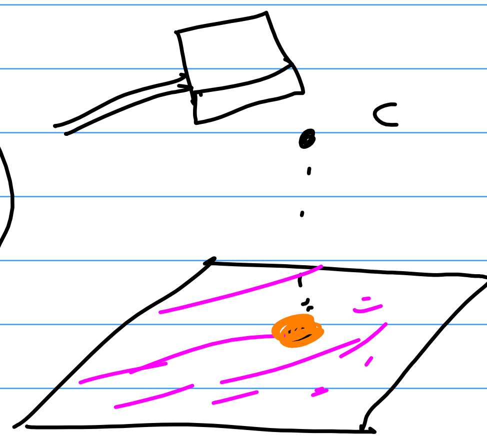
2DL60I - 201015.

- Questions? I like to hear them.
- Exercises - Theory doesn't matter

A $(n \times n)$ rang$(A) = n$

$x \in \mathbb{R}^n$ proj van x op $R(A)$

$$= A(A^TA)^{-1}.A^T x$$



$$\begin{pmatrix} | & & | \\ | & \cdots & | \\ | & & | \end{pmatrix} \sim x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$
$$\quad a_1 \qquad a_n$$

inproduct??

$$(A(A^TA)^{-1}A^T)$$

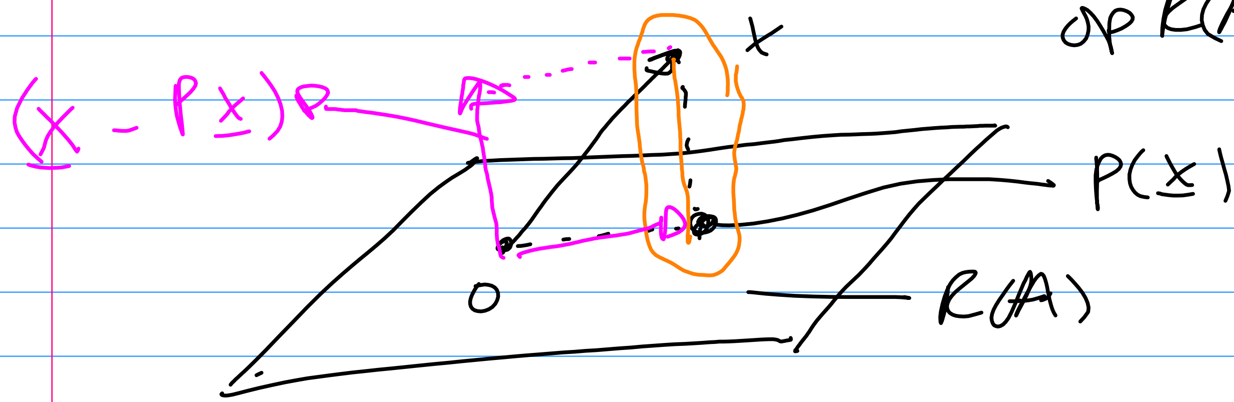
projectie??

$$(A(A^TA)^{-1}A^T)(A(A^TA^{-1})A^T) =$$

$$A(A^TA)^{-1}.(A^TA)(A^TA)^{-1}A^T =$$

$$\boxed{A(A^TA)^{-1}A^T}$$

$\underbrace{A(A^TA)^{-1}A^T \underline{x}}$?? projectie op $R(A)$??



$(x - Px) \perp R(A)$

in produkt kolommen van $A \perp (\underline{x} - P\underline{x})$

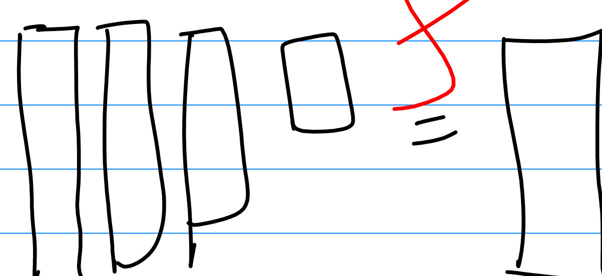
$\underline{\underline{A}}^T(\underline{x} - P\underline{x}) = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$

a^T

$A^T(x - A(A^TA)^{-1}A^T x) =$

$A^T x - \cancel{(A^TA)(A^TA)^{-1}} A^T x = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$

$\boxed{Ax = b}$



(A^TA) invertible $A^T l$.

$$A^TA\underline{x} = A^T l$$

$$\underline{x} = (A^TA)^{-1}A^T l$$

$$Ax = l$$

$$(A^TA)\,x = A^T l$$

$$x = (A^TA)^{-1}A^T l$$

$$\underline{A.\underline{x} \in R(A)}$$

$$A\,\underbrace{(A^TA)^{-1}A^T}\, l$$

$\underline{\underline{P}}$ of $R(A)$

*

orthogonal complement $(\underline{R(A)}) = \underline{N(A^T)}$

$$\boxed{A^T y = \underline{0}}$$

$$A = \begin{pmatrix} a_1 & \cdots & a_n \end{pmatrix}$$

$$(y^T.A = (0 \cdots 0))$$

$y \in$ (orthogonale complement $R(A)$)

$$y \perp \begin{pmatrix} \vdots \\ a_i \\ \vdots \end{pmatrix} \qquad \underline{y}^T. \begin{pmatrix} \vdots \\ a_i \end{pmatrix} = 0$$

$$\underline{(\cdots a_i^T \cdots)} \cdot \begin{pmatrix} \vdots \\ y \\ \vdots \end{pmatrix} = 0$$

$$A^T y = 0$$

$$A = \begin{pmatrix} | & & | \\ a_1 & \cdots & a_k \\ | & & | \end{pmatrix} \longrightarrow A^T \cdot A \quad \text{niet singulier}$$

<u>lin onafh</u>

$$A = \begin{pmatrix} | & & | \\ a_1 & \cdots & a_k \\ | & & | \end{pmatrix}$$

$(k \times k)$ $(A^T A)$

<u>B die singulier</u>: $\boxed{\underline{x} \neq 0 \quad B\underline{x} = 0}$

$(A^T A)\underline{x} = \underline{0}$??

$A^T(A\underline{x}) = 0 \qquad \boxed{A\underline{x} = 0}$

<u>kolommen van A lin onafh.</u>

$$\begin{pmatrix} | & & | \\ a_1 & \cdots & a_k \\ | & & | \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_k \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$x_1 \cdot \begin{pmatrix} | \\ a_1 \\ | \end{pmatrix} + x_2 \begin{pmatrix} | \\ a_2 \\ | \end{pmatrix} \cdots + x_k \begin{pmatrix} | \\ a_k \\ | \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$$

dan $x_1 = 0, \dots, x_k = 0$ enige opl

$$(A^T A)\underline{x} = \underline{0}$$

enige opl: $\underline{x} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$

Wanneer is A niet singulier

$A\underline{x} = \underline{0} \Rightarrow \underline{x} = 0$ de enige opl

!

