201123 – 2DL04 I

• Questions, let hear!

(12.3)(17)

$$f(x,y) = \frac{x}{x^2+y^2} \quad \text{at } (1,2)$$

$$\frac{\partial f}{\partial x} =$$

$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f'(x) \cdot g(x) + f(x)\, g'(x)$$

$$\frac{d}{dx}\left(x \cdot \left(\frac{1}{x^2+y^2}\right)\right) =$$

$$1 \cdot \frac{1}{(x^2+y^2)} + x \cdot \left((-1) \cdot (x^2+y^2)^{-2} \cdot 2x\right)$$

$$(x^2+y^2)^{-1} \quad \frac{d}{dx}\left(-1 \cdot (x^2+y^2)^{-2} \cdot 2x\right)$$

$$\frac{1}{x^2+y^2} - \frac{2x^2}{(x^2+y^2)^2} =$$

$$\frac{(x^2+y^2) - 2x^2}{(x^2+y^2)^2} = \frac{-x^2+y^2}{(x^2+y^2)^2}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

$$\ast \begin{cases} f = x \\ g = x^2 + y^2 \end{cases} \qquad \frac{1\cdot(x^2+y^2) - x\cdot 2x}{(x^2+y^2)^2}$$

($\underline{\text{y diff eq}}$: $y = y_H + y_p$

often in y_p you see y_H back)

$\ast$

$$\int x^3 \cdot e^{(x^2/2)}\, dx = \quad (\text{P.I})$$

$$\left(\int f'\cdot g(x)\, dx - \ldots\right)$$

$$f' = x\cdot e^{x^2/2} \qquad g(x) = x^2$$

$$\int \quad f = e^{x^2/2} \qquad g' = 2\cdot x.$$

$$\int f'\cdot g\, dx = [f\cdot g] - \int f\cdot g'\, dx$$

$$\left[e^{\frac{x^2}{2}}\cdot x^2\right] - \left(\int e^{x^2/2}\cdot 2x\, dx\right)$$

$$= x^2\cdot e^{x^2/2} - 2\cdot e^{(x^2/2)} + C$$

!

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$$\lim_{(x,y)\to(0,0)} \frac{2x^{2}\cdot y^{1}}{x^{4}+y^{2}}$$

(I) : $\left(\frac{0}{0}\right)$

(II) : $y=0 \rightsquigarrow \frac{0}{x^4}=0 \quad \forall x\neq 0$.

$y=x^{(\alpha)} \quad \alpha$??

$y=x$: $\frac{2\cdot x^2\cdot x}{x^4+x^2} = \frac{2\cancel{x^2}\cdot x}{\cancel{x^2}(1+x^2)} \to 0$

$x\to 0 \rightsquigarrow x=\frac{1}{10} \quad x^4=\frac{1}{10000}$

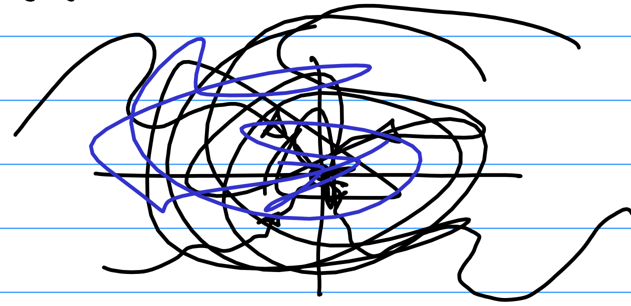
$\rightsquigarrow x^2=\frac{1}{100}$

$y=x^2 \rightsquigarrow \frac{2\cdot x^2\cdot x^2}{x^4+x^4} = \frac{2}{2} = 1$

limit doesn't exists

2 ways $\rightsquigarrow$ 2 different values.

"If limit exists"



$$\lim_{(x,y)\to(0,0)} \frac{x^{3}}{(x^{2}+y^{2})} \overset{?}{=} 0$$

limit

$$\left| \frac{x^3}{x^2+y^2} - 0 \right| =$$

$$x^3 = x^2 \, x$$

$$\left| x \cdot \underbrace{\frac{x^2}{x^2+y^2}}_{\leq 1} \right| \leq |x| \to 0$$

(because $x \to 0$)

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(18.6) (12)

$$y'' + 2y' + y = 0 \qquad e^{-x}, \; x e^{-x}$$

$$y'' + 2y' + y = (x \cdot e^{-x}) \quad f(x)$$

$$\cancel{y_p(x) = c \cdot x^2 e^{-x}}$$

$$\begin{cases} y'' + 2y' + y = (e^{-x}) \\ y_p(x) = c\, x^2 (e^{-x}) \end{cases}$$

