2DL40 I - 201116:

Questions about theory?

Let here! Questions about exercises can also be done!

By ask

exc. 4: $z = a + ib$ solution

$\Rightarrow \bar{z} = a - ib$ also

Question: When??

real coëfficients

$\rightsquigarrow (i \cdot z + 1)^2 = 0$; no real coëff

$z = i$ $\quad$ $z = \cancel{-i}$?

$-z^2 + 2iz + 1 = 0$

$z = i$: $+1 - 2 + 1 = 0$

$z = -i$; $+1 + 2 + 1 \neq 0$

$-z^2 + 2iz + 1 = -(\bar{z})^2 - 2i\bar{z} + 1$

$(z=-i)$

$$(iz+1)^2 \cdot (-iz+1) = 0$$

*

$z = 3i$
$z = 1-i$

$-3i$
$1+i$

real coeff

$$(z-3i)(z+3i)(z-(1-i))\cdot(z-(1+i))$$

$$(1-i)(1+i) = 2$$

$$(z-(1-i))(z-(1+i)) =$$

$$z^2 - z(1+i) - z(1-i) + 2$$

$$z^2 - 2z + 2$$

$$((z-1)^2+1)$$

$$(z^2+9)(z^2-2z+2)$$

$$(z^4 + \ldots\ldots)$$

$\sqrt[3]{-1} \to z_1, z_2, z_3$

$z \to \operatorname{Ang}(z)$

$\cancel{\left(\frac{2}{3}\pi\right)3 = 2\pi}$

$z^3 = -1$ $\qquad$ $R^3 = 1$ $\qquad$ $(R = 1)$

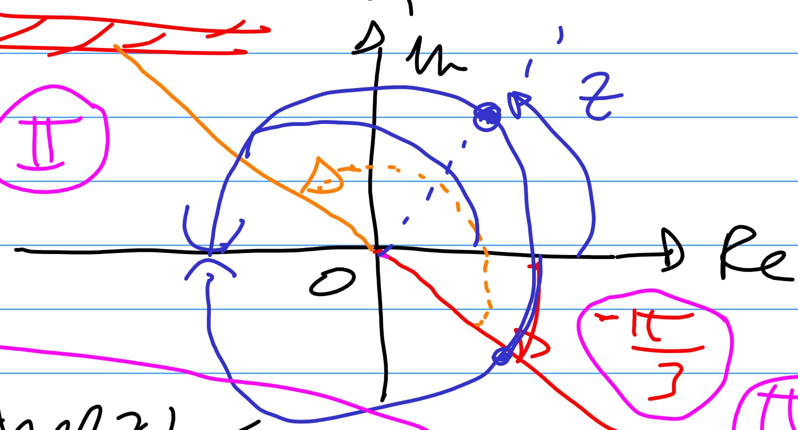
$z = R \cdot e^{i\varphi}$

$-1 = e^{(\pi i + 2k\pi i)}$

$\arg(z^3) = 3 \cdot \varphi = (\pi + k \cdot 2\pi)$

$\varphi = \frac{\pi}{3} + k \cdot \frac{2}{3}\pi$

$(z = -1$ Im $(-1)^3 = -1)$



$\frac{\pi}{3}$

$10\pi + \frac{\pi}{3} = \frac{31\pi}{3}$

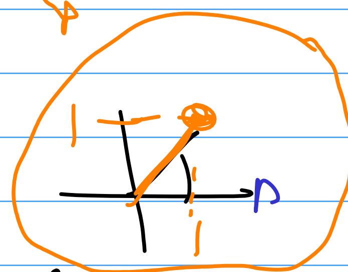
$-\pi < \operatorname{Ang}(z) \le \pi$

Brasch

2)
$$e^{iz} = \frac{2}{(1-i)} \cdot \frac{(1+i)}{(1+i)} = \frac{2(1+i)}{2} = 1+i$$

$$e^{i \cdot z} = 1+i$$

$$\Rightarrow \boxed{z = a + i \cdot b}$$



$$\exp(i(a+ib)) = 1+i$$

$$\exp(i \cdot a - b) = (1+i) = \sqrt{2} \cdot e^{(\frac{\pi}{4} + k \cdot 2\pi) i}$$

