2DL60 I — 20101 2;

(2.2)(11)

$A(n \times n)$ $\quad A^2 + I = 0$??

$|A|$ $\quad |A^2| = |-I|$

$0 \leq |A|^2 \quad (-1)^n \cdot 1$

n = even, (n oneven, kan niet)

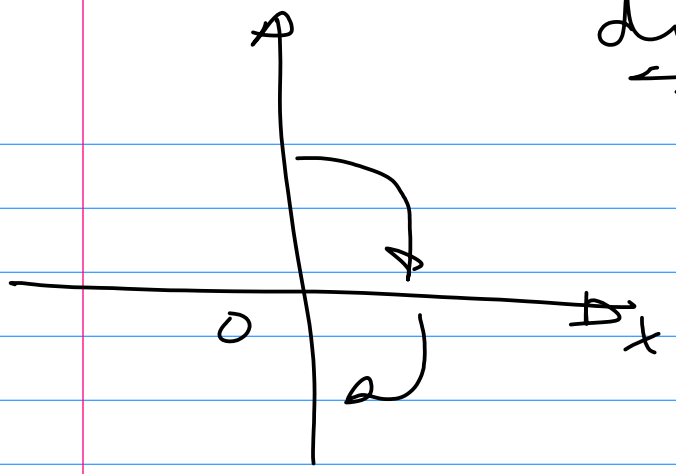
$$\begin{cases} \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ a^2 + bc = -1 \qquad ab + bd = 0 \\ ca + dc = 0 \qquad cb + d^2 = -1 \\ a = 1 \qquad b \cdot c = -2 \qquad b + d = 0 \\ d = 1 \end{cases}$$

$(-I)$ $\quad (|A| = \det(A))$

$|(-1)| = -1$ $\quad (1 \times 1)$

$\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} = +1$ $\quad (2 \times 2)$

$\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -1$ $\quad (3 \times 3)$



drawing $\left(\frac{\pi}{2}\right)$

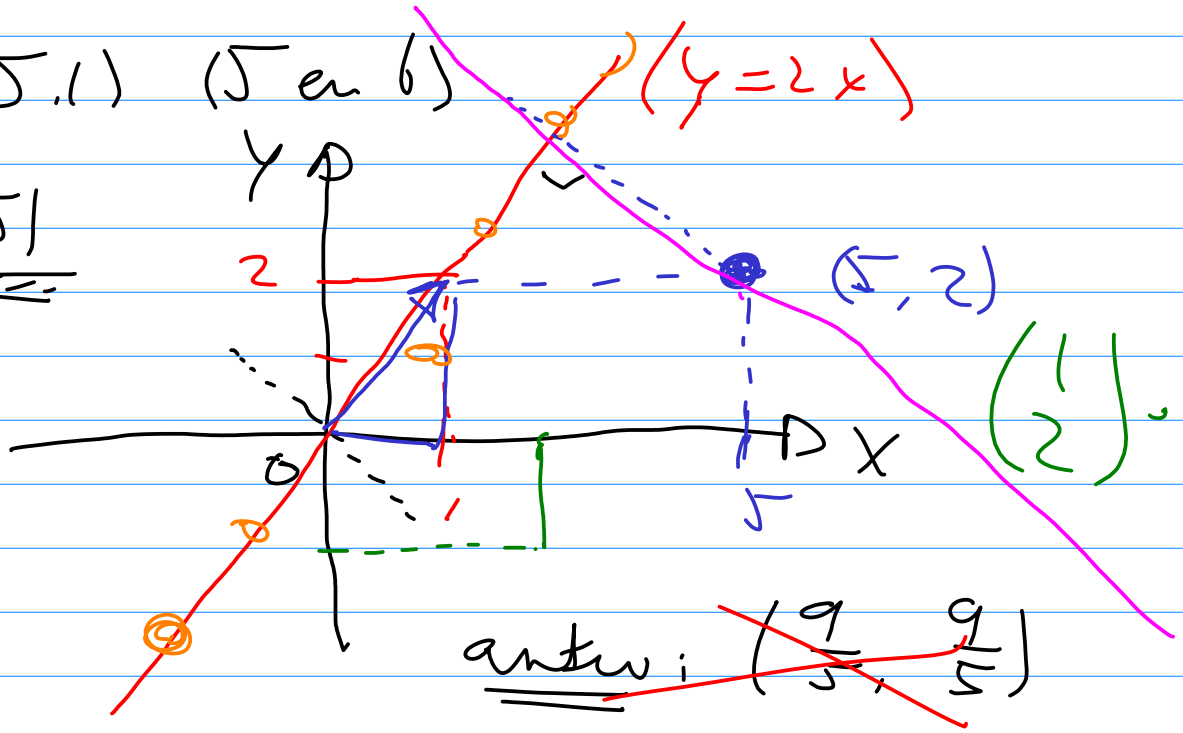
$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I$$

