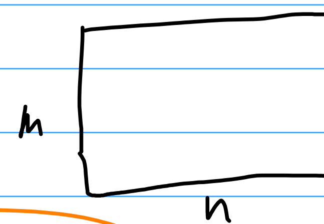
2DL60I − 201008

- Now I'm present!! Niet vergeten.
- Vragen? Stel ze maar!
- Eerst nog even koffie halen! 8

(3.6) 10)

$m \times n$



$\mathrm{rank}(A) = n$

$Ac = Ad \Rightarrow c \overset{?}{=} d$

→ (aantal onafh. rijen)

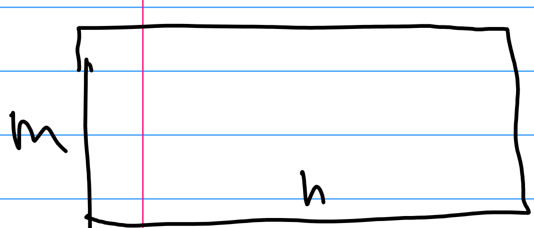
$\Rightarrow \underbrace{Ac - Ad}_{} = 0$

$A(c-d) = 0 \rightarrow \cancel{c-d=0} \; ?!$

nee { A na vegen

$$\begin{pmatrix} 1 & \cdot & \cdot & 0 & & \\ 0 & 1 & & 0 & & \\ 0 & 0 & & - & - - & \cdots \end{pmatrix}$$

$c \neq d$?? indien kolommen van A lin. afh. zijn



$(n > m)$

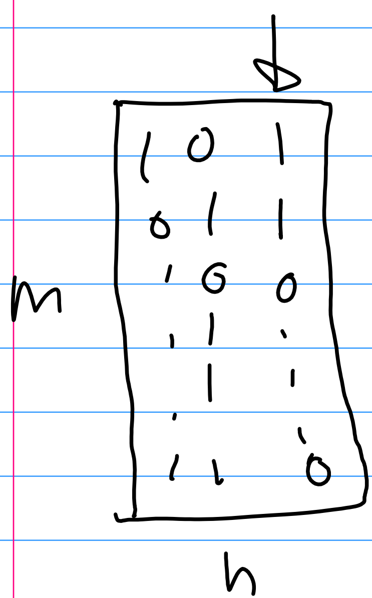
$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

rang(A) = 2 $\neq 4$.

$$A(\underbrace{c-d}) = 0$$

eis een opl $c - d \neq 0$.

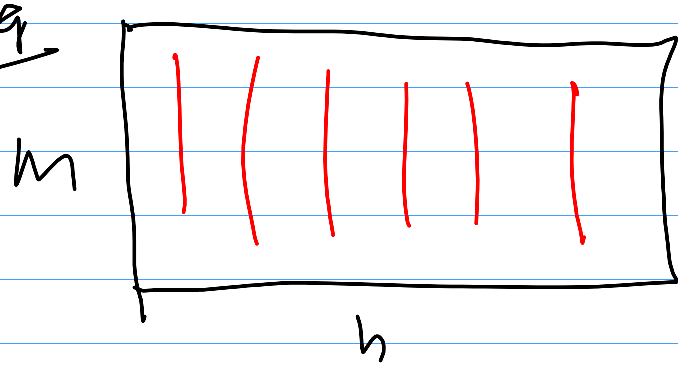
$c \neq d$.



$\longleftarrow$ (rang(A) = 2 $\neq$ 3)

$(m > n?)$

vraag



rang(A) = n

kolommen van A?

zijn lin. onafh.

dus dan $c = d$

(rang(A) < n dan kan $c \neq d$ i.h.a.)

$$4 \ \boxed{a_1,\ a_2,\ a_3} = A \quad (3 \text{ columns})$$

(3.6) (13)

$$z_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \quad z_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$N(A) = \langle z_1, z_2 \rangle$$

$$b = a_1 + 2a_2 + a_3$$

$$Ax = b \qquad A\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 1a_1 + 2a_2 + a_3 = b$$

$$\boxed{x_p = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}}$$

$$Ax = 0 : \quad x =$$

$$\boxed{z \in \langle z_1, z_2 \rangle}$$

alle lin. combinatie

$$z = \alpha \cdot z_1 + \beta \cdot z_2, \quad \alpha, \beta \in \mathbb{R}$$

$$A z = 0$$

$$A x_H = 0 \qquad x_H = \alpha \cdot z_1 + \beta \cdot z_2$$

$$x_A = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \alpha z_1 + \beta z_2 .$$

$A(x+y) =$

$A(x_A) = A\left(\underbrace{\begin{pmatrix}1\\2\\1\end{pmatrix} + \alpha \cdot z_1 + \beta z_2}\right)$

$= \underbrace{A\begin{pmatrix}1\\2\\1\end{pmatrix}} + A(\alpha z_1) + A(\beta z_1) =$