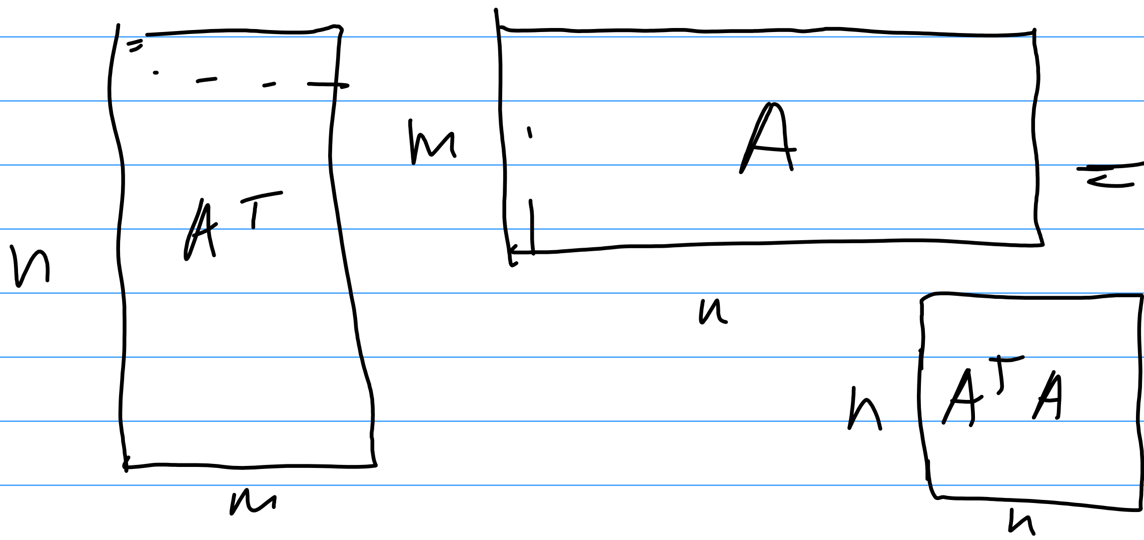
$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad A^T A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} =$$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ A niet singulier

$m > n$

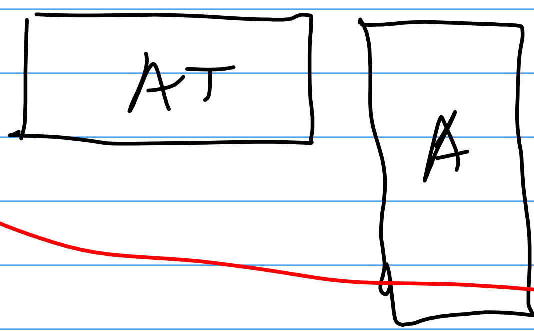
$A \ (m \times n)$, $\quad \mathrm{rang}(A) = m$

$\underline{x} \in \mathbb{R}^n \qquad \mathrm{proj}_x(R(A)) = A(A^T A)^{-1} A^T x$



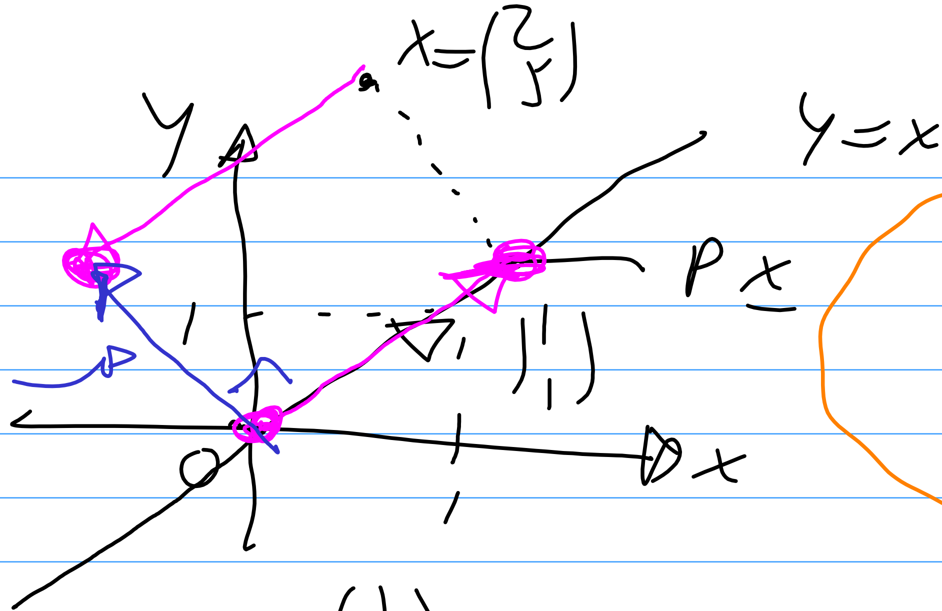
$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad \mathrm{rang}(A) = 2$$

$$A^T A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$= (A^T A)$

$\underline{(m \times m)}$



??

$\left(\frac{7}{2}\right)\begin{pmatrix}1\\1\end{pmatrix}$

$A = \begin{pmatrix}1\\1\end{pmatrix}$ $\qquad x = \begin{pmatrix}2\\5\end{pmatrix}$

$\underline{A(A^TA)^{-1}A^T\begin{pmatrix}2\\5\end{pmatrix}}$

$\underbrace{\begin{pmatrix}1\\1\end{pmatrix}}_{A}\underbrace{\left((1\ \ 1)\begin{pmatrix}1\\1\end{pmatrix}\right)^{-1}}_{(A^T A)^{-1}}\underbrace{(1\ \ 1)\begin{pmatrix}2\\5\end{pmatrix}}_{A^T \;\to\; 7} =$

$(2)^{-1} = \left(\frac{1}{2}\right)$ $\qquad 2 - \frac{7}{2} = -\frac{3}{2}$

$\begin{pmatrix}1\\1\end{pmatrix}\cdot\frac{1}{2}\cdot 7 = \left(\frac{7}{2}\right)\begin{pmatrix}1\\1\end{pmatrix}$

$\left(\begin{pmatrix}2\\5\end{pmatrix} - \left(\frac{7}{2}\right)\begin{pmatrix}1\\1\end{pmatrix}\right)\cdot\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-\frac{3}{2}\\\frac{3}{2}\end{pmatrix}\cdot\begin{pmatrix}1\\1\end{pmatrix}$

$= 0$

$A \qquad \underline{x}$

$$\underbrace{\text{proj}_x(R(A))}_{} = A(A^TA)^{-1}A^Tx$$

$$\boxed{\left(x - A(A^TA)^{-1}A^Tx\right) \perp R(A)}$$

$$A = \begin{pmatrix} | & | & | & | \end{pmatrix}$$

$x \perp R(A)$?!

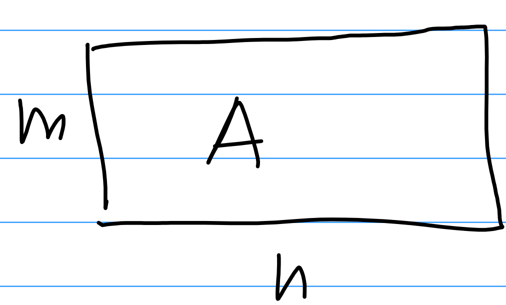
$A^Tx = 0$

$$A^T\left(x - A(A^TA)^{-1}A^Tx\right) =$$

$$A^Tx - \underbrace{(A^TA)(A^TA)^{-1}}_{I}A^Tx =$$

$$A^Tx - A^Tx = \underline{0}$$

$$\boxed{\text{rang}(A) = m}$$



$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\underline{\underline{A^T A}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}}$$

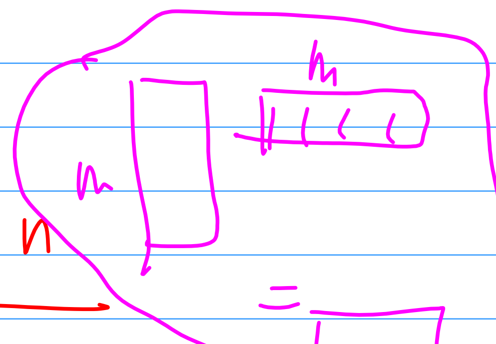
$$A\underline{x} = \begin{array}{c} \\ m \end{array} \overset{n}{\begin{pmatrix} | & & | \\ a_1 & \cdots & a_m \\ | & & | \end{pmatrix}} \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} =$$