(5.1) (5 en 6)

(5)



$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix} = 0$$

antw: ~~$\left(\frac{9}{5}, \frac{9}{5}\right)$~~

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 2\lambda \\ -\lambda \end{pmatrix} = \begin{pmatrix} \alpha \\ 2\alpha \end{pmatrix}$$

$$\begin{cases} \alpha - 2\lambda = 5 \\ 2\alpha + \lambda = 2 \end{cases}$$ A

ps 222

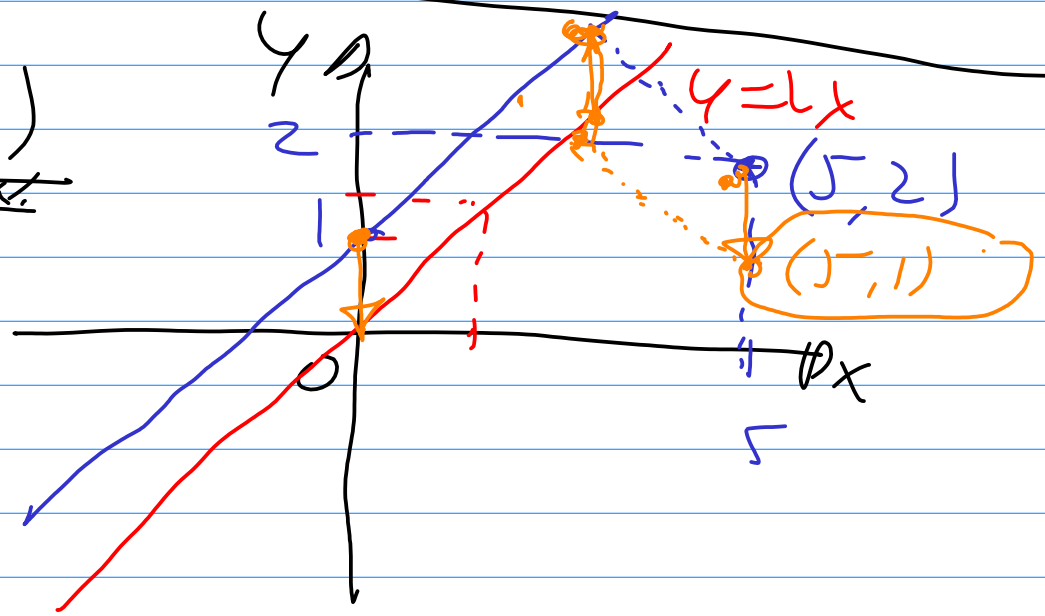
$v = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $w = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$\left(\frac{v^T \cdot w}{w^T \cdot w} \right) \cdot w = \frac{\begin{pmatrix} 5 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}}{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} =$$

$$\left(\frac{9}{5} \right) \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$ ✓

$x \cdot y = \|x\| \cdot \|y\| \cdot \cos(\text{theek tussen } x \text{ en } y)$

(6)



A $(n \times n)$

$B = S^{-1} \cdot A \cdot S$

non singular

B no singular

$|B| = |S^{-1}AS| = |S^{-1}||A||S|$

$|S^{-1}\cdot S| = |I| = 1 = |S^{-1}||S|$

$1 = a \cdot \ell$

$|S^{-1}| = \frac{1}{|S|}$

$\ell = \frac{1}{a}$

$0 \neq |B| = |A|$

&

$Ax = b$ consistent

en is een oplossing

$A \cdot$ vector $=$ ------ ??