$\rightarrow \alpha A(z_1) =$

$z_1 \in N(A)$

$A z_1 = 0$

$= b + 0 + 0$

$= b$

$N(A)$?? $Ax = 0$

$z \in N(A)$ $\quad Az = 0$

$p(x) = (x-1)\cdot(x^{10} + x^9 + x^8 - x^7 - \dots)$

$p(x) = 0$?? $\quad x = 1$ nulp. van p

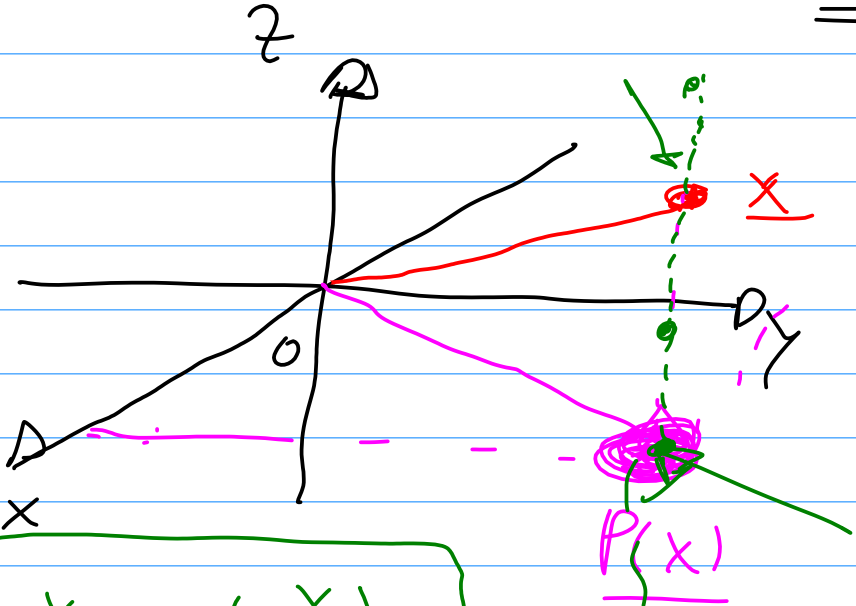
$\rightarrow p(1) = 0$

$A, \quad x \in N(A), \quad Ax = 0$

$$10\left\{\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix}\right.$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & 0 \\ & 1 \\ & 1 \end{pmatrix} \qquad \boxed{|A|\,??}$$

$Ax = 0 \qquad \underline{x = \binom{0}{0}}$



A^{-1} niet bepaald

$$\boxed{P\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}}$$

$A: \mathbb{R}^3 \to \mathbb{R}^3$

Th 3.6.6 toelichting:

$A = \square$? $\square$? $\square$?

$(m \times n)$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \qquad (2 \times 3)$$

dim (rijruimte van A) = 2

dim. (kolomruimte van A) = 2

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

leidende variabel

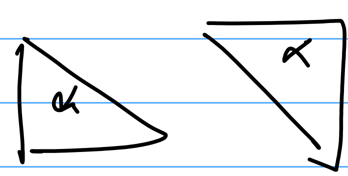
vrije variabelen

$$\begin{matrix} \to & 1 & 0 & 1 \\ \to & 0 & 1 & 1 \end{matrix} \qquad A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 1 & 3 & 4 \end{pmatrix} \quad \cancel{4}$$

A rref : $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

rij rank van A = 2.

$$\boxed{A = LU}$$



$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$

dim kolom $A^T = 2$

dim rij $A^T = 2$

$\boxed{p = 2}$ bewijs??

$\boxed{p \geq 2}$??

$\boxed{p \leq 2}$

? kolomruimte van A = vectorspace ?

$A = (a_1 \ldots a_n)$ a_i kolommen

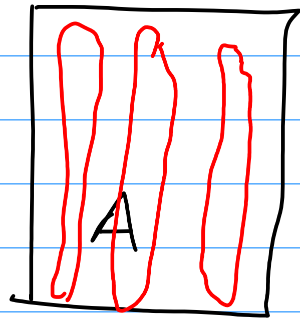
kolomruimte van A = span$(a_1 \text{----} a_n)$

altijd

$\boxed{0 \in \text{span}(a_1, \ldots a_n)}$

(3.6.) 8)

5



3

x_1, x_2, x_3 basis van $\mathbb{R}^3$

rang $(A) = 3$

$(x_1, x_2, x_3 \in \mathbb{R}^5)$

$x \in$ kolomruimte van A

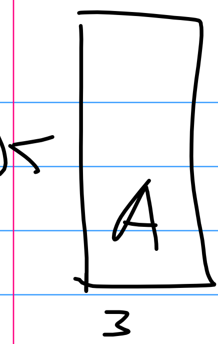
$$x = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3$$

$$(\alpha_1, \alpha_2, \alpha_3) \in \mathbb{R}^3$$

$N(A) = \{0\}$, $\Rightarrow$ $\boxed{Ax = 0}$

a) $x = 0$ de enige opl. omdat x_1, x_2, x_3 lin. onafh.

b) $y_1 = Ax_1$, $y_2 = Ax_2$, $y_3 = Ax_3$

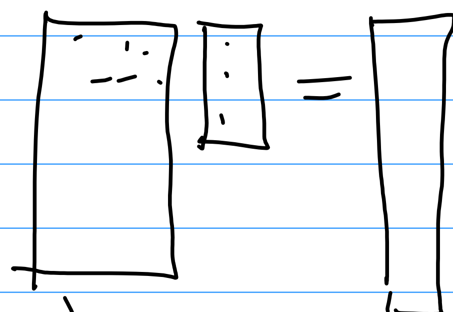
a) 5  rang(A) = 3,

a_1, a_2, a_3 dié lin. onafh.

