20119 - 2DL60 I

- Questions? Let hear!
(Theory / Exercises doesn't matter.)

diffeqn's: step 1: homog. diff eqn. solve y_H

step 2: search particular sol. y_p

general sol: $\underline{\underline{y = y_p + y_H}}$

(and then initial cond.? bound. cond.?)
(or other kind of conditions?

- $y'' + 2y'(t) + y(t) = 0$

$y(t) = e^{\lambda t}$, $(\lambda^2 + 2\lambda + 1) e^{\lambda t} = 0$

$\lambda^2 + 2\lambda + 1 = 0 \qquad (\lambda + 1)^2 = 0$

$\lambda_1 = -1, \lambda_2 = -1$

$y(t) = A e^{-t} + B \cdot t e^{-t}$

$\boxed{y'(t) - y(t) = 0} \to y(t) = A e^{t}$?

$y = e^{\lambda t}$??

? only solution

~~($\sin(t)$. $\min(t)$. $\frac{1}{t}$)~~

$y'(t) - y(t) = 0$ $\quad e^t$??

$f(t) = f(0) + t f'(0) \cdot \; ---$

$$\rightarrow y(t) = \sum_{n=0}^{\infty} a_n \cdot t^n \qquad \text{"Taylor series"}$$

$$y'(t) = \sum_{n=1}^{\infty} n \cdot a_n \cdot t^{n-1}$$

$$\sum_{n=1}^{\infty} n \cdot a_n \cdot t^{n-1} - \sum_{n=0}^{\infty} a_n t^n = 0$$

$$\sum_{n=0}^{\infty} (n+1) a_{(n+1)} t^n - \sum_{n=0}^{\infty} a_n t^n = 0$$

$$\sum_{n=0}^{\infty} \boxed{\left((n+1) a_{n+1} - a_n\right)} t^n = 0 \quad \forall t$$

$$(n+1) a_{(n+1)} - a_n = 0 \qquad (n = 0, ---)$$

a_0 is given

$n=0$: $\quad 1 \cdot a_1 - a_0 = 0$

$$a_1 = a_0$$

$n=1$; $\quad 2 \cdot a_2 - a_1 = 0$

$$a_2 = \left(\frac{a_0}{2}\right)$$

$n=2$; $\quad 3a_3 - a_2 = 0$

$$a_3 = \frac{a_2}{3} = \frac{a_0}{3 \cdot 2}$$

$$y(t) = a_0 \cdot \boxed{\sum_{n=0}^{\infty} \frac{t^n}{n!}} = \underline{a_0 \cdot e^t}$$

$\rightarrow y''(t) + 2y'(t) + y(t) = 0$

$$\left.\begin{array}{l} y_1(t) = e^{-t} \\ y_2(t) = \underline{t \cdot e^{-t}} \end{array}\right\| \quad \underline{\text{both are}} \ \underline{\text{solutions}}$$

$$\left(\begin{array}{l} y_2'(t) = 1 \cdot e^{-t} - t \cdot (e^{-t}) \\ y_2''(t) = \ldots \end{array} \right)$$

$\rightarrow y'''(t) - \ldots = 0$

$\lambda_1 = -1$, $\lambda_2 = -1$, $\lambda_3 = -1$

$$y(t) = A e^{-t} + B t \cdot e^{-t} + C \underline{t^2} e^{-t}$$

$\rightarrow$ (wolfram alpha)

$y(x)$, $x = e^t$

(18.5)(14)