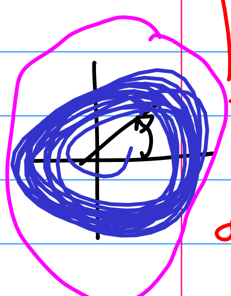
$$= e^{-b} \cdot e^{ia} = \sqrt{2} \cdot e^{(\frac{\pi}{4} + k \cdot 2\pi) i}$$

$$|\cdot| = |\cdot| \Rightarrow \boxed{e^{-b} = \sqrt{2}} \qquad -1 = e^{\pi i}$$

ln

$$-b = \ln(\sqrt{2})$$

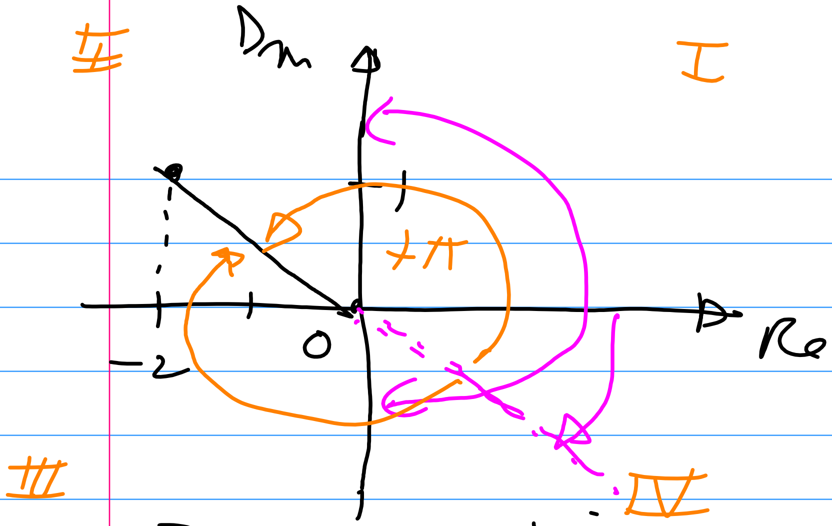
$$b = \ln\left(\frac{1}{\sqrt{2}}\right)$$



$$\arg(\cdot) = \arg(\cdot) \Rightarrow$$

$$a = \left(\frac{\pi}{4} + k \cdot 2\pi\right)$$

$$\Rightarrow z = \left(\frac{\pi}{4} + k \cdot 2\pi\right) + i \ln\left(\frac{1}{\sqrt{2}}\right) \quad k \in \mathbb{Z}$$



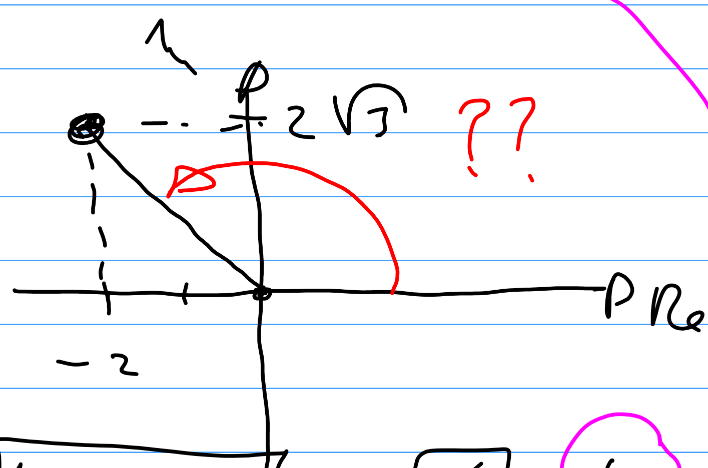
$$z = -2 + i$$

$$-\frac{\pi}{2} < \arctan\left(\frac{1}{-2}\right) < \frac{\pi}{2}$$

(7 b) $\quad (\underbrace{iz+2}_{w})^4 = -2 + 2i\sqrt{3}$

$$w^4 = -2 + 2i\sqrt{3}$$

4 diff. solutions



$w = a + ib$

$$\boxed{w = R\cdot e^{i\varphi}}$$

$$|-2 + 2i\sqrt{3}| = \sqrt{4 + 4\cdot 3} = \sqrt{16} = 4$$

$$\arg(-2 + 2i\sqrt{3}) = \left(\frac{2}{3}\pi\right) + k\cdot 2\pi$$

$$\arctan\left(\frac{2\sqrt{3}}{-2}\right) = \arctan(-\sqrt{3}) \quad (k \in \mathbb{Z})$$

$$w^4 = \underbrace{R^4 \cdot e^{i4\varphi} = 4 \cdot e^{i\left(\frac{2\pi}{3} + k 2\pi\right)}}$$

$$R^4 = 4 \qquad \boxed{R = \sqrt{2}}$$

$$4\varphi = \frac{2}{3}\pi + k \cdot 2\pi$$

$$\varphi_k = \left(\frac{\pi}{6} + k \cdot \frac{\pi}{2}\right) \quad \underline{k = 0, 1, 2, 3}$$

$$w_k = \sqrt{2} \cdot \exp(i\varphi_k) \qquad k = 0, 1, 2, 3$$

$$w = iz + 2$$

$$(-i)(iz = (w - 2))$$

$$z = (-i \cdot w + 2i)$$

$$z_k = -i \cdot w_k + 2i \qquad \underline{k = 0, 1, 2, 3}$$

$\varphi_k = \ldots k \ldots$ k an integer multiplication

↑ k an index

$$\boxed{z^2 - 2iz} - 1 = 0$$

$$\boxed{(z-i)^2 + 1} - 1 = 0$$

$$(z-i)^2 = 0 \qquad \begin{cases} z_1 = i \\ z_2 = i \end{cases}$$

(solutions "fall" together)