$Ax = 0$ alleen mog. voor $x = 0$
omdat kolommen lin. onafh.

b) x_1, x_2, x_3 basis in $\mathbb{R}^3$

lin. onafh.

$Ax_1 =$  $= y_1$

$A x_2 = y_2$

$A x_3 = y_3$

y_i lin onafh:

$$\alpha_1 \cdot y_1 + \alpha_2 y_2 + \alpha_3 y_3 = 0$$

α_i ? $\Leftrightarrow$ $\alpha_1 A x_1 + \alpha_2 A x_2 + \alpha_3 A x_3 = 0$

$(c \cdot (Ax) = A(cx))$

$A \neq 0$

$\underline{A}(\alpha_1 \cdot x_1) + \underline{A}(\alpha_2 x_2) + \underline{A}(\alpha x_3) = 0$

$\underline{A}(\alpha_1 \cdot x_1 + \alpha_2 x_2 + \alpha_3 x_3) = 0$

$\text{rang}(A) = 3$

basis van $\mathbb{R}^3$ lin onafh

$\Rightarrow (\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 = 0)$

$\Rightarrow (\alpha_1 = 0, \quad \alpha_2 = 0, \quad \alpha_3 = 0)$

↳ de enige oplossing

dus: y_1, y_2, y_3 lin onafh

c) $y_i = A\underline{x}_i = \begin{pmatrix} \alpha \\ \alpha \\ \alpha \\ \alpha \\ \alpha \end{pmatrix}$ $\quad y_i$ 3x

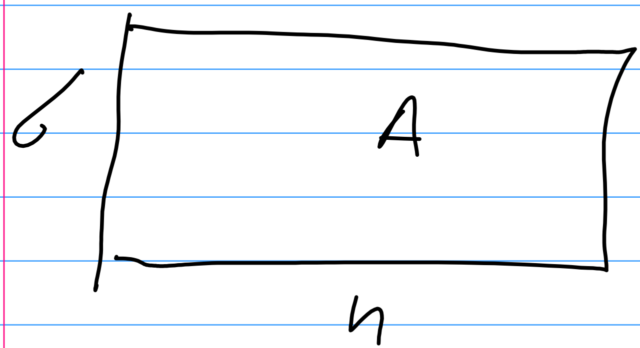
nee

|| y_i basis van $\mathbb{R}^5$??

$\boxed{\dim \mathbb{R}^5 = 5}$

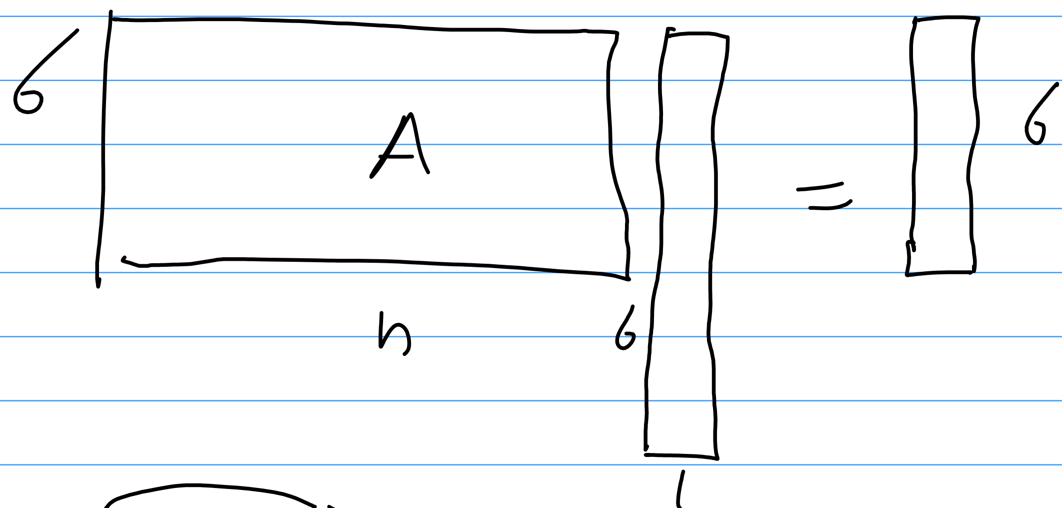
$(3 \times y_i)$ dus niet

(3.6)(7)