$x = e^t$, $z = y(e^t)$

$$\frac{dz}{dt} = y'(e^t) \cdot e^t$$

$$\frac{d^2z}{dt^2} = \left(y''(e^t) \cdot e^t\right) \cdot e^t + y'(e^t) \cdot e^t$$

$$a \cdot \left(y''(e^t) \cdot (e^t)^2 + y'(e^t) \cdot e^t\right) + (b-a)\left(y'(e^t) \cdot e^t\right) + c \cdot y(e^t) = 0$$

$$a \cdot (e^t)^2 \cdot y''(e^t) + b\, e^t \cdot y'(e^t) + c\, y(e^t) = 0$$

$$a \cdot x^2 \cdot y''(x) + b \cdot x \cdot y'(x) + c \cdot y(x) = 0$$

(18.6)(10)

$$y''(t) + 2y'(t) + 2y(t) = e^{-x} \cdot \sin(x)$$

$$y_H(x) = A e^{-x} \cdot \cos(x) + B e^{-x} \cdot \sin(x)$$

$$\rightsquigarrow y_p(x) = A \cdot x \, e^{-x} \cos(x) + B \cdot x \, e^{-x} \cdot \sin(x)$$

$$y'(t) + y(t) = e^{t}$$

$$y_H(t) = e^{-t}$$

$$\rightsquigarrow y_p(t) = \frac{1}{2} e^{t}$$ is a solution

$$y_p'(t) = \frac{1}{2} e^{t}$$

$$\rightsquigarrow y_p(t) = \left(\frac{1}{2} e^{t} + \cancel{10 \cdot e^{-t}}\right)$$

*

$$\frac{d}{dt}\left(e^{\lambda t}\right) = \lambda e^{\lambda t}$$ λ: constant

$$y'(t) - 2y(t) = 0 \qquad y_H(t) = A e^{+2t}$$

$$y_H'(t) = A \cdot 2 \, \boxed{e^{2t}}$$

*

$$\begin{cases} a\lambda^2 + b\lambda + c = 0 \rightarrow \text{solutions are eigenvalues of diff. eq.} \\ a y'' + b y' + c y = 0 \end{cases}$$

(10.5)(2)

$$y^{(4)}(t) - 2y'' + y = 0$$

$$\lambda^4 - 2\lambda^2 + 1 = 0$$

$2.2 = 4$

$\lambda^4, \lambda^2 \qquad (\lambda^2)^2 = \lambda^4$

$$\lambda^2 = u$$

$$\rightarrow u^2 - 2u + 1 = 0$$

$$(u-1)(u-1) = 0$$

$u = 1, \quad u = 1$

$\lambda^2 = 1 \qquad \boxed{\lambda = \pm 1}$

$\lambda = 1$ (2x)

$\lambda = -1$ (2x)

$$y_h(t) = A e^{t} + B\, t e^{t} + C\, e^{-t} + D\, t e^{-t}$$

e^{t^2} ? $\underline{y(t) = e^{t^2}}$

$y'(t) =$

$\boxed{\begin{aligned} &\rightsquigarrow \underline{y(t) = e^{(\lambda t)}} \\ &y'(t) = \underline{\underline{\lambda}}\, e^{\lambda t} \end{aligned}}$? $\cancel{\underline{y'(t) = (\lambda(t)) e^{\lambda t}}}$?

$\cancel{\begin{aligned} &y(t) = e^{(t^2)} \\ &y'(t) = \underline{\underline{2 \cdot t e^{t^2}}} \end{aligned}}$

* (10.6) (11)

$y'' + y' = 4 + 2x + e^{-x}$

$y_H(x) = A + B \cdot e^{-x}$

$\rightsquigarrow \boxed{y_p(x) = A \cdot x e^{-x} + B \cdot x + C \cdot x^2}$

$y_p'(x) = A e^{-x} - A x e^{-x} + B + 2 \cdot C x$

$y_p''(x) = -A e^{-x} - A(\underline{e^{-x} - x e^{-x}}) + 2 \cdot C$

$$A + 2A + 1 = A + 1$$

(12) $y'' + 2y' + y = x \cdot e^{-x}$

$$y_H(x) = A e^{-x} + B x e^{-x}$$

$$y_p(x) = C \cdot x^2 e^{-x} + E x^3 e^{-x}$$

$$y'_p(x) = 2 C x e^{-x} - C \cdot x^2 e^{-x}$$

$$y''_p(x) = 2 C e^{-x} - 2 C x e^{-x}$$
$$- C\, 2 x e^{-x} + C \cdot x^2 \cdot e^{-x}$$

