2DL60I - 200928:

Questions exerc. / theory ask them!

$A_{(n \times n)}$ $\quad A(x-y)=0 \quad x-y \neq 0$

$\boxed{Ax = Ay}$ én $x \neq y$

$\Rightarrow A$ is singular $\quad$ true

$\boxed{A \text{ singulier}} \Rightarrow \boxed{\exists x \neq 0 ; Ax = 0}$

A niet singulier $Ax = 0 \Rightarrow x = 0$ de enige opl.

$Ax = Ay$

$\underbrace{Ax - Ay} = 0$

$A(x-y) = 0$

$x \neq y \quad (x-y) \neq 0$

* $A_{(7 \times 7)}$ met $A^T = -A$:

$\Rightarrow$ A is singulier

$A \quad n \times n \qquad \det(A) \neq 0$

$\det(A) = 0$ singulier

$\underline{\det(A^T)} = \det(A)$

$\underline{\det(-A)} = (-1)^7 \cdot \det(A) =$

$\det(3A) = \qquad\qquad -\det(A)$

$A^T = -A$

$\det(A^T) = \det(-A)$

$\qquad \| \qquad\qquad\qquad \|$

$\underline{\det(A) = (-1) \cdot \det(A)}$

$\det(A) + \det(A) = 0$

$2 \det(A) = 0 \qquad\qquad 2 \cdot aap = 0$

$\boxed{\underline{\underline{\det(A) = 0}}} \qquad\qquad \overline{aap} = 7$

$\rightarrow$ dus A is singulier

$$\boxed{A\ (7\times 7)\quad A^T = -A}$$

$$\det(A^T) = \det(-A) = (-1)^7 \det(A)$$

$$\parallel$$

$$\det(A) = -\det(A) \Rightarrow$$

$$2\det(A) = 0 \Rightarrow \underline{\det(A) = 0}$$

$\Rightarrow$ <u>A is singulier</u>

$\underline{(6\times 6)}$: $\quad A^T = -A$

$$\det(A^T) = \det(A)$$

$$\det(-A) = (-1)^6 \cdot \det(A) = \det A$$

$$\underline{\det(A) = \det(A)}$$

<u>geen idee</u>

$$(2\times 2)\quad A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad A^T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\boxed{A^T = -A}\ \&\ |A| = +1$$

<u>niet singulier</u>

ontkennen

$A \ (9\times 9)$ dan $\boxed{A^2 + I \neq 0}$

<u>vraag: bestaat er zo'n A??</u>

Stel

$\cancel{A^2 + I = 0}$ $\quad A^2 = -I$

$\det(A^2) = \det(-I) = -1$

$\|$

$\det(A \cdot A) = (\det(A))^2$

$0 \leq (\det A)^2 = -1$ ↯

<u>Conclusie:</u> $A^2 + I \neq 0$

$$\boxed{(AC = BC \text{ én } C \neq 0)}$$

$\Rightarrow A = B$?? <u>niet waar</u>

$AC - BC = 0 \qquad (A - B)C = 0$

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(\underset{A}{\begin{pmatrix}1&0\\0&0\end{pmatrix}} + \underset{B}{\begin{pmatrix}0&-1\\-1&1\end{pmatrix}}\right)\underset{C}{\begin{pmatrix}1\\1\end{pmatrix}} = \begin{pmatrix}0\\0\end{pmatrix}$$

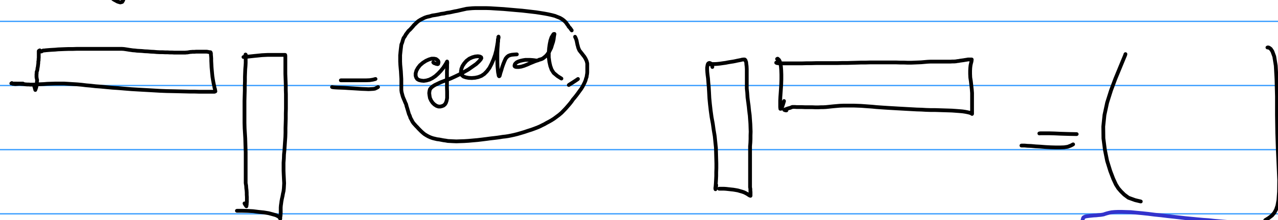
$$\underset{A}{\begin{pmatrix}1&0\\0&0\end{pmatrix}}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} \quad \underbrace{-\begin{pmatrix}0&-1\\-1&1\end{pmatrix}}_{B}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}+1\\0\end{pmatrix}$$

$$A = \begin{pmatrix}1&0\\0&0\end{pmatrix} \qquad B = \begin{pmatrix}0&1\\1&-1\end{pmatrix}$$

$$A\begin{pmatrix}1\\1\end{pmatrix} = B\begin{pmatrix}1\\1\end{pmatrix}$$

$$A x = \begin{pmatrix}0\\0\end{pmatrix}$$ ← idee

- If $AB = 0$ ~~then~~ $BA = 0$



$$\underset{A}{(1\ \ 1)}\underset{B}{\begin{pmatrix}1\\-1\end{pmatrix}} = \underset{(1\times 1)}{(0)} \qquad \underset{B}{\begin{pmatrix}1\\-1\end{pmatrix}}\underset{A}{(1\ \ 1)} = \underset{(2\times 2)}{\begin{pmatrix}1&1\\-1&-1\end{pmatrix}} \neq 0$$