$$x_1 a_1, \dots + x_m a_m$$

$\boxed{\underline{\underline{m > n}}}$

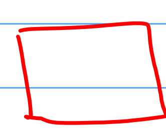
$\underline{\underline{m > n}}$

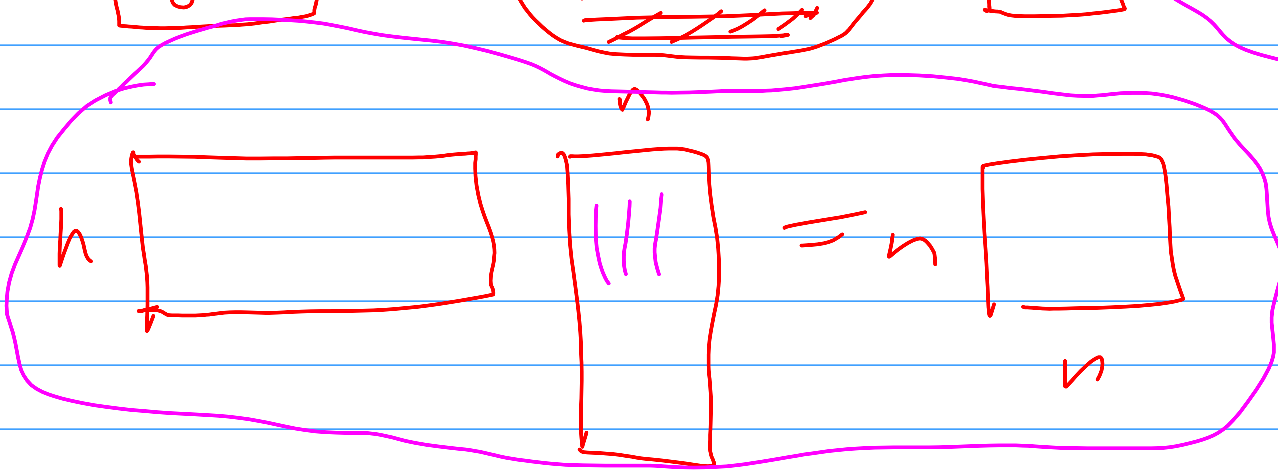
$$\begin{array}{cc} & n \\ m & \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \\ 0 & & 0 \\ 0 & & 0 \end{pmatrix} \end{array}$$



$\underline{\text{rang}(A^\cdot) \leq n}$

$$\underline{\underline{(A^T A)^{-1}}}$$





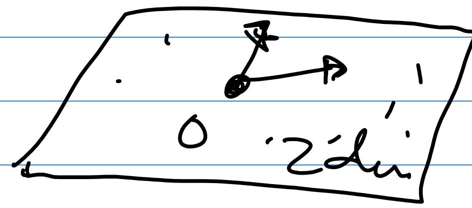
(5,2) (2a)

(2a)

$\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}-1\\0\\1\end{pmatrix}, \begin{pmatrix}2\\3\\1\end{pmatrix}, \begin{pmatrix}0\\2\\2\end{pmatrix}, \ldots$ ∞ veel

basis van $S^\perp$:

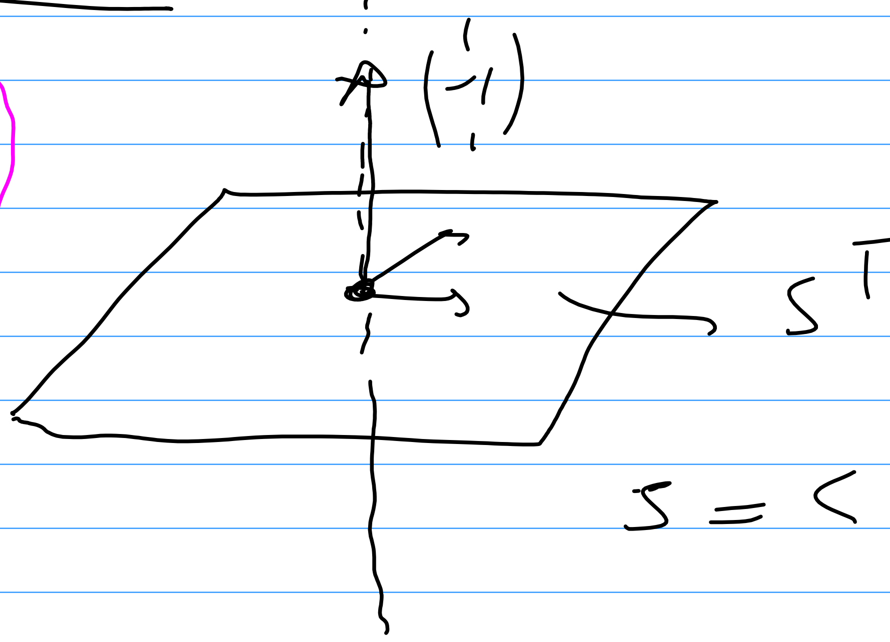
"klein" ~ "minst" aantal



$S = \begin{pmatrix}1\\-1\\1\end{pmatrix}$

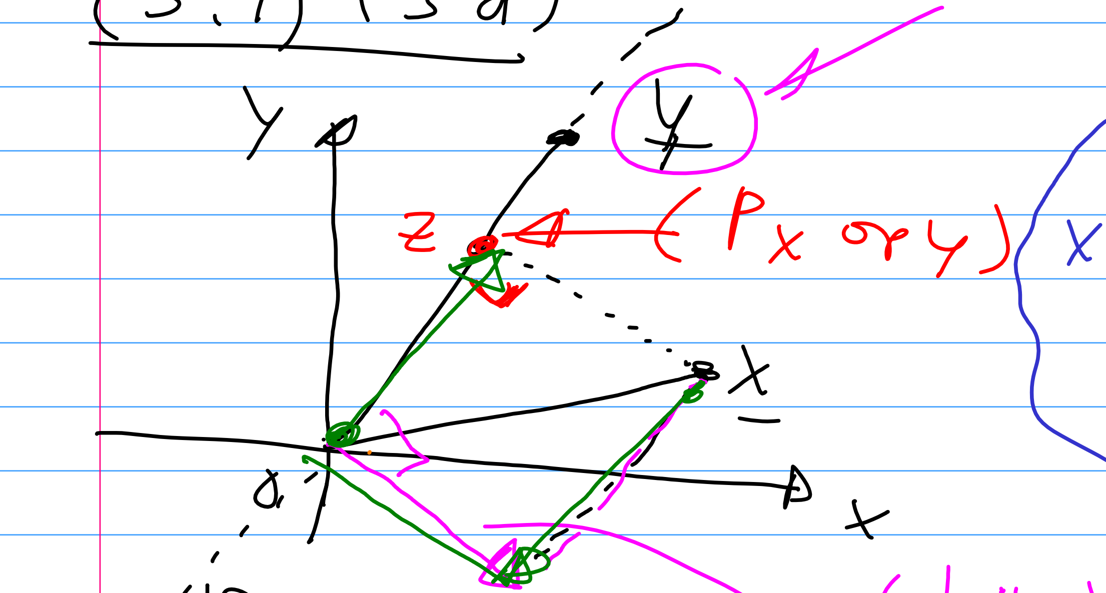
$\dim(S^\perp) = 2$

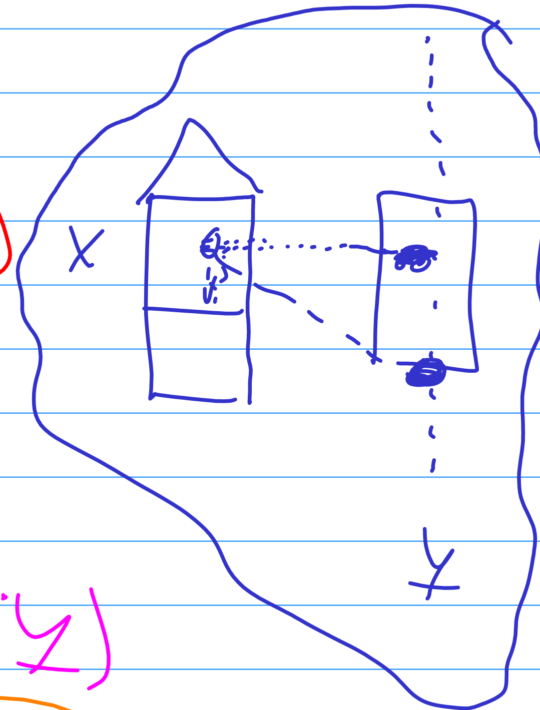
wacom intuos



$S = \left\langle \begin{pmatrix}1\\-1\\1\end{pmatrix} \right\rangle$

(5.1) (3d)