$$\left(2 \cdot C e^{-x} - 2 C x e^{-x} - 2 x C e^{-x}\right.$$
$$\left. + \cancel{C \cdot x^2 e^{-x}}\right) + 2\left(2 C x e^{-x} - \cancel{C x^2 e^{-x}}\right)$$
$$+ \cancel{C x^2 \cdot e^{-x}} = x e^{-x}$$

$$y_H(x) = A e^{-x} + B x e^{-x}$$
$$y_p(x) = x^2 \cdot (\dots)$$
$$x^2 e^{-x} \qquad e^{-x}, x e^{-x}$$

$$y'' + 2y' + y = e^{-x} + x e^{-x}$$
$$y_p(x) = A \cdot x^2 e^{-x} + B x^3 e^{-x}$$
$$y_p(x) =$$

$(\lambda+1)^2=0$ ℓ

$$y''(x)+2y'(x)+y(x)=x e^{-x}$$

$$y_p(x)=\underline{A\cdot x^2 e^{-x}+B\cdot x^3 e^{-x}}$$

need end

"I will search solution"

(7.9) (7)

(non-linear)

$$\left(\frac{dy}{dx}=(1-y^2)\right)\cdot dx$$

$$\left(dy=(1-y^2)\cdot dx\right)/(1-y^2)$$

$$\int\frac{dy}{(1-y^2)}=\int dx=x$$

$$\int\frac{1}{(1-y^2)}dy=-\frac{1}{2}\cdot\ln|1-y|+\frac{1}{2}\cdot\ln|1+y|+c$$

$$\frac{\frac{1}{2}}{(1-y)}+\frac{1/2}{(1+y)}=\frac{1}{1-y^2}$$

$$=\frac{1}{2}\cdot\ln\left|\frac{1+y}{1-y}\right|+C_1$$

$$e^{a+b} = e^{a} \cdot e^{b}$$

$$e^{\left(\ln\left|\frac{1+y}{1-y}\right| + C = 2\cdot x\right)}$$

?y?

$$C\cdot\left(\frac{1+y}{1-y}\right) = e^{2x}$$

$$C\cdot(1+y) = (1-y)\,e^{2x}$$

$$C\cdot y + y\,e^{2x} = (e^{2x} - C)$$

$$C\cdot y(1+e^{2x}) = (e^{2x} - C)$$

$$y(x) = \frac{(e^{2x} - C)}{C(1+e^{2x})}$$

(10.5)(3)

$$y^{(4)} + 2\cdot y'' + y = 0$$

$$(\lambda = -i), (2x) \qquad (\lambda = i)(2x)$$

$$(\lambda^2+1)^2 = 0 \qquad \underline{\lambda^2 = -1}$$

<u>real</u> solutions

complex solution e^{ix}, e^{-ix},

$x \cdot e^{ix}$, $x \cdot e^{-ix}$

$$e^{ix} = \cos(x) + i \sin(x)$$

$$y_H(x) = A \cos(x) + B \cdot \sin(x)$$

$$C \cdot x \cdot \cos(x) + D x \cdot \sin(x)$$

$$\cos(x) = \left(\frac{e^{ix} + e^{-ix}}{2}\right)$$

$$\sin(x) = \left(\frac{e^{ix} - e^{-ix}}{2i}\right)$$

$$y_H(x) = x \cdot \cos(x)$$

$$y_H'(x) = \cos(x) - x \cdot \sin(x)$$

$$y_H''(x) = -2\sin(x) - x \cdot \cos(x)$$

$$y'''(x) = -3 \cdot \cos(x) \qquad + x \cdot \sin(x)$$

$$y^{4}(x) = \boxed{+4 \cancel{\sin(x)} \qquad + x \cos(x)}$$

$$2 \cdot (-2 \cancel{\sin x} - x \cdot \cancel{\cos(x)})$$

$$+ x \cdot \cos(x) = 0$$

(18.6) (1)

unknown : $y(t)$??