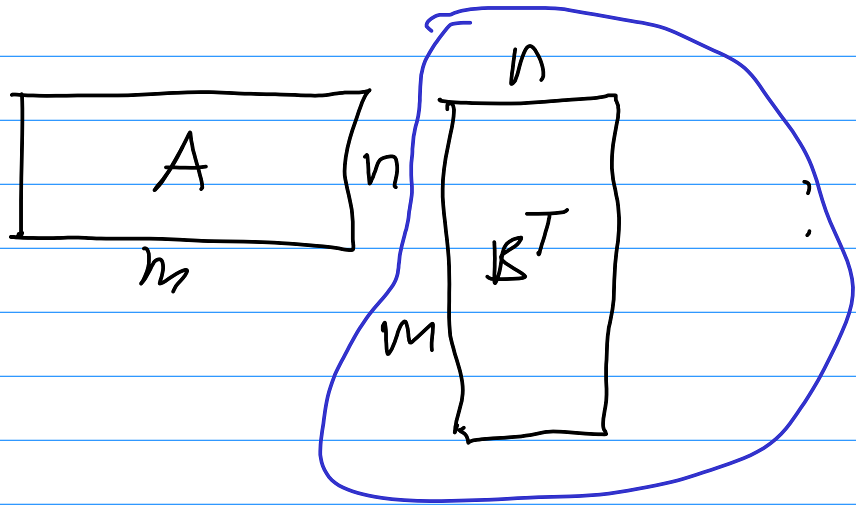
(1×1)

$(2)\cdot(3) = (3)\cdot(2)$

(ciha: $AB \neq BA$)

as A, B $(n \times m)$

$\det(AB^T) = \det(BA^T)$



$(AB^T) = (n \times n)$

$(B \cdot A^T) = (n \times n)$

$(A \cdot B)^T = B^T A^T$

$\det((AB)^T) = \det(B^T A^T)$

$$\det((AB)^T) = \det(B^T A^T)$$

$$\boxed{A = C^T}$$

$$\det((C^T \cdot B)^T) = \underline{\det(B^T \cdot C)}$$

$$\det(A) = \det(A^T)$$

$$\det(\underbrace{A\,B^T}_{n\times n}) = \det((A \cdot B^T)^T) =$$

A $n \times n$ $\det(A) = \det(A^T)$

$(A \cdot B^T)$ is $(n \times n)$

nu gevonden:

$\det(B \cdot A^T) = \det(A B^T)$

$$= \det(\underbrace{(A \cdot B^T)}^T) = \det(A \cdot B^T)$$

$$(A \cdot B^T)^T = (B^T)^T \cdot A^T = B \cdot A^T$$

$$(A \cdot B^T)^T = (B^T)^T \cdot A^T = \boxed{B \cdot A^T}$$

$$= \det(B \cdot A^T)$$

$$\det((A \cdot B^T)) = \det((A \cdot B^T)^T) = \det(B \cdot A^T)$$

A 5 x 4 matrix

$A = (a_1, a_2, a_3, a_4)$

if $b = a_2 - a_4$ then

$\boxed{A x = b}$ consistent

$$\begin{pmatrix} \begin{bmatrix} a_1 \end{bmatrix} & \begin{bmatrix} a_2 \end{bmatrix} & \begin{bmatrix} a_3 \end{bmatrix} & \begin{bmatrix} a_4 \end{bmatrix} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} = \begin{bmatrix} a_2 \end{bmatrix} - \begin{bmatrix} a_4 \end{bmatrix} = b$$

A x kolom = lin comb van k
van A

$Ax = b$ heeft opl. $\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$

dus consistent

$A(n \times n)$ A heeft gelijke kolommen

aftrekken

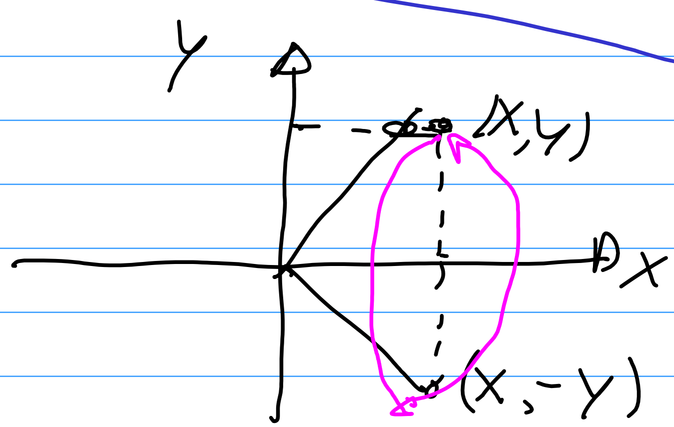
$$\det\left(\begin{array}{ccc} \cdots & a_i & \cdots & a_j & \cdots \end{array} \right) = $$

$$\ominus$$

$$a_i = a_j \qquad \det\left(\begin{array}{cc} a_i & \begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \end{array} \right) = 0$$

$$\begin{vmatrix} 1 & 2 & 1 \\ 1 & 4 & 1 \\ 1 & 5 & 1 \end{vmatrix} = 0$$

? $\boxed{A = A^{-1}}$ ~~dan~~ $\underline{A = I}$ of $\underline{A = -I}$



$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A^2 = I$$

$$AA = I = A \cdot A^{-1}$$

$$\boxed{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

A

$I \neq$

$-I \neq$

$$\boxed{A^{-1} = A}$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$A \neq I$

$A \neq -I$

$$A^{-1} = A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$|A| = |A^{-1}|$$

$$|A| = \frac{1}{|A|} \qquad |A|^2 = 1$$

$$|A| = \pm 1 \sim \cancel{A = I} \text{ or } \cancel{A = -I}$$

echelon form free variables

vrij

$$\begin{pmatrix} 1 & x & 0 & \cdots & & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \qquad \begin{pmatrix} -2y \\ y \\ 0 \end{pmatrix} =$$

$x + 2y = 0 \qquad x = -2y \qquad y\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

$z = 0$

∞ veel opl