P van x op y

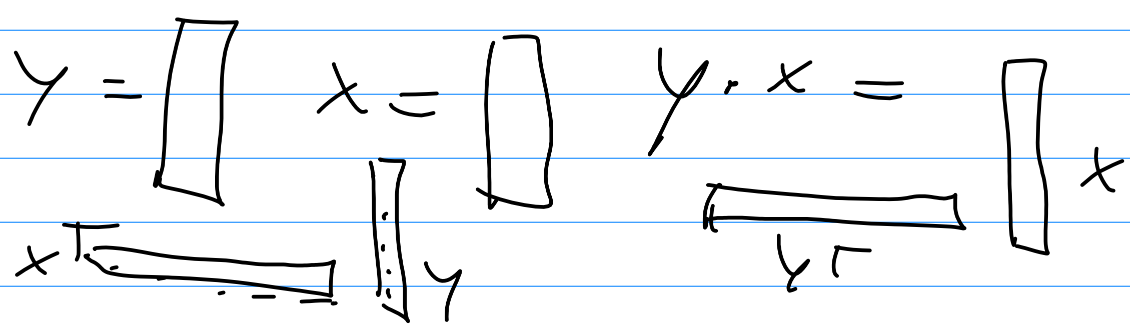
$$(x - z) \qquad z = \lambda y$$

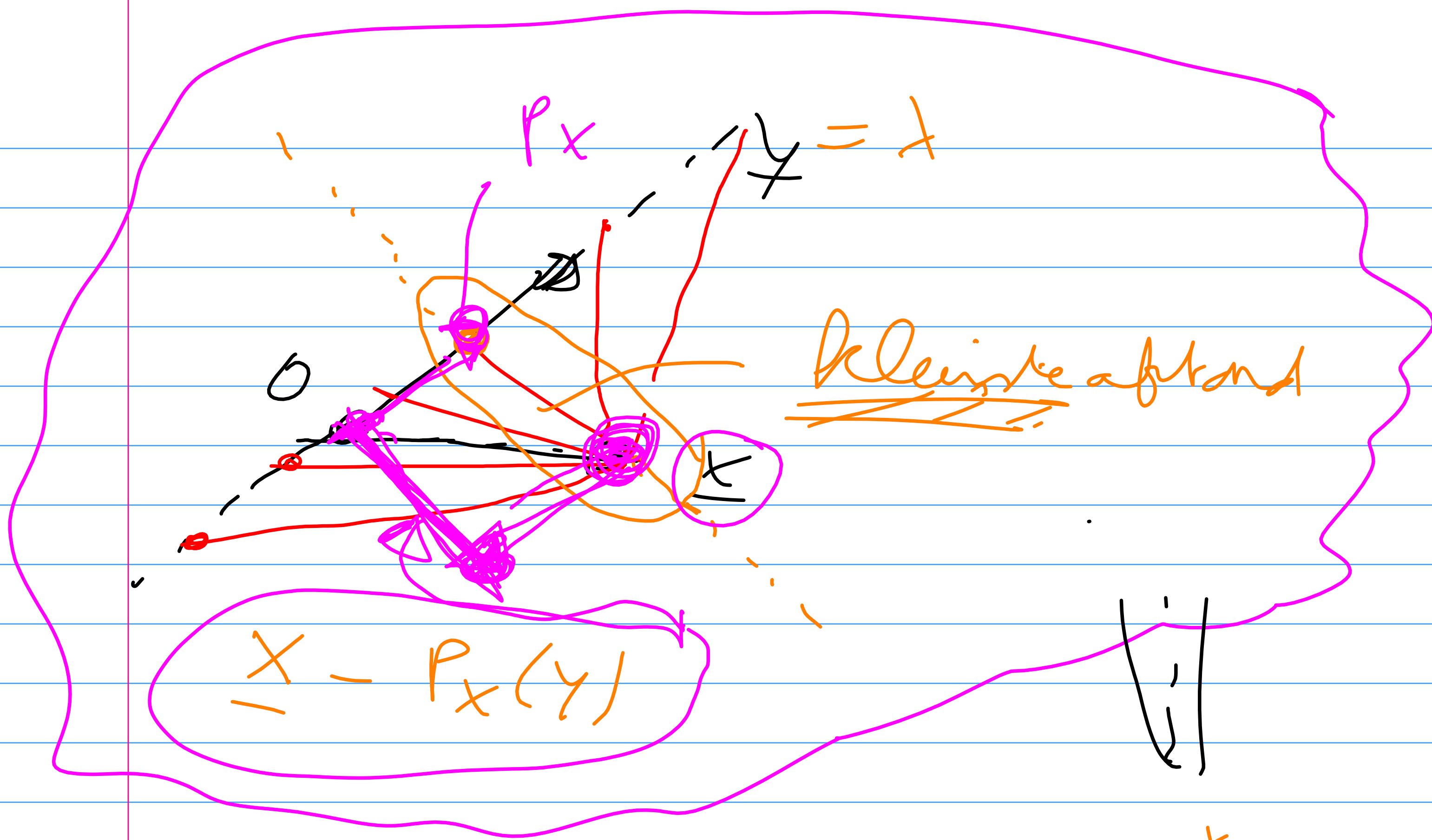
$$(x - \lambda y) \perp y$$

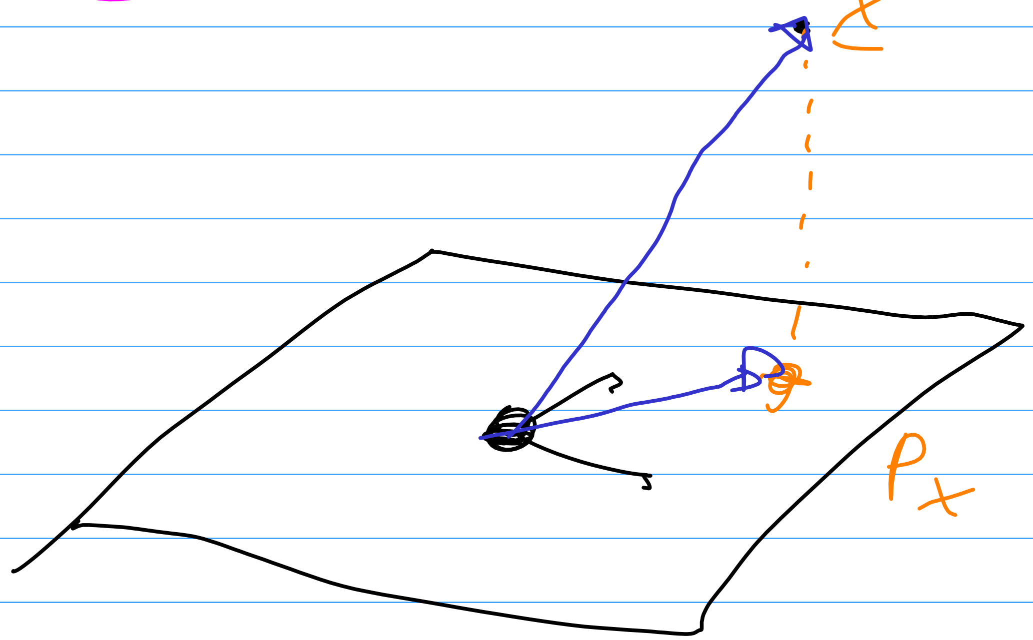
$$y^T(x - \lambda y) = 0$$

$$y^T \cdot x - \lambda\, y^T y = 0$$

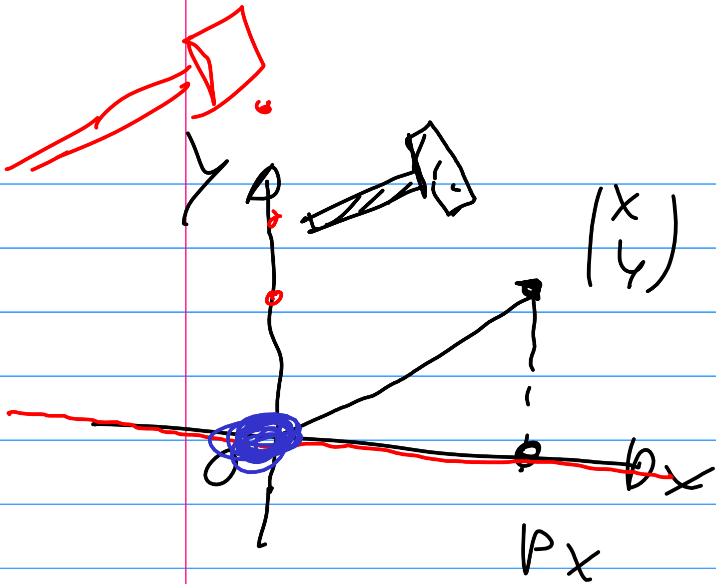
$$\lambda = \frac{y^T \cdot x}{(y^T \cdot y)} \qquad z = \lambda \cdot y$$







$$(x - P_x) \perp \text{vlak}$$



$$P = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

$$P\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x\\0\end{pmatrix}$$

$$P\underline{v} = \underline{v}$$

$$\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$P\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} = 0\cdot\begin{pmatrix}0\\1\end{pmatrix}$$

$$P\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} = 1\cdot\begin{pmatrix}1\\0\end{pmatrix}$$

