) codes. Additionally, it is possible to show that they can achieve the hashing bound and violate the quantum Hamming bound. The stabilizer formalism for qubits QUENTA codes was created by Brun *et al.* in 2006 [1], where they showed that QUENTA codes paradigm does not require the dual-containing constraint as standard quantum error-correcting code does. The generalization to any finite fields has been recently published by Galindo, *et al.* [2].

After this paper of Brun *et al.*, many papers have focused on the construction of QUENTA codes based on classical linear codes. However, the analysis of q -ary QUENTA codes was taken into account only recently. The majority of them utilize constacyclic codes or negacyclic codes as the classical counterpart. However, little attention has been paid to maximal entanglement QUENTA codes. Using the well-known class of cyclic codes, we describe a method to construct QUENTA codes that have maximal entanglement. This results in quantum codes with better parameters when compared with the ones in the literature and codes that are good candidates to achieve the hash bound.

First of all, we show that the amount of entanglement in a QUENTA code is related to the intersection of the two codes used in the construction. With this characterization and after showing that intersecting two cyclic codes gives us another cyclic code, a few families of QUENTA codes are constructed via BCH and Reed-Solomon codes. In addition, it is showed that one of the families has maximal entanglement and is maximal distance separable. In the end, some numerical examples are given and comparisons with the best quantum codes in the literature are given.

References

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- [2] C. Galindo, F. Hernando, R. Matsumoto, D. Ruano, “Entanglement-assisted quantum error-correcting codes over arbitrary finite fields,” *Quantum Information Processing* 18(116), 1–18, April & 2019.

*Partially funded by the Brazilian funding agencies CNPq (201223/2018-0)