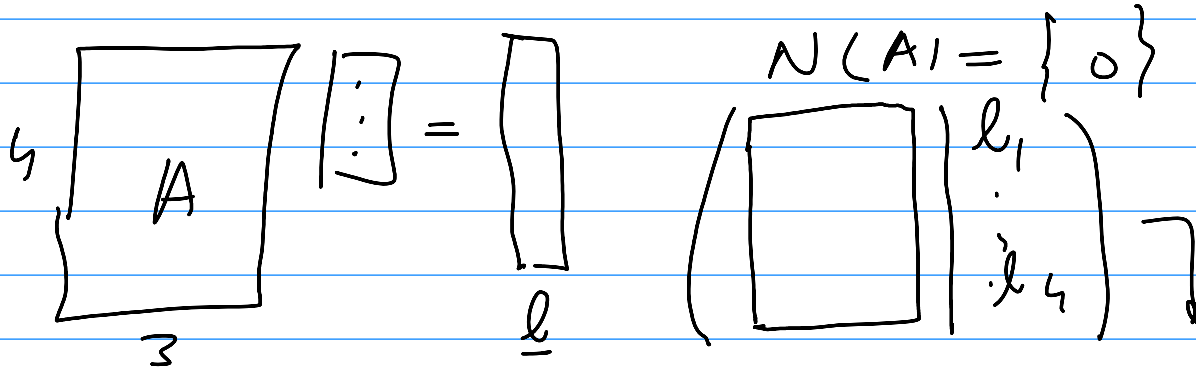
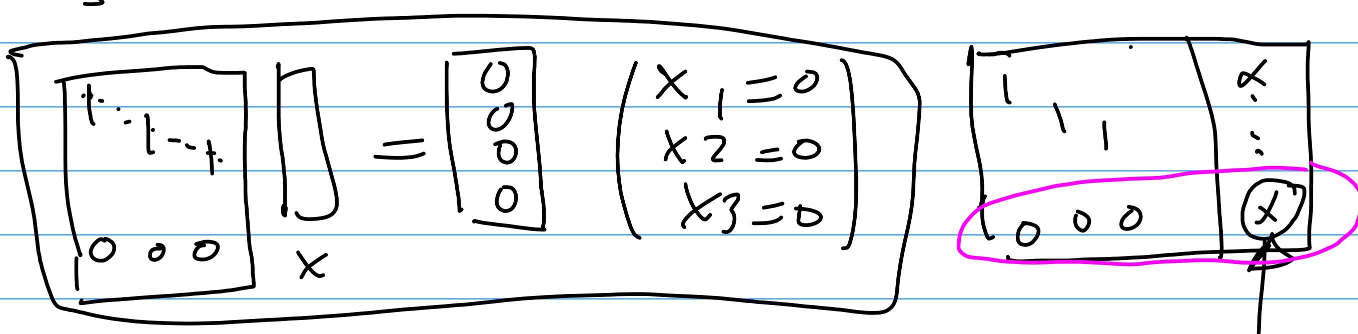
2DL60I – 20/0/9:

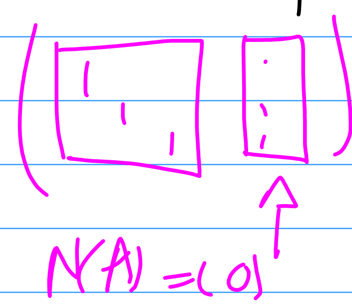
Vragen? Stel ze maar!

pg. 145, 147

$A0 = 0 \quad (N(A) \neq \{0\})$

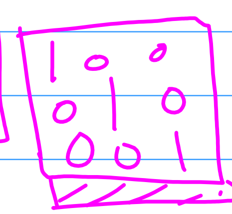




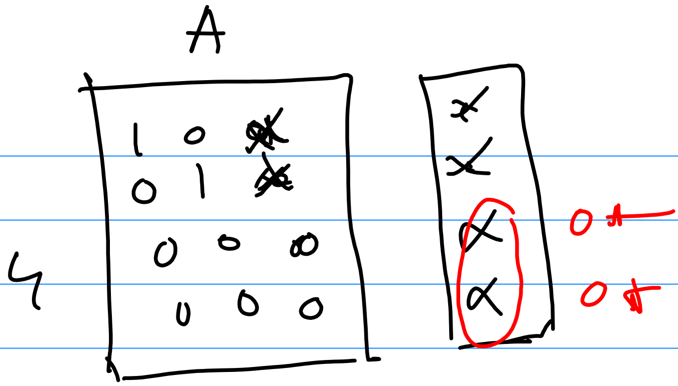


$\begin{cases} \underline{b} \notin R(A) & \text{géén oplossing} \\ \underline{b} \in R(A) & \text{1 oplossing} \end{cases}$

$A\underline{x} = \underline{0} \quad A\ (3\times3) \quad N(A) = \{0\}$



$N(A) \neq \{0\}$



$b \in R(A)$

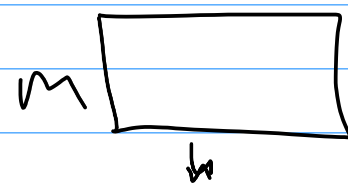
$Ax = b$

$N(A) \neq \{0\}$ $\quad Ay = 0$

$A(x+y) = Ax + Ay = b + 0 = b.$

(5.2) (13)

$A \quad (m \times n)$



$x \in N(A^TA)$

$Ax = x_1 a_1 + \cdots + x_n a_n \in R(A)$

$(A^TA)(x) = A^T(Ax)$

$y \in N(A^T)$?? $\quad A^Ty = 0$

$x \in N(A^TA)$ gesua $\Rightarrow$

$(A^TA)x = 0 \qquad A^T(Ax) = 0$

$Ax \in N(A^T) \qquad \in N(A^T)$

$$N(A^TA) = N(A) \,?$$

$x \in N(A^TA)$ $\quad$ $(?\, x \in N(A)\, ?)$

$(A^TA)\,x = 0$

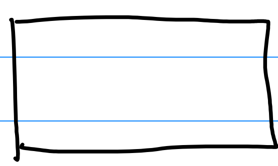
$A^T(Ax) = 0$

$x \in N(A) \Rightarrow Ax = 0 \Rightarrow$

$A^T(Ax) = A^T(0) = 0$

$(A^TA)\,x = 0$ $\quad$ $\underline{x \in N(A^TA)}$

$(N(A) \subseteq N(A^TA))$ ← ?!