0, 1 eigenwaarden van P

eigenruimten: $E_0 = \left\langle \begin{pmatrix}0\\1\end{pmatrix} \right\rangle$

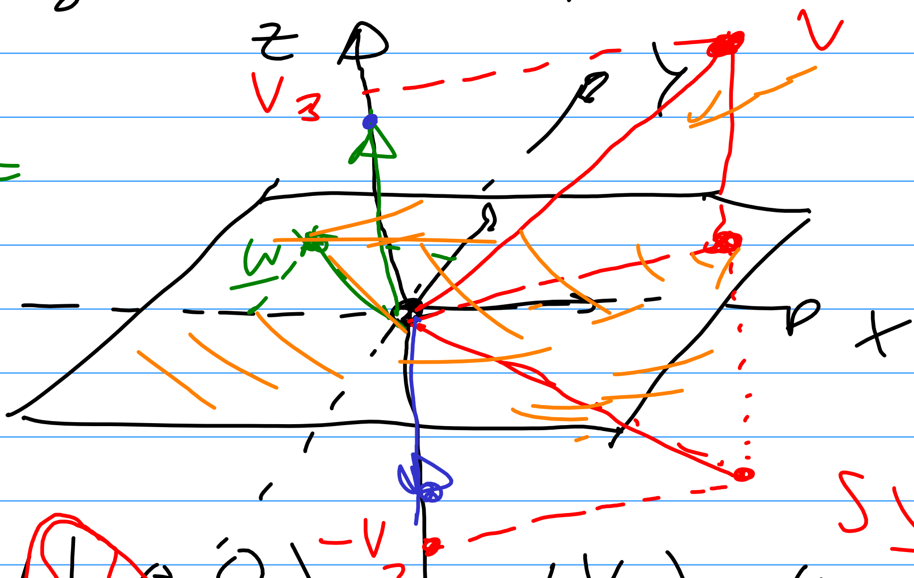
$$E_1 = \left\langle \begin{pmatrix}1\\0\end{pmatrix} \right\rangle$$

Spiegelen:

$\mathbb{R}^3$, $(x-y)$ vlak.

spiegelen in $(x-y)$ vlak:

$S\underline{w} = \underline{w} = 1 \cdot \underline{w}$



$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$S\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ -v_3 \end{pmatrix}$$

eigenwaarden??

$\lambda_1 = 1$, $\lambda_2 = 1$ $x-y$ vlak

$\lambda_3 = -1$ z-as

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

① eigenwaarden →

② eigenvectoren

$$\boxed{Ax = \lambda x}$$

$(x \neq 0)$

λ is dan ew

x is bijbeh. e.v. $(x \neq 0)$

$$Ax - \lambda x = 0$$

$$(A - \lambda I)\, x = 0 \qquad x \neq 0$$

$A - \lambda I$ moet singulier zijn

$$\boxed{|A - \lambda I| = 0} \rightsquigarrow \text{ew}$$

$$\left| \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} (1-\lambda) & 1 \\ 1 & (-1-\lambda) \end{vmatrix} = 0$$

$$(1-\lambda)(-1-\lambda) - 1 = 0$$

$$-1 \cancel{-\lambda} \cancel{+\lambda} + \lambda^2 - 1 = 0$$

ew. vgl.

$$\lambda^2 - 2 = 0 \quad \text{ew. vgl}$$

$$\boxed{\lambda_1 = \sqrt{2}, \quad \lambda_2 = -\sqrt{2}}$$ ew. van A

$$\boxed{\begin{array}{ll} A & |A - \lambda I| = 0 \\ (n \times n) \quad ? & \text{nulpunten} \to \underline{\underline{\text{ew}}} \end{array}}$$

$Ax = \lambda x \qquad x \neq 0$

$(A - \lambda I)x = 0$

$\to$ singulier. $|A - \lambda I| = 0$

$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \to \lambda_1 = \sqrt{2}, \lambda_2 = -\sqrt{2}$

eigenvector bep. bij $\lambda_1 = \sqrt{2}$:

$Ax = \sqrt{2}\,x$ zoek uit x

$|A - \lambda I| = 0 \rightsquigarrow \lambda^2 = 2 \to \lambda_i$

$(A - \lambda_1 I)x = 0 \qquad (x \neq 0)$

$\left(\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} - \sqrt{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)x = 0$

$\begin{pmatrix} (1 - \sqrt{2}) & 1 \\ 1 & (-1 - \sqrt{2}) \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$-1\quad\left(\begin{array}{cc|c} (1-\sqrt{2}) & 1 & 0 \\ 1 & (-1-\sqrt{2}) & 0 \end{array}\right)$$

$$\sqrt{2}\quad\left(\begin{array}{cc|c} -\sqrt{2} & 2+\sqrt{2} & 0 \\ 1 & -1-\sqrt{2} & 0 \end{array}\right)$$

$$-1\quad\left(\begin{array}{cc|c} -2 & 2\sqrt{2}+2 & 0 \\ 2 & -2-2\sqrt{2} & 0 \end{array}\right)$$

$$\boxed{-2x_1 + (2\sqrt{2}+2)x_2 = 0}$$

$$x_1 = \frac{(2\sqrt{2}+2)}{2}\cdot x_2$$

$$x_1 = (1+\sqrt{2})\,x_2$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} (1+\sqrt{2}) \\ 1 \end{pmatrix} x_2$$

ev. behorend $\boxed{\sqrt{2}}$: $\begin{pmatrix} 1+\sqrt{2} \\ 1 \end{pmatrix}$

goed

Jor

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1+\sqrt{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 2+\sqrt{2} \\ \sqrt{2} \end{pmatrix}$$

$$= \sqrt{2}\begin{pmatrix} 1+\sqrt{2} \\ 1 \end{pmatrix}$$

$$\boxed{\lambda_2 = -\sqrt{2}}$$

$$\begin{pmatrix} -(1+\sqrt{2}) \\ 1 \end{pmatrix} \quad (\underline{\underline{\text{denk}}})$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -(1+\sqrt{2}) \\ 1 \end{pmatrix} =$$

$$\begin{pmatrix} -1-\sqrt{2}+1 \\ -1-\sqrt{2}-1 \end{pmatrix} = \begin{pmatrix} -\sqrt{2} \\ -2-\sqrt{2} \end{pmatrix}$$

$$= -\sqrt{2}\begin{pmatrix} 1 \\ - \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ -(1+\sqrt{2}) \end{pmatrix} = \begin{pmatrix} -\sqrt{2} \\ 2+\sqrt{2} \end{pmatrix}$$

$$\lambda_2 = -\sqrt{2}\begin{pmatrix} 1 \\ -(1+\sqrt{2}) \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\lambda_1 = \sqrt{2} \quad E_{\lambda_1} = \left\langle \underbrace{\begin{pmatrix} 1+\sqrt{2} \\ 1 \end{pmatrix}}_{\underline{ev}_1} \right\rangle$$

$$\lambda_2 = -\sqrt{2} \quad E_{\lambda_2} = \left\langle \underbrace{\begin{pmatrix} -1 \\ 1+\sqrt{2} \end{pmatrix}}_{\underline{ev}_2} \right\rangle$$

$\underline{ev}_1$, $\underline{ev}_2$
zijn lin. onafh.