$$y'' + y' - 2y = 1$$

$$y_H(t) = A e^{t} + B e^{-2t} ;$$

$$y_p(t) = \left(-\frac{1}{2}\right) \qquad -t,\ t,\ \ldots$$

$$y'' + y' - 2y = (1+t) \quad \text{polynomial}$$

$$y_p(t) = A + B \cdot t$$

$$y_p'(t) = B$$

$$y_p''(t) = 0$$

$$y(t) = y_H(t) + y_p(t)$$

$$0 + (B) - 2(A + Bt) = 1 + t$$

$$(B - 2A) - 2Bt = 1 + t$$

$$\forall t \quad (B - 2A - 1) + t(-2B - 1) = 0$$

$$B - 2A - 1 = 0, \quad -2B - 1 = 0$$

$$-\frac{1}{2} - 2A - 1 = 0 \qquad B = \left(-\frac{1}{2}\right)$$

$$2A = -\frac{3}{2} \qquad A = -\frac{3}{4}$$

(7.9)(16)

$$\frac{dy}{dx} + 2e^{x} y = e^{x}$$

$\rightsquigarrow$ (I) $\frac{dy}{dx} + 2e^{x} y = 0$

$$\int \frac{dy}{y} = \int -2e^{x}\,dx$$

$$\ln|y| = (-2e^{x} + c)$$

$$\underline{\underline{y_H(x)}} = c \cdot e^{(-2e^{x})}$$

$$\left(e^{f(x)}\right)' = f' \cdot e^{f}$$

$f = -2e^{x}$

(II)

$$y(x) = c(x) \cdot e^{(-2e^{x})}$$

$$y'(x) = c'(x) \cdot e^{(-2e^{x})} + c(x) \cdot (-2e^{x}) \cdot e^{(-2 \cdot e^{x})}$$

$$c'(x) \cdot e^{(-2e^{x})} - \cancel{2 \cdot c(x) \cdot e^{x} e^{(-2e^{x})}} + \cancel{2 \cdot e^{x} \cdot \left(c(x) e^{(-2e^{x})}\right)} = e^{x}$$

$$c'(x) = 1 \cdot e^{x} \cdot e^{(+2e^{x})}$$

$$\cancel{C(x)} = \left(\frac{1}{2} e^{(2e^x)} + C_1\right)$$

$$y(x) = \left(\frac{1}{2} e^{(2e^x)} + c\right) \cdot e^{(-2e^x)}$$

$$y(x) = \frac{1}{2} + c\, e^{(-2e^x)}$$

$y_H(x)$

$\int$

$\underline{(18.5)}$ $\underline{(5)}$

$$\underline{y = e^{2t}}$$

Show a solution:

$$y' = 2e^{2t} \qquad 8e^{2t} - 2 \cdot 2e^{2t} - 4e^{2t} = 0$$

$$y'' = 4e^{2t}$$

$$y''' = 8e^{2t}$$

$\int$

$\times$

$$y(t) = e^{\lambda t}$$

$$\lambda^3 - 2\lambda - 4 = 0 \rightsquigarrow \boxed{\lambda = 2}$$

$$(\lambda^3 - 2\lambda - 4) : (\lambda - 2) = \cdots$$

(18.6) (12)

$$y'' + 2y' + y = xe^{-x}$$

$$y_H(x) = Ae^{-x} + B \cdot xe^{-x}$$

$$(\lambda+1)^2 = 0$$

if $y'' + 2y' + y = e^{-x}$

then $y_p(x) = \underline{C \cdot x^2 \cdot e^{-x}}$

$$y_H \approx 1 \cdot e^{-x},\ x \cdot e^{-x},\ x^2 \cdot e^{-x}$$

if $y'' + 2y' + y = xe^{-x}$

$$y_H \sim 1 \cdot e^{-x},\ x \cdot e^{-x}$$

then $y_p(x) = B \cdot x^2 \cdot (xe^{-x})$

<u>Gana:</u> $(B = \frac{1}{6})$