3d) $z^3 - 3iz^2 - 2z = 0$

$$z(z^2 - 3iz - 2) = 0$$

$$\boxed{z_1 = 0}$$

$$\boxed{z^2 - 3iz} - 2 = 0$$

$$\boxed{\left(z - \frac{3}{2}i\right)^2 + \left(\frac{9}{4}\right)} - 2 = 0$$

$$z^2 - 3i + \underbrace{\left(-\frac{3}{2}i\right)^2}_{\leadsto\ -\frac{9}{4}}$$

$$\left(-\frac{3}{2}\right)^2 \cdot i^2 =$$

$$\left(z - \frac{3}{2}i\right)^2 + \frac{1}{4} = 0$$

$$\left(z - \frac{3}{2}i\right)^2 = -\frac{1}{4}$$

$$\left(z - \frac{3}{2}i\right) = \pm\frac{1}{2}i$$

$$z = \frac{3}{2}i \pm \frac{1}{2}i$$

$$z = 2i, \quad z = i$$

$z = 0$
$z = 2i$
$z = i$

$$(z - 2i)(z - i) =$$

$$z^2 - i z - 2i z - 2$$

$$z^2 - 3i z - 2$$

(b) $p(z) = z^5 + 3z^4 + 4z^3 + 4z^2 + 3z + 1$

real coeff!

$\underline{z_1 = i}$, $\underline{z_2 = -i}$

$$\boxed{\begin{gathered}(iz - 1)(iz + 1) = 0 \\ \underline{z = i}, \quad \underline{z = -i}\end{gathered}}$$

<u>facts</u>: $(z - i)(z + i) = (z^2 + 1)$

$$\cancel{z^5}+3z^4+4z^3+4z^2+3z+1 : (z^2+1) = z^3+3z^2+3z+1$$

$$\cancel{z^5} \quad +z^3 \quad -$$

$$3z^4+3z^3$$

$$3z^4 \quad +3z^2 \quad -$$

$$3z^3+z^2+3z$$

$$3z^3 \quad +3z \quad -$$

$$z^2+1$$

$$z^2+1 \quad -$$

$$0$$

$$1z^3+3z^2+3z+1=0$$

$z=-1$ $\quad$ $z+1$

1 1

1 2 1

1 3 3 1

$z+1$ $\quad$ 1 1

$(z+1)^2$ $\quad$ 1 2 1

$(z+1)^3$ $\quad$ 1 3 3 1

$z=i$

$z=-i$

$z=-1$

$z=-1$

$z=-1$

$$(z+1)(z^2+2z+1)=0$$

$$(z+1)^3=0 \qquad z=-1\ (3x)$$

$p(z) = (z-i)(z+i)(z+1)(z+1)(z+1)$

! don't forget

(7a) $(iz+i)^3 = i$

$(-i)($ $\to$ $w = iz + i$ $)$ $\boxed{w_k}$ known

z ??

(asked)

$-iw = z + 1$

$z_k = (-1 - i \cdot w_k)$

$(iz_k + i)^3 = (i(-1 - iw_k) + i)^3 =$

$(w_k)^3 = i$ $\qquad i^3 = -i$

$(i(z+1))^3 = i$

$-(z+1)^3 = 1$ $\qquad ((z+1)^3 = -1)$

((4) $(z-3i)(z+3i)(z-(1-i))(z-(1+i))$)

(18.5) Adams

1) $y''' - 4.y'' + 3y' = 0$

$$y(t) = e^{(\lambda t)}$$

$$\lambda^3 \cdot e^{\lambda t} - 4.\lambda^2 e^{\lambda t} + 3.\lambda e^{\lambda t} = 0 \qquad \underline{\underline{\forall t}}$$

$$(\lambda^3 - 4\lambda^2 + 3\lambda)e^{\lambda t} = 0$$

$$(\lambda^3 - 4\lambda^2 + 3\lambda) = 0$$

$$\lambda(\lambda^2 - 4\lambda + 3) = 0$$

$$\lambda \cdot (\lambda - 3)(\lambda - 1) = 0$$

$$\lambda_1 = 0, \quad \lambda_2 = 3, \quad \lambda_3 = 1$$

$$y_1(t) = A, \quad y_2 = B e^{3t}, \quad y_3(t) = Ce^{t}$$

$$y(t) = A + B.e^{3t} + Ce^{t}$$

$$y(t), \quad \frac{dy}{dt}, \quad \cancel{\frac{d^2y}{dt^2} \neq \left(\frac{dy}{dt}\right)^2}$$