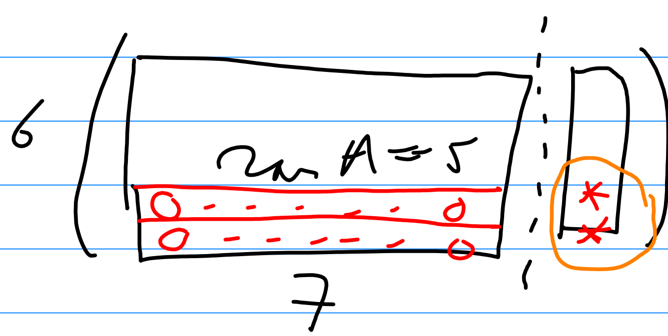
, rang(A) = r

$b \in \mathbb{R}^6$ $\qquad A\underline{x} = \underline{b}$



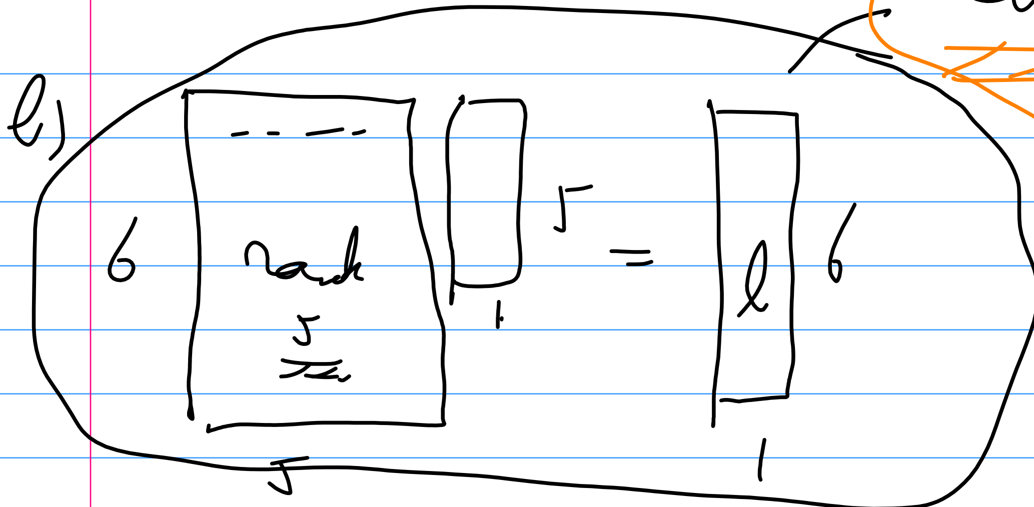
a) $n = 7$, $\text{rank}(A) = 5$

$\underline{Ax = b\ ??}$ $\qquad \underline{\dim(N(A)) = 7 - 5 = 2}$



← $\underline{\text{géén opl}}$

b)



één oplossing

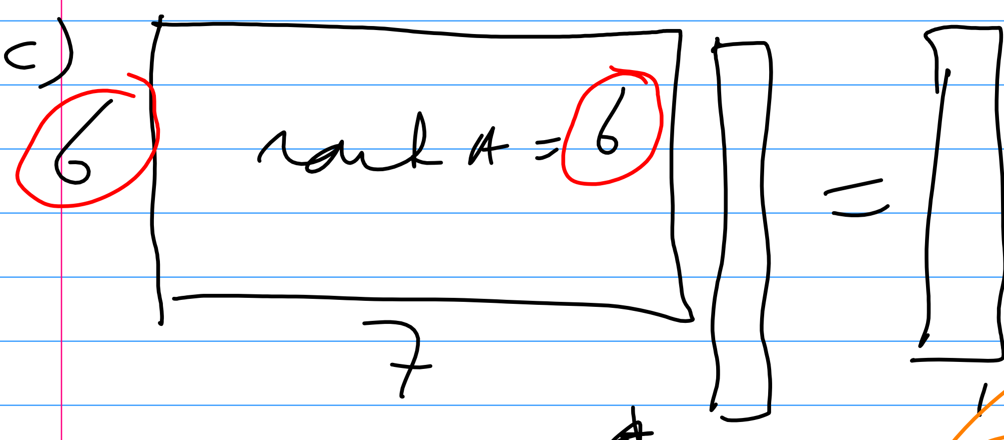
$b \in$ kolomruimte A

kolommen lin onafh

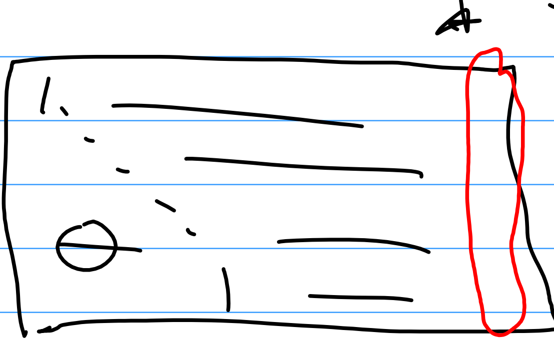
na vegen

$N(A) = 0$

c)



altijd opl.