↳ spannen basis op.

basis : $\langle \underline{ev}_1, \underline{ev}_2 \rangle = \underline{\underline{EV}}$

A tov. E.V.

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

matrix(A) tov EV $= \underline{\underline{B}}$

$$B\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \sqrt{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$B\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\sqrt{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ t.o.v $\mathbb{R}^2 = \left\langle \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix} \right\rangle$

Hoe ziet A er uit tov de basis van eigenvectoren

nieuwe basis $= \langle \underline{ev}_1, \underline{ev}_2 \rangle$??

$B(\underline{ev}_1) = \sqrt{2}\,(\underline{ev}_1)$

$B(\underline{ev}_2) = -\sqrt{2}\,(\underline{ev}_2)$

$\underline{\underline{B}} = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & -\sqrt{2} \end{pmatrix}$

A tov de basis van eigenv

bepaal je met ev

$T = \begin{pmatrix} 1+\sqrt{2} & -1 \\ 1 & 1+\sqrt{2} \end{pmatrix}$

$$\boxed{T^{-1} A \cdot T} = D = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & -\sqrt{2} \end{pmatrix}$$

$A \qquad |A - \lambda I| = 0$

$(A - \lambda_i I)x = 0$

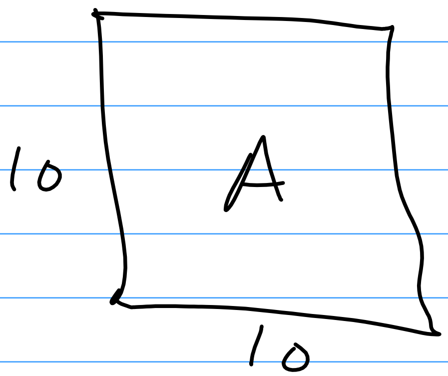
$p(\lambda) = 0, \quad \lambda_1 \ldots \lambda_n \; \underline{\underline{ev}}$

$\underline{\underline{ev_i}}$

$$T = \begin{pmatrix} | & & | \\ ev_1, & \cdots & ev_n \\ | & & | \end{pmatrix}$$

$$T^{-1} A T = D = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$$

$$T = \begin{pmatrix} | & & | \\ ev_1 & \cdots & ev_n \\ | & & | \end{pmatrix}$$



T

$\xrightarrow{T}$

andere basis

$$D = \begin{pmatrix} \bullet & & \\ & \ddots & \\ & & \bullet \end{pmatrix}$$

$$y''(t) = y(t) \qquad \underline{\underline{y(t) = e^t}}$$

$$\frac{d^2}{dt^2} e^t = 1\, e^t$$

$$y(t) = e^{\lambda t}$$

$$y'(t) = 2\, y(t) \qquad e^{2t}$$

$$\lambda e^{\lambda t} = 2 e^{\lambda t} \qquad \underline{\lambda = 2}$$

$$y''(t) = y(t) \qquad y(t) = e^{t}$$

$$\frac{d^2}{dt^2} e^{t} = (1) e^{t} \qquad y(t) = e^{\lambda t}$$

$$y'(t) = 2y(t) \qquad e^{2t}$$

$$\lambda e^{\lambda t} = 2 e^{\lambda t} \qquad \lambda = 2$$

$$(\lambda - 1)(\lambda - 2) = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$y''(t) - 3y'(t) + 2y(t) = 0$$

$$y(t) = e^{\lambda t}$$

$$\lambda^2 - 3\lambda + 2 = 0 \quad (\lambda - 2)(\lambda - 1) = 0$$

$$\lambda_1 = 1, \quad \lambda_2 = 2 \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$y_1(t) = e^{t}, \quad y_2 = e^{2t} \quad \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$y(t) = c_1 e^{t} + c_2 e^{2t} \quad \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\langle e^t, e^{2t} \rangle \qquad |c_1 e^t + c_2 e^{2t}|$$

$$\left\langle \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix} \right\rangle \qquad \begin{pmatrix}c_1\\c_2\end{pmatrix}$$

$$f(t) = e^t$$

f is exp. function $\quad f \quad e^{(\cdot)}$

$$y''(t) - 3y'(t) + 2y(t) = 0$$

$$\begin{pmatrix}v\\v'\end{pmatrix}' = \begin{pmatrix}v'\\v''\end{pmatrix} = \begin{pmatrix}v'\\3v' - 2v\end{pmatrix}$$

$$v = y(t) \qquad y''(t) = 3y' - 2y$$

$$v' = y'(t) \qquad = 3 \cdot v' - 2v$$

$$\boxed{\begin{pmatrix}y\\y'\end{pmatrix}' = \begin{pmatrix}0 & 1\\3 & -2\end{pmatrix}\begin{pmatrix}y\\y'\end{pmatrix}}$$

$$A = \begin{pmatrix} 0 & 1 \\ 3 & -2 \end{pmatrix} \qquad \| \; \lambda^2 - 3\lambda + 2 = 0$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 1 \\ 3 & (-2-\lambda) \end{vmatrix} = 0$$

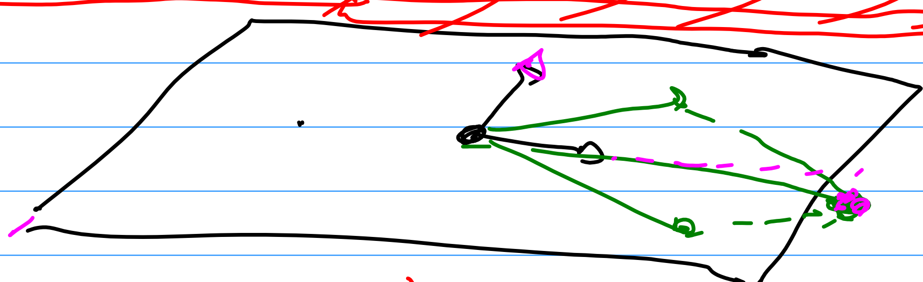
$$-\lambda(-2-\lambda) - 3 = 0$$

$$\lambda^2 + 2\lambda - 3 = 0$$

$\underline{A}$ afhankelijk v.d. diff vgl ??

eigenruimte : $\boxed{\text{lin deelruimte}}$

P xy vlak

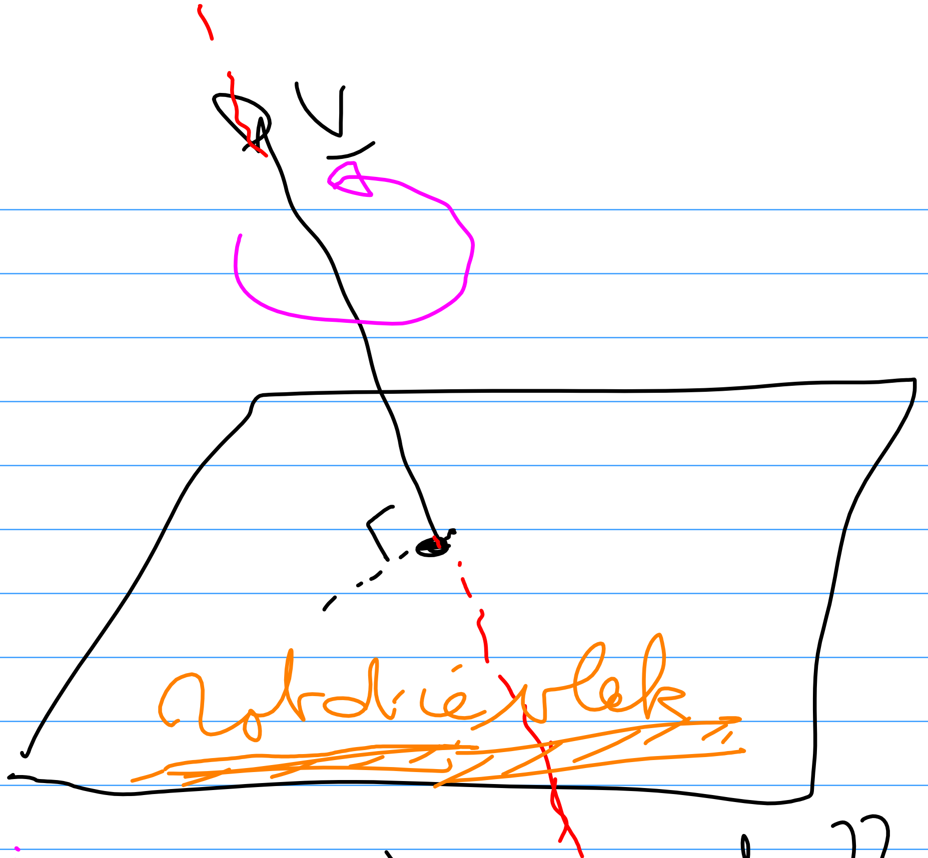


$$A(c\underline{x}) = c \cdot A\underline{x} = c\lambda\underline{x} = \lambda(c\underline{x})$$