$A \cdot \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} =$

$A s = \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} =$

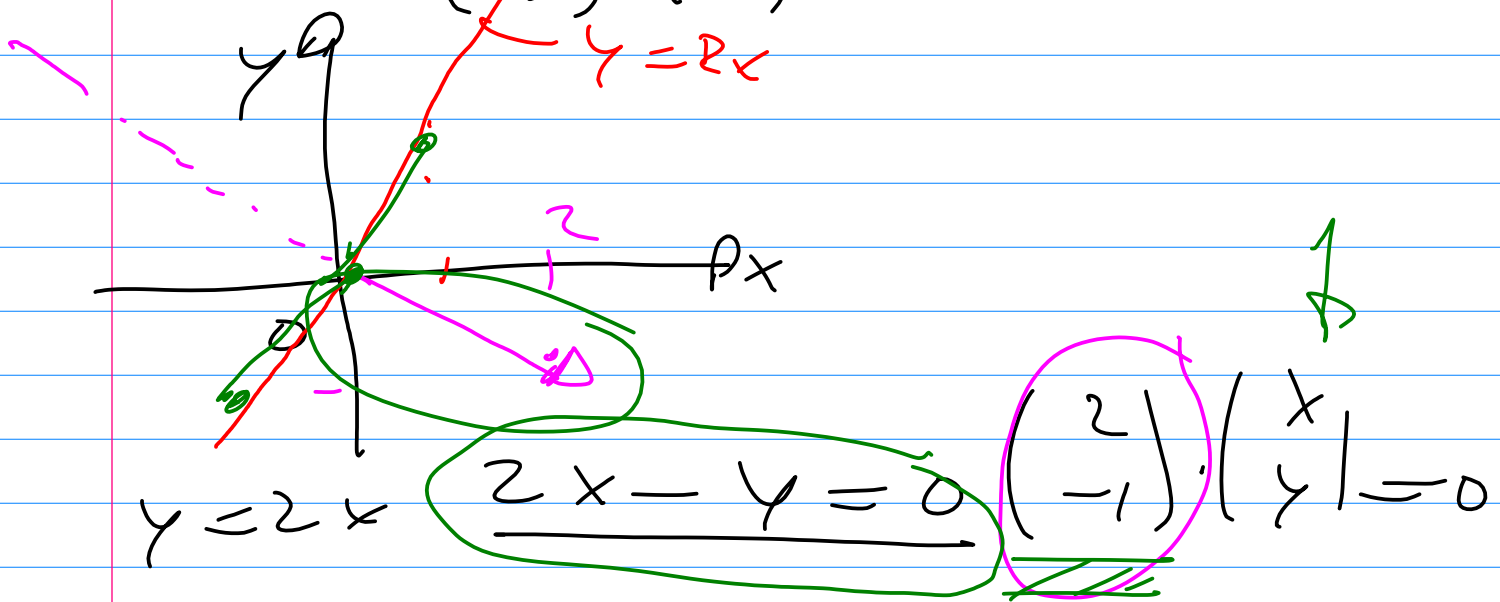
$s_1 a_1 + s_2 a_2 + s_3 a_3 = b$

matrix x vector =
lin comb. van kolommen van matrix

(5.1) (8)

calc. $2x + 4y + 3z = 0$

$$\begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$



$$\begin{pmatrix} 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

(l) $$\begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x-4 \\ y-2 \\ z+5 \end{pmatrix} = 0$$

$$n \cdot (\underline{x} - P) = 0$$

$-3(x-4)+6(y-2)+2(z+5)=0$

$$\underline{N}\cdot(\underline{x}-\underline{p})=0$$

(5.11)

(13) $\quad \underline{u},\underline{v}\in\mathbb{R}^2$

$$\|\underline{u}+\underline{v}\|^2 \le (\|\underline{u}\|+\|\underline{v}\|)^2$$