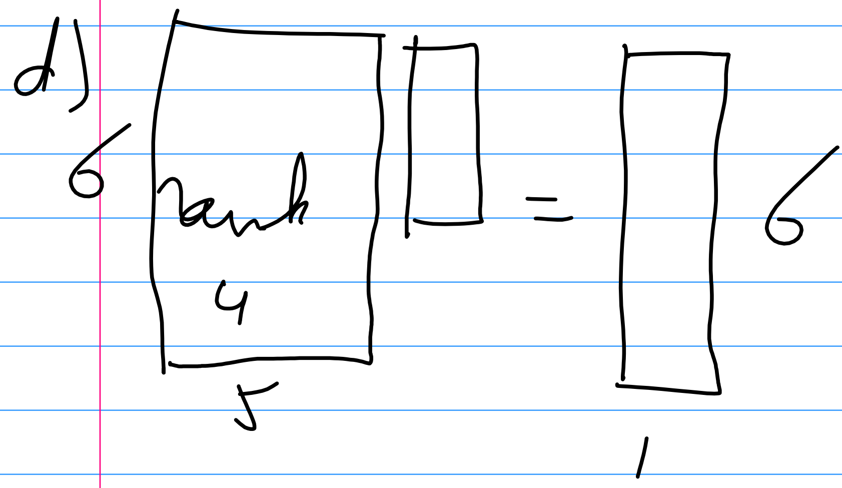


$N(A) \neq 0$

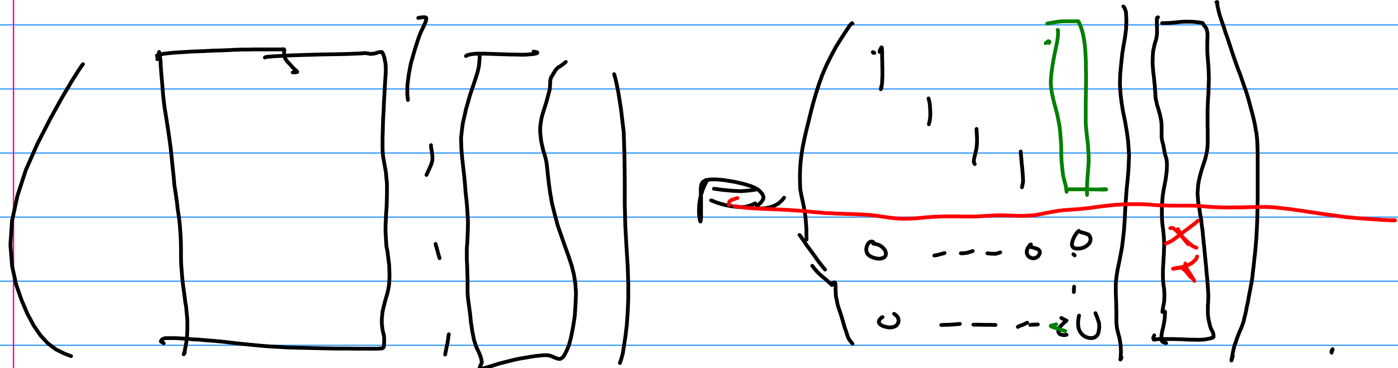
hoeveel oplos.??

∞

d)



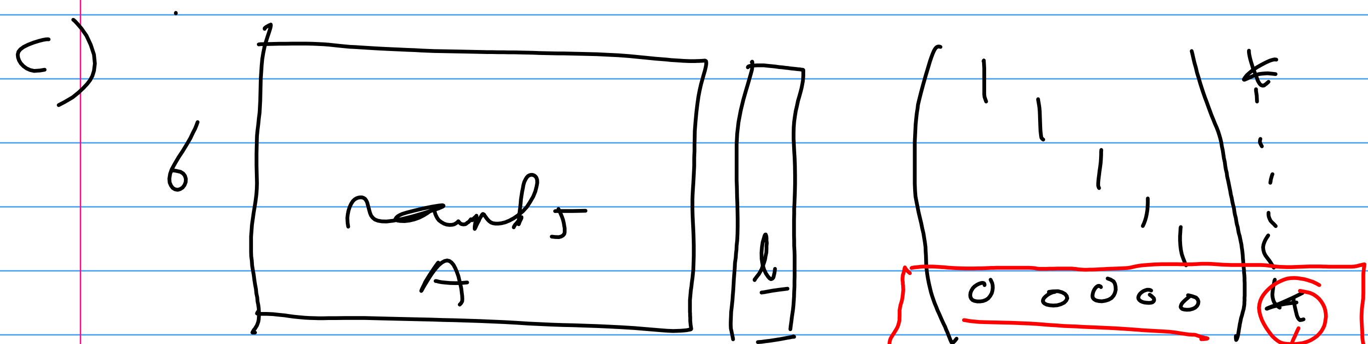
N(A) meer dan 0



N(A) = 0 of één

(↳ veel opl)

c)

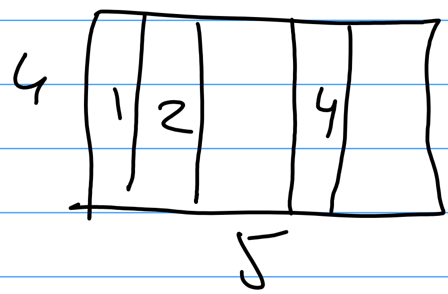


!!
geen opl ⇐ ≠ 0

(a) → (e)
(c) (d))

⚠ pas op antwoorden transponeren!

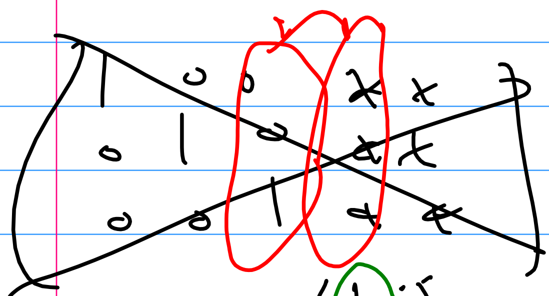
(3.6)(17)



$a_3 = a_1 + 2a_2$

$a_5 = 2a_1 - a_2 + 3a_4$

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



$$a_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad a_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