$$A\underline{x} = \lambda \cdot \underline{x}, \quad A\underline{y} = \lambda\underline{y}$$

$$A(\underline{x} + \underline{y}) = A\underline{x} + A\underline{y} =$$

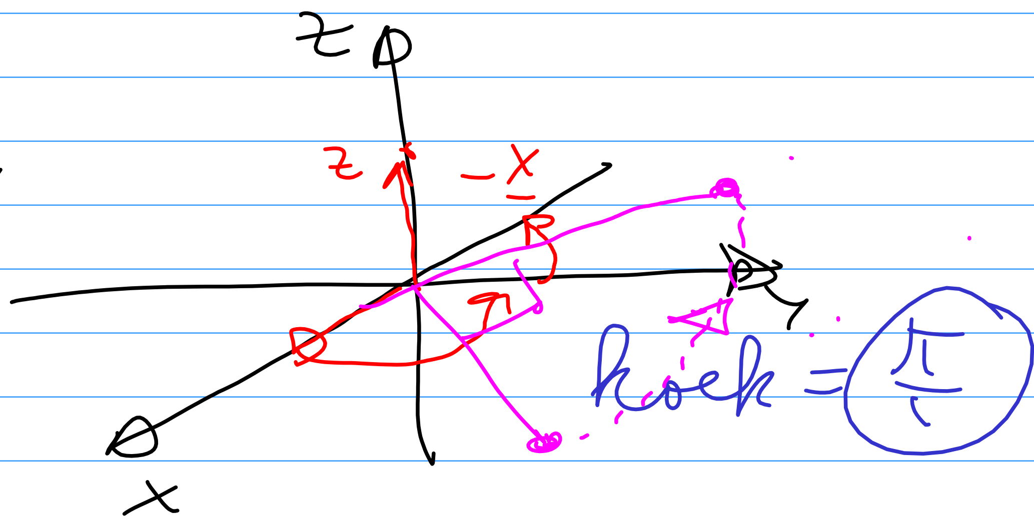
$$\lambda \cdot \underline{x} + \lambda\underline{y} = \lambda(\underline{x} + \underline{y})$$



Rotatie : eigenwaarden??

rotatie-as : $\mathrm{Rot}(as) = 1 \, as$ ew

$\mathbb{R}^3$:



$$R\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix} \qquad R\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{bmatrix}-1\\0\\0\end{bmatrix}$$

$$R\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} \qquad \underline{R} = \begin{pmatrix}0 & -1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{pmatrix}$$

$$|R-\lambda I|=0 \quad \begin{vmatrix} -\lambda & -1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\cdot\begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$A\underline{x} = 5\cdot\underline{x}$

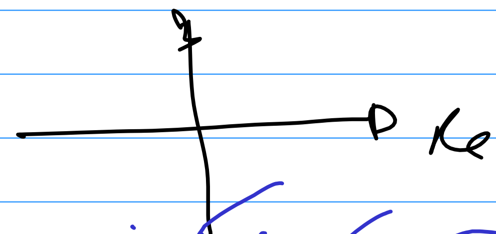
$$(1-\lambda)(\lambda^2+1)=0$$

$\boxed{\lambda_1 = 1}$ → forgatás (z - as)

$\lambda^2+1=0 \qquad \lambda = \pm i$

$\lambda_2 = i \qquad \lambda_3 = -i$

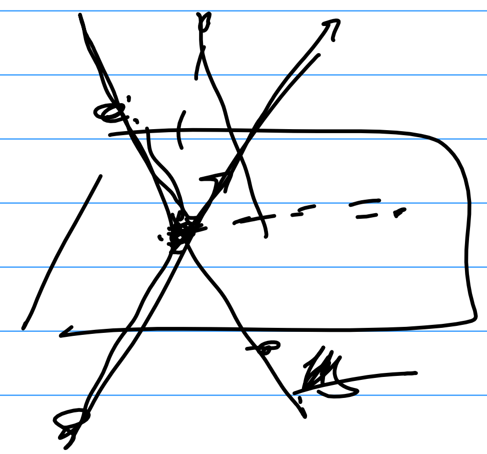
$\frac{\pi}{2} \qquad -\pi/2$



$\underline{e^{i\varphi}\cdot e^{i\theta}} = e^{i(\varphi+\theta)}$

$A\underline{x} = 10\cdot\underline{x}$

$A\underline{y} = -10\cdot\underline{y}$



$$R=\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\lambda_1 = i \qquad \lambda_2 = -i$

$$(R - iI)\underline{x} = 0 \qquad \underline{\underline{E_i}}$$

$$i\left(\begin{array}{ccc|c} -i & -1 & 0 & 0 \\ 1 & -i & 0 & 0 \\ 0 & 0 & 1-i & 0 \end{array}\right) \qquad (1-i)z = 0$$

$$\underset{\neq 0}{1-i} \qquad \underset{\neq 0}{(1-i)}$$

$$\boxed{z = 0}$$

$$-ix - y = 0 \qquad y = -ix$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ -ix \\ 0 \end{pmatrix} = x\begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} = \begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} = i\begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

$$E_i = \left\langle \begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} \right\rangle \qquad \underline{\underline{E_{-i} = \left\langle \begin{pmatrix} -i \\ 1 \\ 0 \end{pmatrix} \right\rangle}}$$

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -i \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -i \\ 0 \end{pmatrix} = -i\begin{pmatrix} -i \\ 1 \\ 0 \end{pmatrix}$$

$$Re(z) = \frac{1}{2}\left(\begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -i \\ 1 \\ 0 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

$$\begin{cases} z = a + ib \\ \bar{z} = a - ib \end{cases} +$$

$$\frac{z + \bar{z}}{2} = a$$

$$Re(z) = a = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

vector uit

rotatievlak

$$Im(z) = \frac{z - \bar{z}}{2i} = \frac{1}{2i}\left(\begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -i \\ 1 \\ 0 \end{pmatrix}\right)$$

$$= \frac{1}{2i}\begin{pmatrix} 2i \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

orthogonaliteit

$$A = \begin{pmatrix} a_1 & \cdots & a_k \end{pmatrix} \qquad a_i \cdot a_k = 0 \quad i \neq k$$

$$A^T A \qquad (a_i \neq 0)$$

$$\boxed{A^T} \; \boxed{A} = \begin{pmatrix} \|a_1\|^2 & 0 & \cdots & 0 \\ 0 & \|a_2\|^2 & \cdots & 0 \\ \vdots & 0 & \ddots & \\ \vdots & \vdots & & \|a_n\|^2 \end{pmatrix}$$

$A = (a_1 \; \cdots \; a_n)$

$$a_1^T a_1 = a_1 \cdot a_1 = \underline{\underline{\|a_1\|^2}}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \det(A^T) = -1$$