$$\frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right); \quad \frac{d^3y}{dt^3} = \frac{d}{dt}\left(\frac{d^2y}{dt^2}\right) \cdots$$

adams

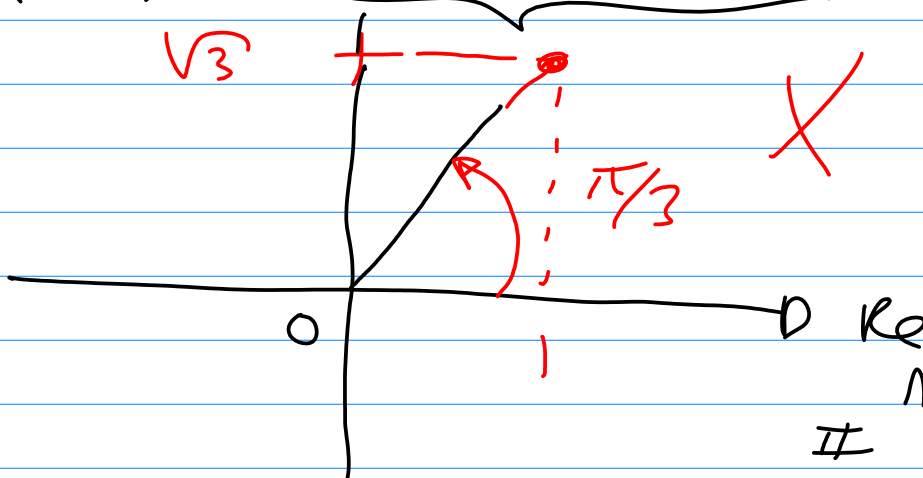
$$z^4 = -1 + i\sqrt{3}$$

(55) $z^4 + 1 - i\sqrt{3} = 0$

$$z_k = 2^{1/4} \cdot e^{i\left(\cancel{\frac{\pi}{12}} + k\cdot\frac{\pi}{2}\right)}$$

$\frac{\pi}{6}$

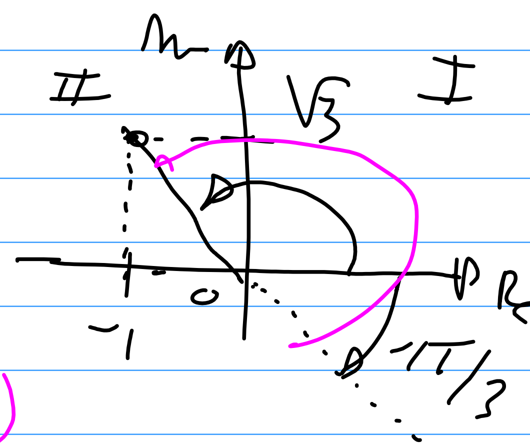
$$(z_k)^4 = \underbrace{2 \cdot e^{i\left(\frac{\pi}{3}\cdot 2 + k\cdot 2\pi\right)}}$$



$$z^4 = \left(-1 + i\sqrt{3}\right)$$

$\frac{2\pi}{3}$

arg(·)

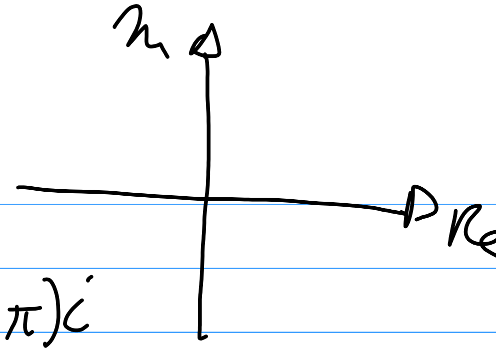


1) a) $e^z = 1$ $\qquad$ $\ln(a+ib)$??

$\ln(e^z = 1)$ $\qquad$ $\boxed{z = 0}$

$(z = Re^{i\varphi})$ $\qquad$ $\boxed{z = a + ib}$

Bwasch



① $e^{(a+ib)} = 1$

$e^{a} \cdot e^{ib} = e^{(0+k\cdot 2\pi)i}$

$|\cdot| = |\cdot|$ $\quad e^{a} = 1 \Rightarrow a = 0$

$\arg(\cdot) = \arg(\cdot)$ $\quad b = 0 + k\cdot 2\pi$

$z_k = 0 + (k\cdot 2\pi)i \quad , k \in \mathbb{Z}$