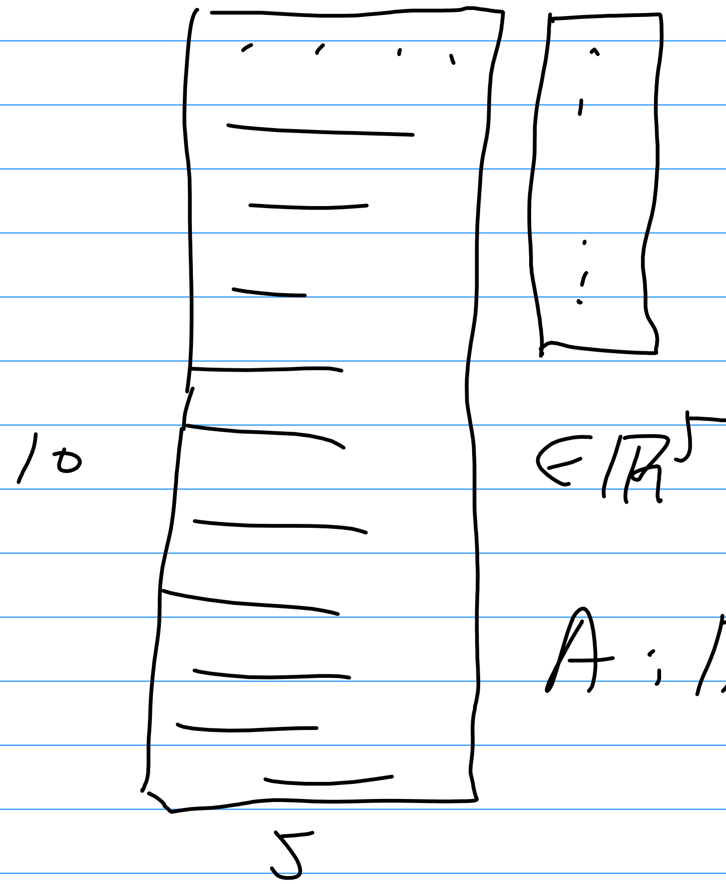
$$a_3 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix} \quad a_5 = \begin{pmatrix} 2 \\ -1 \\ 3 \\ 0 \end{pmatrix} \quad 3a_4 = \begin{pmatrix} 0 \\ 0 \\ 3 \\ 0 \end{pmatrix}$$

$$A = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \end{pmatrix} \begin{bmatrix} \; \\ \; \\ \; \end{bmatrix} = \begin{bmatrix} \; \\ \; \\ \; \end{bmatrix}$$

$A : \mathbb{R}^5 \to \mathbb{R}^4$

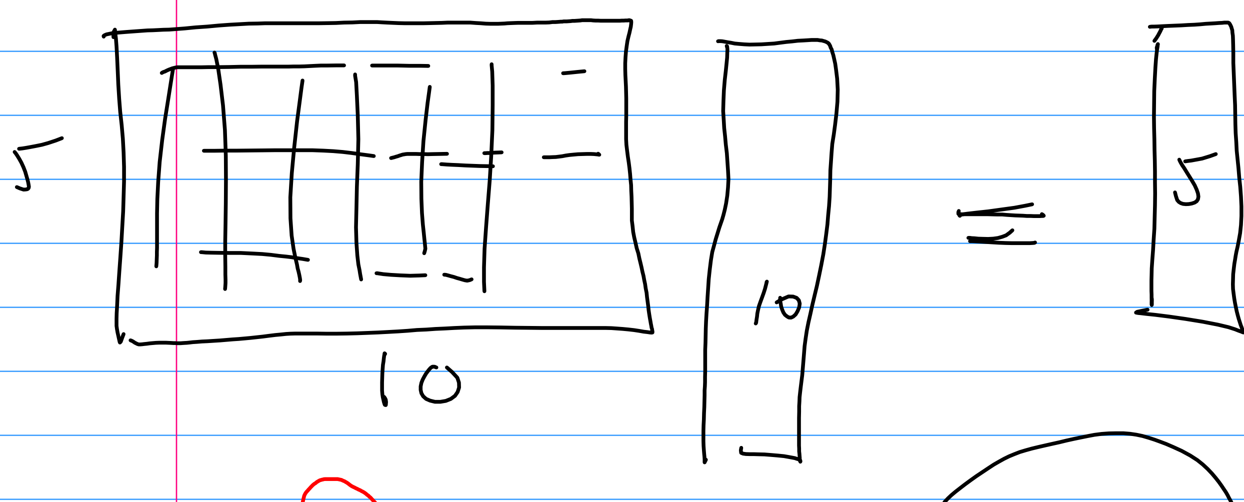
$A = \begin{pmatrix} | & | & | \end{pmatrix}$

$A : \cancel{\mathbb{R}^3} \to \mathbb{R}^5$



$\in \mathbb{R}^5$

$A : \mathbb{R}^5 \to \underline{\mathbb{R}^{(10)}}$



$$5\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & & \\ 0 & 0 & & \\ 0 & 0 & & \\ 0 & 0 & & \end{pmatrix}_{10} = \begin{pmatrix} x \\ 0 \\ \vdots \\ 0 \end{pmatrix} \in \mathbb{R}^3$$