$\rightarrow (|a+b| \le |a|+|b|)$

driehoeks ongelijkheid

$\|\underline{u}\|^2 = \underline{u}\cdot\underline{u}$ ↓ in product

$\|\underline{u}+\underline{v}\|^2 = (\underline{u}+\underline{v})\cdot(\underline{u}+\underline{v}) =$

$\underline{u}\cdot\underline{u} + \underline{v}\cdot\underline{v} + \underline{u}\cdot\underline{v} + \underline{v}\cdot\underline{u} =$

$\|\underline{u}\|^2 + \|\underline{v}\|^2 +$

$2\cdot(\underline{u}\cdot\underline{v}) \le \|\underline{v}\|^2 + \|\underline{u}\|^2 + 2\|\underline{u}\|\|\underline{v}\|$

$|\underline{u}\cdot\underline{v}| \le \|\underline{u}\|\cdot\|\underline{v}\|$

$(a^2+b^2+2ab = (a+b)^2)$

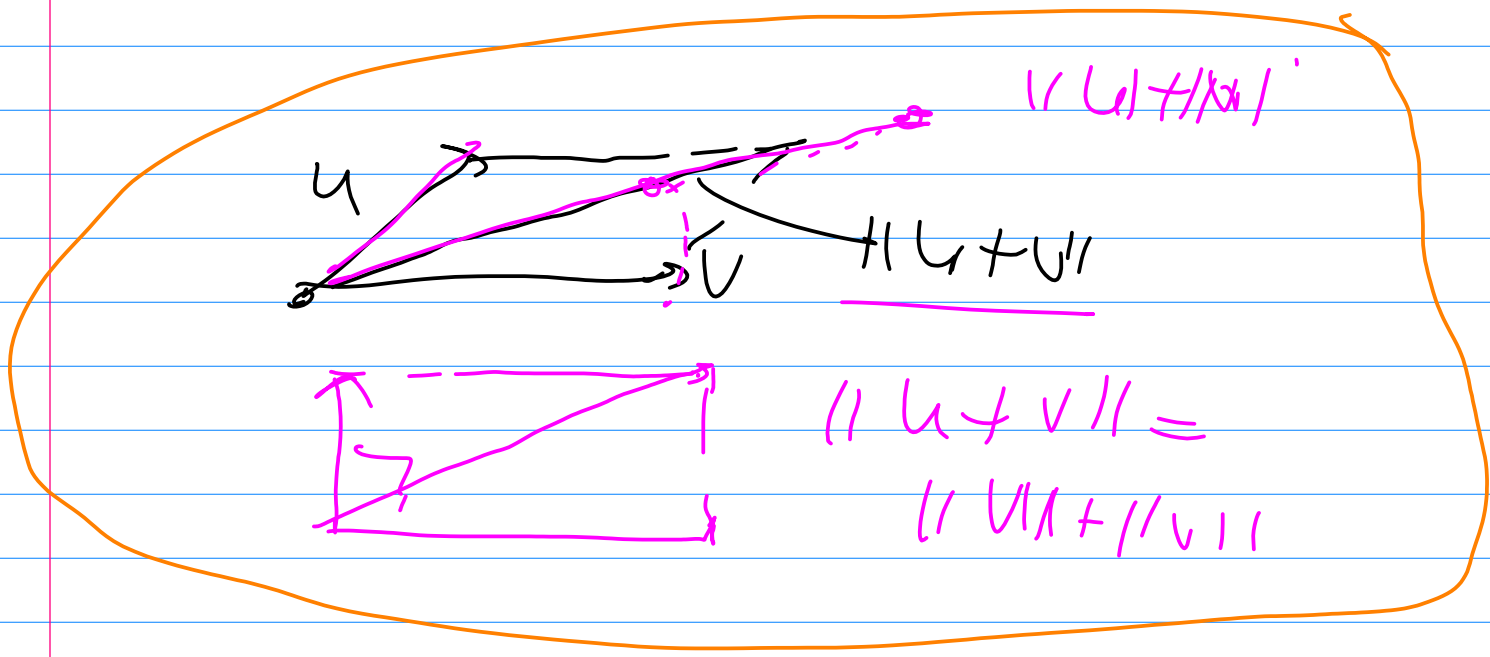
$$\frac{(\|u\|+\|v\|)^2}{\sqrt{\|u+v\|^2}} \leq \sqrt{(\|u\|+\|v\|)^2}$$

