2DL60I - 200914:

(notes will be saved at my homepage:
www.win.tue.nl/~rvhassel/)

(1.5)(4) $\qquad E = \begin{pmatrix} | & | \end{pmatrix}$

$$\begin{pmatrix} 2 & 4 \\ 1 & 6 \end{pmatrix} E = \begin{pmatrix} 2 & -2 \\ 1 & 3 \end{pmatrix}$$

matrix × kolom =

$$\begin{pmatrix} | & & | \\ a_1 & \cdots & a_k \\ | & & | \end{pmatrix} \begin{pmatrix} l_1 \\ \vdots \\ l_k \end{pmatrix} =$$

$$l_1 \cdot \begin{pmatrix} | \\ a_1 \\ | \end{pmatrix} + l_2 \begin{pmatrix} | \\ a_2 \\ | \end{pmatrix} \cdots + l_k \begin{pmatrix} | \\ a_k \\ | \end{pmatrix}$$

$$\begin{pmatrix} 2 & 4 \\ 1 & 6 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ 1 & 3 \end{pmatrix}$$

$$\alpha \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$$

$\alpha = -3$
$\beta = 1$

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 6 \\ -2 \end{pmatrix} =$$

$$6\begin{pmatrix} 1 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \ldots$$

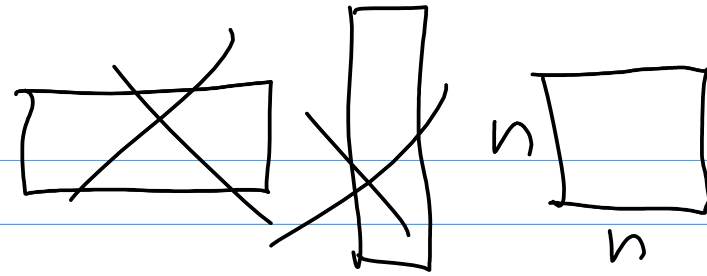
$$\begin{pmatrix} 4 & 1 & 3 \\ 2 & 1 & 4 \\ 1 & 3 & 2 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 4 \\ 4 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}$$

?E?

$$\begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 + 2(-4) \\ 4\cdot 1 + 1\cdot(-4) \end{pmatrix}$$

$$= 1\cdot\begin{pmatrix} 1 \\ 4 \end{pmatrix} - 4\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

(1.4) (19)



$\underline{A \ (n \times n)}$

$A^2 = 0$

$(I - A)$ niet singulier?

$(1 - x)(1 + x) = 1 - x^2 \longleftarrow$??

$(I - A)(I + A) =$

$I - A + A - A^2 = I$

$$(I - A)^{-1} = (I + A)$$

$C, \ C^{-1}$ $\quad \begin{pmatrix} C \cdot C^{-1} = I \\ C^{-1} \cdot C = I \end{pmatrix}$

$\cancel{A^{-1} = \frac{1}{A}}$

(1.5) (, 2 a)

$$A \cdot X + B = C \qquad X\,??$$

$$AX = (C - B)$$

$$A^{-1}(AX = (C - B))$$

$$\underbrace{A^{-1}A}_{I} \cdot X = A^{-1}(C - B)$$

$$A = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix} \qquad (A|I)$$

$$\begin{array}{c} 3 \\ 5 \end{array} \cdot \left(\begin{array}{cc|cc} 5 & 3 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right) \sim (I\,|\,A^{-1})$$

12 c

$$AX + B = X$$

Ç,

$$B = X - AX$$

$$B = (I - A)X$$

$$(I - A)X = B$$

$$\cancel{(I - A)^{-1}} \cdot \cancel{(I - A)}\, X = (I - A)^{-1} \cdot B$$

$$X = (I - A)^{-1} B$$

$$(I - A)^{-1} \;??$$

(d) $XA + C = X$

$$\boxed{A \cdot B \neq B \cdot A!}$$

i.h.a

maar wel $A \cdot I = I \cdot A$

$$X - XA = C$$

$$X \cdot (I - A) = C$$

$$X = C \cdot (I - A)^{-1}$$

$(1.5)(1\ 5)$ $\quad A = 3 \times 3$

$$A = (a_1\ a_2\ a_3)$$

$$\boxed{2 \cdot a_1 + 1 \cdot a_2 - 4 a_3 = 0}$$

$$\underline{AX = 0}$$

$\underline{A \times \text{kolon}}$ = ?

$$\begin{pmatrix} 4 & 2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \cdot 1 + 2 \cdot 4 \\ 1 \cdot 1 + 0 \cdot 4 \end{pmatrix}$$

$$= \boxed{1 \cdot \begin{pmatrix} 4 \\ 1 \end{pmatrix} + 4 \begin{pmatrix} 2 \\ 0 \end{pmatrix}}$$

1e

$$A \underline{X} = \underline{0}$$

$$\boxed{(a_1\ a_2\ a_3) \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}$$

$$2 \cdot \begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$2 \cdot (\underline{a}_1, \underline{a}_2, \underline{a}_3)\begin{pmatrix}2\\-1\\4\end{pmatrix} = 2 \cdot \begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$(\underline{a}_1, \underline{a}_2, \underline{a}_3)\begin{pmatrix}4\\-2\\8\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

Is A nonsingular?!

(als A inverteerbaar is

$A\ (n \times n)$ $\qquad$ $A\underline{x} = \underline{0} \Rightarrow$

A^{-1} $\qquad$ $A^{-1}(A\underline{x}) = A^{-1}(\underline{0}) = \underline{0}$

$$\underline{x} = \underline{0}$$

$\underline{x} = \underline{0}$
de enige opl

$$A\begin{pmatrix}2\\-1\\4\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$A\underline{x} = \underline{0}$

A niet-singulier $\Rightarrow$ $\underline{x} = \underline{0}$ de enige!!

$$A\begin{pmatrix}2\\-1\\4\end{pmatrix} = \underline{0} \quad \begin{pmatrix}2\\-1\\4\end{pmatrix} \neq \underline{0}$$

~~A niet singulier~~ ??

A singulier

$1 \cdot x = 0 \Rightarrow x = 0$ enige opl

$0 \cdot x = 0 \Rightarrow$ heleboel

$A\underline{x} = 0$, $\underline{x} = 0$ de enige $\Rightarrow$ A is niet singulier

$A\underline{x} = 0$, heleboel $\Rightarrow$ A singulier

18 (1.5) A, B, C $(n \times n)$

$$C = A \cdot B$$

als B singulier $\Rightarrow$ C singulier

$\Rightarrow$ er is $\underline{x} \neq \underline{0}$ met $B\underline{x} = \underline{0}$

??

$$C\underline{x} = (AB)\underline{x} = A(B\underline{x}) = A\underline{0} = \underline{0}$$

$\neq \underline{0}$

C singulier !?

$\exists \underline{x}$ met $C\underline{x} = \underline{0}$

$\neq \underline{0}$

$$(A(B\underline{x})) = \underline{0}$$

B is sing.

$x \neq 0$ $\rightarrow$ $B\underline{x} = \underline{0}$??!!