$\underline{\dim(R(A)) = 1}$

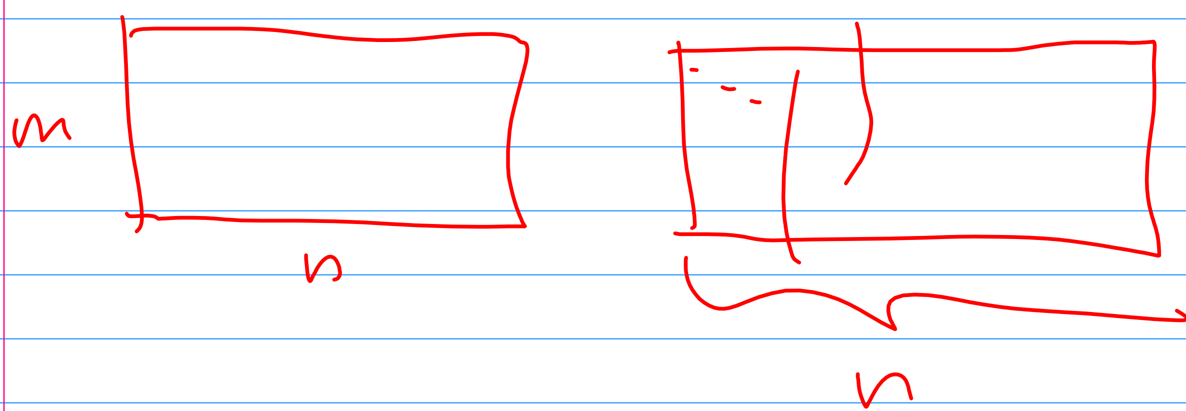
aantal leidende

aantal vrije var.

$$A : \mathbb{R}^n \to \mathbb{R}^m$$

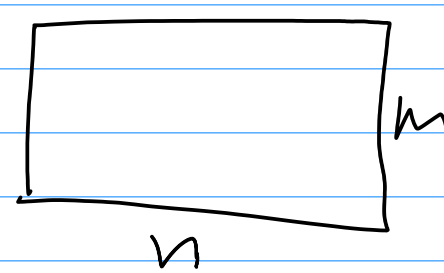
$$\dim(R(A)) + \dim(N(A)) = n$$

dim kolom ruimte van A

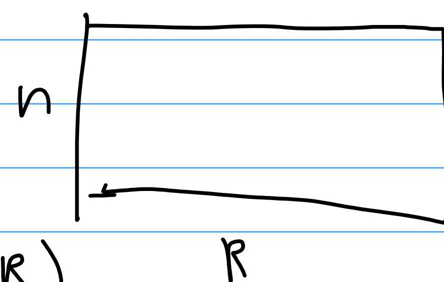


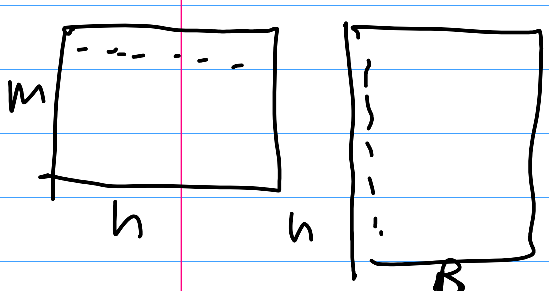
(28) $\mathbb{R}$ → reële getallen

$A \in \mathbb{R}^{(m \times n)}$?

$A =$ 

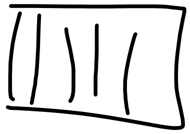
$B \in \mathbb{R}^{n \times R}$

$B =$ 



$= C$

$C\ (m \times R)$



$A \cdot B = C$

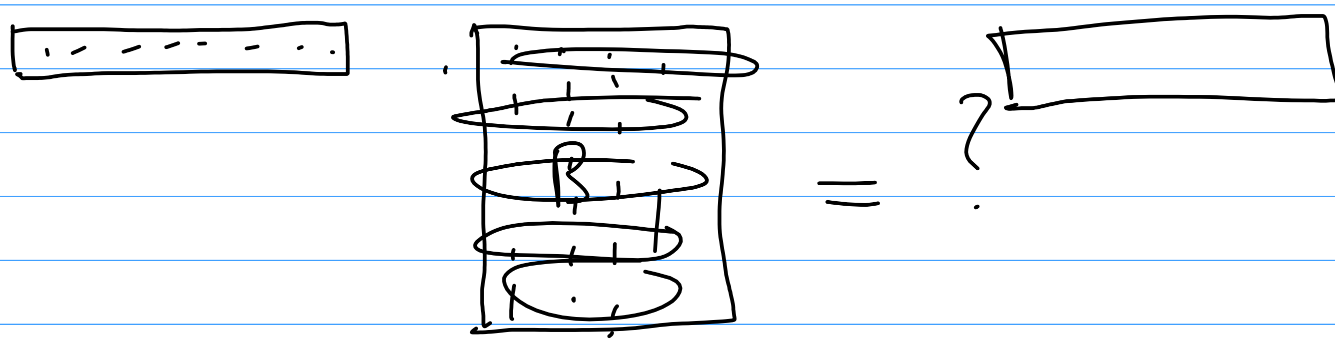
$A \cdot \boxed{\ } =$ lin comb. van de kolommen van A