$$\|u+v\| \leq \|u\| + \|v\|$$

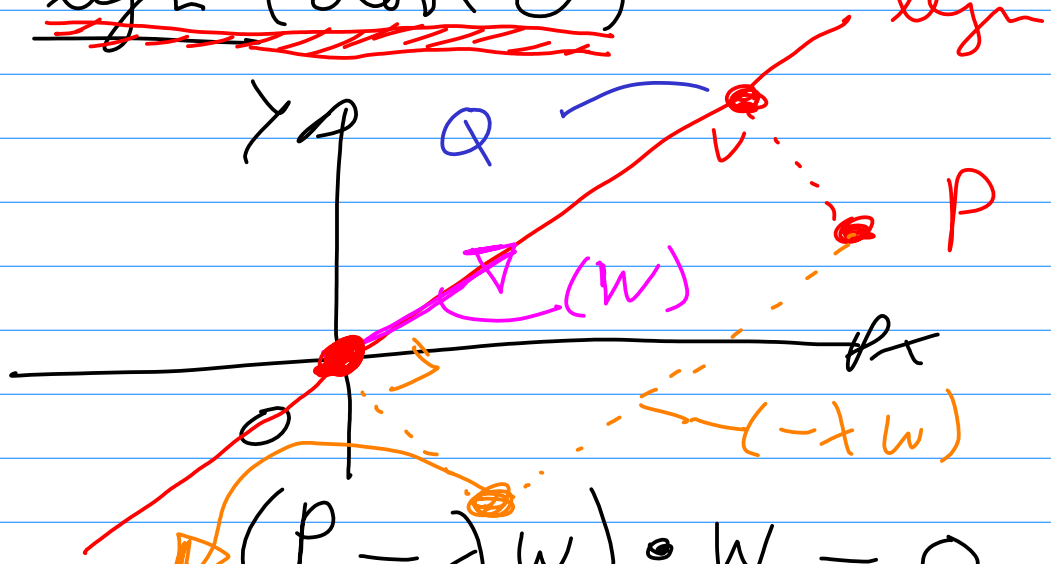
Wannee

$\|u+v\| = \|u\| + \|v\|$?

$(u \perp v)$



Projectie van punt P op een lijn (door O)



$$(P - \lambda W) \cdot W = 0 \Rightarrow$$

$$P \cdot W - \lambda (W \cdot W) = 0$$

$$\left(\lambda = \left(\frac{P \cdot W}{W \cdot W} \right) \right)$$

$(P - \lambda \cdot W)$

$$Q = \lambda \cdot W = \left(\frac{P \cdot W}{W \cdot W} \right) \cdot W$$

(λ (getal) W (vecto))

(projectie van P op lijn)

? $A\underline{x} = \underline{l}$?

$A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ (x eengetal)

$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot x = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ geen oplossing

$\rightsquigarrow A^T A x = A^T l$

$(1 \quad 2)\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \tilde{x} = (1 \quad 2)\begin{pmatrix} 5 \\ 2 \end{pmatrix}$

$5 \cdot x = 9 \qquad \tilde{x} = \begin{pmatrix} 9/5 \end{pmatrix}$

$A\tilde{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{pmatrix} 9/5 \end{pmatrix}$

$\left(\begin{pmatrix} 5 \\ 2 \end{pmatrix} - A\tilde{x}\right) \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} =$

$\left(\begin{pmatrix} 5 \\ 2 \end{pmatrix} - \frac{9}{5}\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right) \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} =$

$\left(5 - \frac{9}{5}\right) + 2\left(2 - \frac{10}{5}\right) =$

$9 - \frac{45}{5} = 0$

$(A\tilde{x} - l)$ "1" A

$(A\tilde{x} - b) \perp$ op kolommen ruimte van A

$$A^T \cdot (A\tilde{x} - b) = 0$$

$A\tilde{x}$ dichtstbijzijnde punt

$\frac{9}{5} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$