$x^2 \quad x^2 \quad x \cdot e^{-x}$

try: $y_p(x) = c \cdot x^3 \cdot \exp(-x)$

$$y_p'(x) = 3x^2 \cdot \exp(-x) - x^3 \cdot e^{-x}$$

$$y_p''(x) = 6xe^{-x} - 3x^2e^{-x} - 3x^2e^{-x} + x^3 \cdot e^{-x}$$

$$c\big(\cancel{x^3 \cdot e^{-x}} \cancel{- 6x^2e^{-x}} + 6xe^{-x}$$

$$2\cdot(\cancel{3x^2e^{-x}} \cancel{- x^3e^{-x}}) + \cancel{x^3e^{-x}}\big) = xe^{-x}$$

$$\underline{6 \cdot C = 1} \quad (C = 1/6)$$

$*$

$$y_g(x) = c \cdot x^2 e^{-x}$$

$$y'' + 2y' + y = \underline{\underline{("c")}}\, e^{-x}$$

$$y'' + 2y' + y = \underline{x \cdot e^{-x}}$$

$$y_g(x) = x^2(\underline{\underline{A \cdot x \cdot e^{-x}}} + \underline{Bxe^{-x}}$$

$$+ \underline{C \cdot e^{-x}})$$

$$\underline{\underline{(e^{-x}, xe^{-x})}}$$

$*$

$$y_g(x) = \underline{\underline{B \cdot x^2 e^{-x} + C \cdot x^3 e^{-x}}}$$

$*$ $$\Rightarrow (\underline{\underline{B = 0}}, \underline{\underline{C = 1/6}})$$

construction of guess:

$$y'' + 2y' + y = f(x)$$

y_H : $x^{0} e^{-x}$, $x^{1} e^{-x}$

$y_P \simeq x^{2} (f(x))$

on $f(x) = x e^{-x}$

we know in our guess certainly

$x^2 (x e^{-x})$ no diff. or will

give terms: $x^2 e^{-x}$ may be to

work them away on some way and

here by taking the term: $B \cdot x^2 e^{-x}$

$$y_g(x) = B \cdot x^2 e^{-x} + C \cdot x^3 \cdot e^{-x}$$

(e^{-x}, $x e^{-x}$ have no sense, they will give zero: solutions of homogen. equation.)

* with wolfram alpha, just to fill in: $y'' + 2 \cdot y' + y = x \cdot \exp(-x)$

goes very easy! (Also de of CH 7.9!)

(wolframalpha: $\sqrt[x]{e} = e^{\left(\frac{1}{x}\right)}$)

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Everything goes well??

It is quiet today,

a "black" Monday??

* Indeed Stefan! Better than yesterday.

Weather forecast is good.

* (18,6) (10)

$$y'' + 2y' + 2y = e^{-x} \cdot \sin(x)$$

$$y_H(x) = (A \cdot \cos(x) + B \cdot \sin(x)) e^{-x}$$

$$(\lambda^2 + 2\lambda + 2 = 0 \qquad (-1 \pm i))$$

$$\rightsquigarrow y_p(x) = ?\ A \cdot x e^{-x} \cdot \cos(x) +$$

$$\rightsquigarrow B \cdot x \cdot e^{-x} \cdot \sin(x)$$

$$((A \cdot x \cdot \cos(x) + Bx \cdot \sin(x)) e^{-x})$$

$$(x \cdot \cos(x))' = (- - - .)$$

$$(x \cdot \sin(x))' = (. - - -)$$

with wolfram alpha:

computation time exceeded.

by calculation of those derivatives

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Maybe some "easier" way

$$e^{-x} \cdot \sin(x) = \mathrm{Im}\left(e^{(-1+i)x}\right)$$

$$y_p(x) = A \cdot x\, e^{(-1+i)(x)} + B \cdot x\, e^{(-1-i)x}$$

$$y_p'' + 2y_p' + 2y_p =$$

$$\left(\frac{e^{(-1+i)\cdot x} - e^{(-1-i)x}}{2i}\right) =$$

$$(e^{-x} \cdot \sin x)$$

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Still a lot of computations to do.

$$y_p'' + 2y_p' + y_p = 2iA\exp((-1+i)x) - 2i \cdot B \cdot \exp((-1-i)x)$$

I don't like this result, I have the feeling that $A = -B$ or something like that

$$2iA = \frac{1}{2i}, \quad -2iB = \frac{-1}{2i}$$

$$2A = -\frac{1}{2}, \quad 2B = -\frac{1}{2}$$

but here $A = B = \frac{-1}{4}$

$$y_p(x) = \frac{-1}{4} \cdot x \cdot \exp((-1+i)x)$$
$$\frac{-1}{4} \cdot x \cdot \exp((-1-i)x) =$$

$$-\frac{1}{4} \cdot x \cdot e^{-x} \left((\cos x + i \sin(x)) + (\cos(-x) + i \sin(-x))\right) =$$

$$-\frac{1}{2} x \cdot e^{-x} \cdot \cos(x).$$

<u>But this goes well!!</u>

$\boxed{\text{(My feeling was wrong!!)}}$