kolom van B

$(A \cdot b_i) = c_i$ kolom van C

kolomruimte C: lin. comb. van de kolommen van A

a_i



rij * matrix = lin com van de rijen van de matrix

matrix * kolom = lin comb van de kolom van matrix

$$\overset{a_i}{\begin{pmatrix} 1 & 2 & -1 \end{pmatrix}} \overset{P}{\begin{pmatrix} 1 & 2 & 1 \\ 5 & 0 & 2 \\ 1 & 0 & 3 \end{pmatrix}} = \begin{pmatrix} \underset{c_i}{\rule{3em}{0.4pt}} \end{pmatrix}$$

$$1 \cdot \begin{pmatrix} 1 & 2 & 1 \end{pmatrix} +$$
$$2 \begin{pmatrix} 5 & 0 & 2 \end{pmatrix} +$$
$$-1 \begin{pmatrix} 1 & 0 & 3 \end{pmatrix}$$

$$\boxed{-a_i- - - -} \cdot \boxed{B} = \boxed{c_i}$$

Projectie op $(x-y)$ vlak

$P: \mathbb{R}^3 \to \mathbb{R}^3$ $\qquad$ rang$(P) = 2$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\dim N(P) = 3 - 2 = 1$

$$P\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad N(P) = \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

p $Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

x z

v6

$$\underset{A}{P} \cdot \underset{B}{Q} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = C$$

rank (C) = { lin onafh kolom / lin onafh rijen

fig. op pg 160.

$\underline{x} = (x_1, x_2, x_3) (\neq \underline{0})$

$\text{span}(\underline{x}) = \{ x \in \mathbb{R}^3 \mid \underline{x} = c \cdot (x_1, x_2, x_3) \}$

één vrijheid $\rightarrow$ $c \in \mathbb{R}$.

$\dim(\text{span}(\underline{x})) = 1$

$\underline{x}, \underline{y} \in \mathbb{R}^3$

$$X = \alpha_1 \underline{x} + \alpha_2 \underline{y}$$

α_1, α_2 vrij

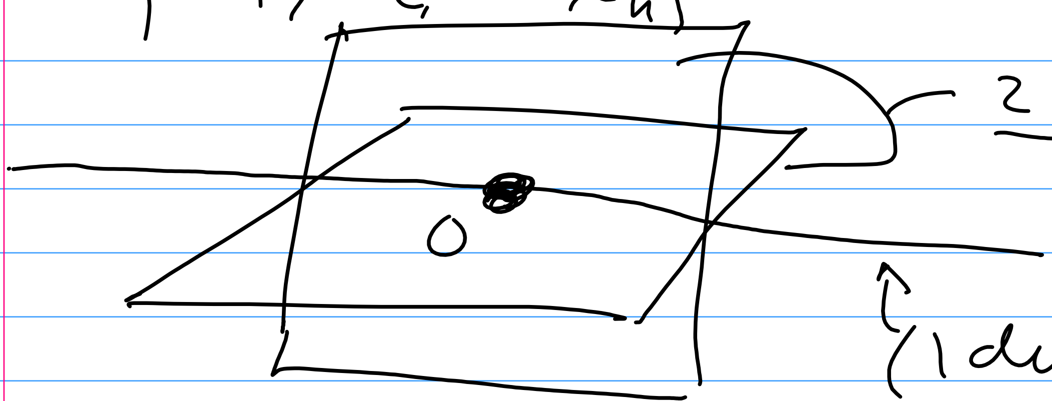
$$\dim(\text{span}\langle \underline{x}, \underline{y} \rangle) = 2$$

$\mathbb{R}^3$: 3-dim. ruimte

$$\underline{x} = (x, y, z)$$

$\rightarrow x + y = 0$

$$\{x_1, x_2, \dots x_n\} \subset \mathbb{R}^n$$



2 deelr. van $\mathbb{R}^3$

(1 dim. deelr.)

vaste vector

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad \underline{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$\alpha \cdot \underline{x} + \beta \underline{y}$$

$\text{span}(\underline{x}, \underline{y}) \rightarrow$

ujhide

$$\underline{p_2(x) = a + b \cdot x + c x^2}$$

$$p_2 \in \underline{\langle 1, x, x^2 \rangle}$$

$$(a, b, c) \in \mathbb{R}^3$$

$$\operatorname{span}(x^2, x^4, 1)$$

$$\underbrace{(a, b, c)}_{\in \mathbb{R}^3} \qquad \boxed{a x^2 + b x^4 + c} \subset \boxed{P_5}$$

$$\underline{\langle 1, x, x^2, x^3, x^4 \rangle}$$

$$\underline{(a, b, c)} \in \mathbb{R}^3$$

$$\boxed{\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\in \mathbb{R}^5} = \begin{pmatrix} a \\ 0 \\ b \\ 0 \\ c \end{pmatrix}}$$

$$\left\langle \begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix} \right\rangle$$

basisdeelruimte van $\mathbb{R}^5$

$$\left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix} \right\}$$

$$\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\end{pmatrix}$$

echte probleem

"tween" probleem

$$\langle 1, x, x^2 \rangle$$

$$\begin{pmatrix}1\\0\\0\end{pmatrix} \quad \begin{pmatrix}0\\1\\0\end{pmatrix} \quad \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$a + bx + cx^2 \; \Big\} \; a\begin{pmatrix}1\\0\\0\end{pmatrix} + b\begin{pmatrix}0\\1\\0\end{pmatrix} + c\begin{pmatrix}0\\0\\1\end{pmatrix}$$