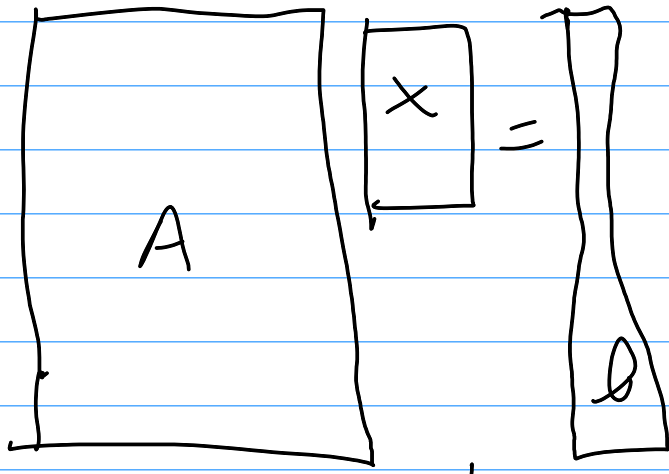
$(\det(A^T A) = +1)$

$$A^T A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

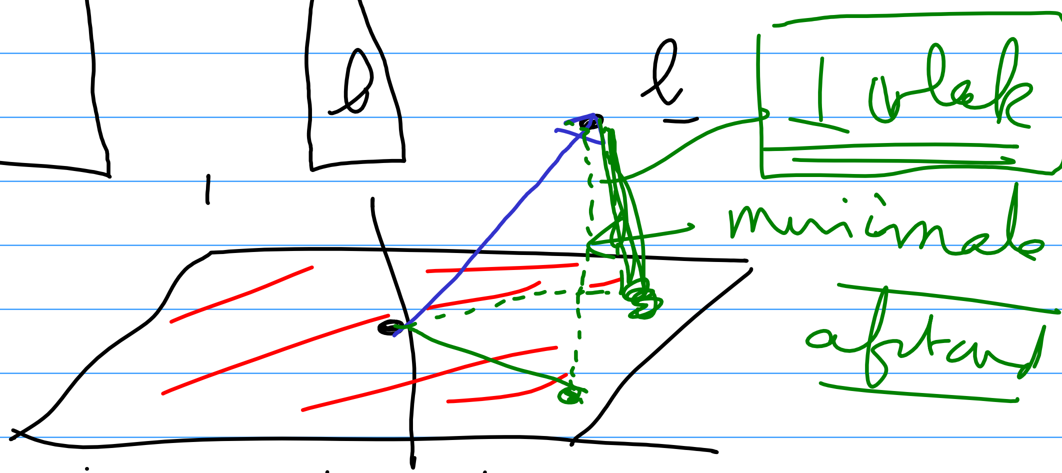
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Least square methode

meer vgl-en dan onbekenden



$b \notin R(A)$



$(x-y)$ vlak $\left\langle \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix} \right\rangle$

$$b = \begin{pmatrix}2\\3\\5\end{pmatrix}$$

$$\boxed{\begin{pmatrix}1 & 0\\0 & 1\\0 & 0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}2\\3\\5\end{pmatrix}}$$

géén oplossing

Stel ik heb $\tilde{x}$ $A\tilde{x}$ het dichtst in de buurt van b

$(b - A\tilde{x}) \perp$ kolommen van A

$$\left(A^T(\underline{\ell} - A\tilde{x}) = \begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}\right)$$

Kleinste kw. methode

$$A^T\left(A\underline{x} = \underline{\ell}\right)$$

$\to$ kolommen lin. onafh.

$$(A^TA)\,\underline{x} = A^T\underline{\ell}$$

(A^TA) niet singulier

$$\tilde{x} = (A^TA)^{-1}\cdot A^T\underline{\ell}$$

$(A^TA)^{-1}$ bestaat

$(\underline{\ell} - A\tilde{x}) \perp$ kolommen van A

$$A^T\left(\underline{\ell} - A(A^TA)^{-1}A^T\ell\right) =$$

$$A^T\underline{\ell} - \underbrace{\cancel{(A^TA)}\cancel{(A^TA)}^{-1}}\,A^T\ell = 0$$

$A\underline{x} = \ell$ kolommen van A lin. onafh.

$$\tilde{x} = (A^TA)^{-1}\cdot A^Tb$$

3 dim space $(1, 0, z_1)$, $(0, 1, z_2)$

$(-1, 0, z_3)$ $(0, -1, z_4)$

→ **vlak:** $z = \alpha x + \beta y + \gamma$ ←

$\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$

som kw: $d(\alpha, \beta, \gamma) = |z_1 - \alpha - \gamma|^2 + |z_2 - \beta - \gamma|^2 + |z_3 + \alpha - \gamma|^2 + |z_4 + \beta - \gamma|^2$

d minimaal

$\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

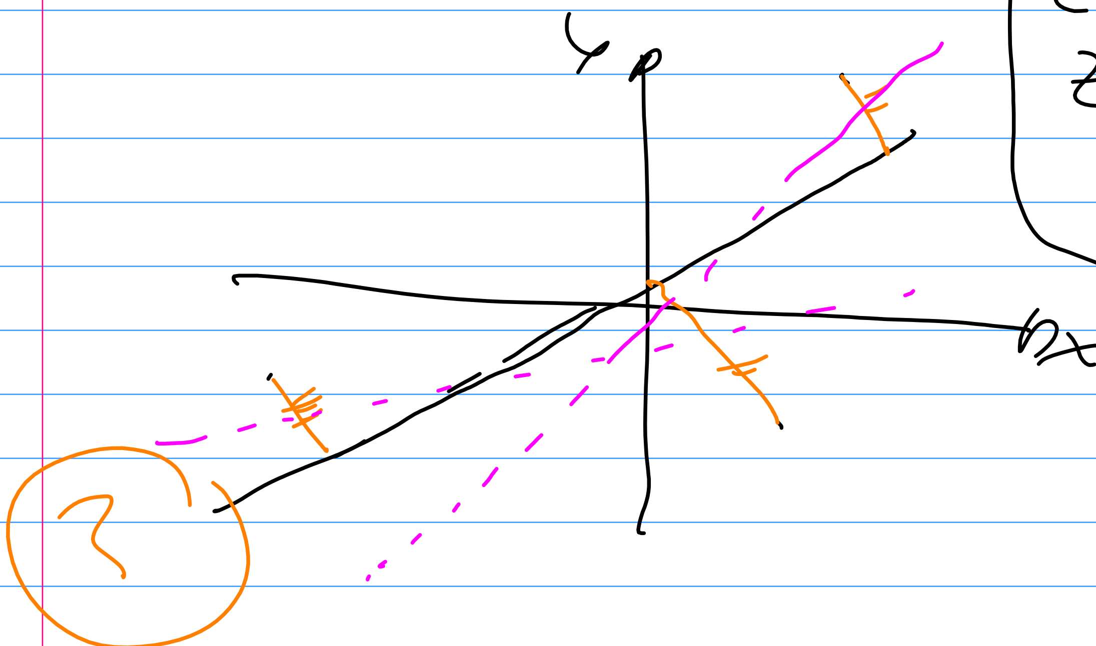
$$A \underbrace{\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}}_{\ell}$$

$$A^T A \underline{x} = A^T \underline{\ell}$$

$$\tilde{\underline{x}} = (A^T A)^{-1} A^T \ell$$

ℓ

$$\left(\begin{array}{ccc} 1 & & \\ & 1 & \\ & & 1 \\ \hline 0 & 0 & 0 \end{array}\right) x = \left(\begin{array}{c} \vdots \\ \hline * \neq 0 \end{array}\right)$$



$$z_1 = \alpha x + y$$
$$z_2 = \alpha x + y$$
$$z_3 = \alpha x + y$$

$$\underbrace{\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}}_{A^T} \underbrace{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}}_{(A^T A)}$$

$$(A^T A)^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ \vdots \\ z_4 \end{pmatrix}}_{A^T z} = \begin{pmatrix} z_1 - z_3 \\ z_2 - z_4 \\ z_1 + \cdots + z_4 \end{pmatrix}$$

$$(A^T A)^{-1} A^T \underline{l} = \frac{1}{2}\begin{pmatrix} z_1 - z_3 \\ z_2 - z_4 \\ z_1 + \cdots + z_4 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{\alpha} \\ \tilde{\beta} \\ \tilde{\gamma} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} z_1 - z_3 \\ z_2 - z_4 \\ z_1 + \cdots + z_4 \end{pmatrix}$$