$A = {}_m$  $\quad$ $\supseteq\,?$ ←

$(x \in N(A) \Rightarrow Ax = 0 \quad A^T(Ax) = A^T(0) = 0)$

$x \notin N(A) \quad A^TAx \quad (Ax \neq 0)$

$x \in N(A^TA) \quad A^T(Ax) = 0$

$Ax = y \quad y \in N(A^T) \quad A^Ty = 0$

$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ $A^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$

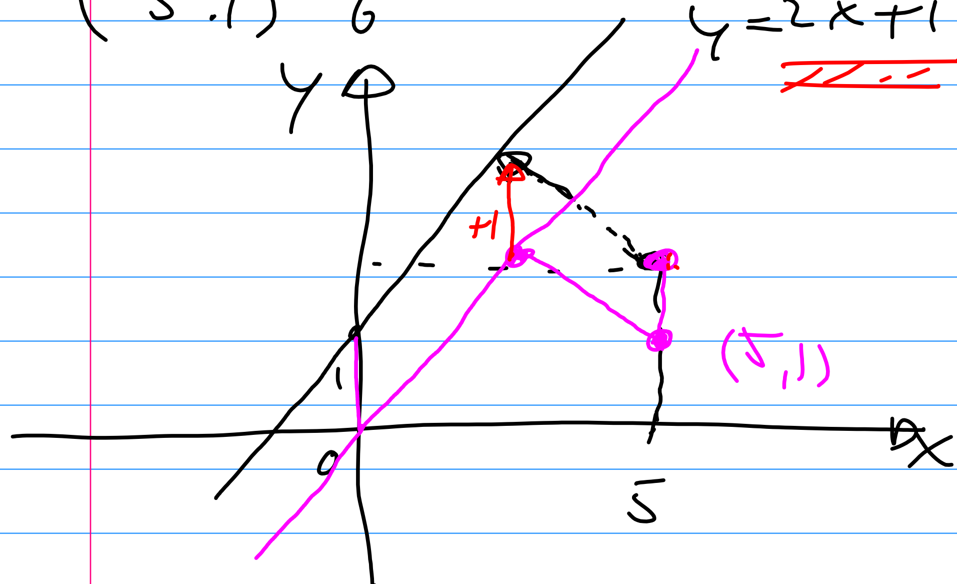
$A^T A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$N(A^T A) = \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$

$\parallel$

$N(A) = \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right.$

(5,1) 6



(5,2) (1 3)

a) $x \in N(A^T A)$ ~ $Ax \in R(A)$

$Ax \in N(A^T)$

$x \in N(A) \qquad Ax = 0 \qquad (A^TA)x = A^T 0 = 0$

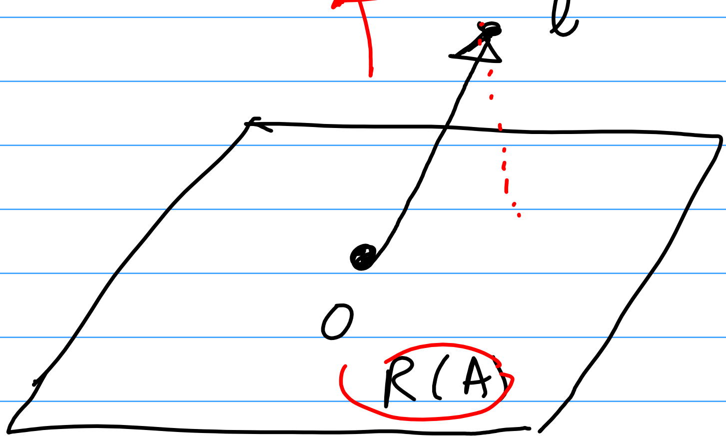
$$\boxed{N(A) \subseteq N(A^TA)}$$

$$??\ N(A^TA) \subseteq N(A)\ ?? \qquad ??$$

(5.3) 4)

$$\boxed{Ax = b}$$

$$\tilde{x} = (A^TA)^{-1}A^Tb$$



$Ax = b$

$\tilde{x} = (A^TA)^{-1}A^Tb$

a) $\begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 6 & 12 \\ 12 & 24 \end{pmatrix}$

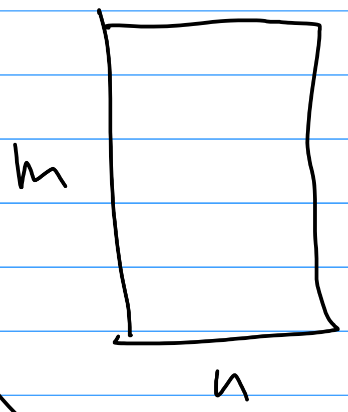
$A^T \cdot A$

$|A^TA| = 0 \qquad$ A^TA not inv

$(A^TA)^{-1}$ not inv ??

$\begin{pmatrix}1\\2\\-1\end{pmatrix}$ $\begin{pmatrix}2\\4\\-2\end{pmatrix}$

kolommen van A moeten lin onafh zijn



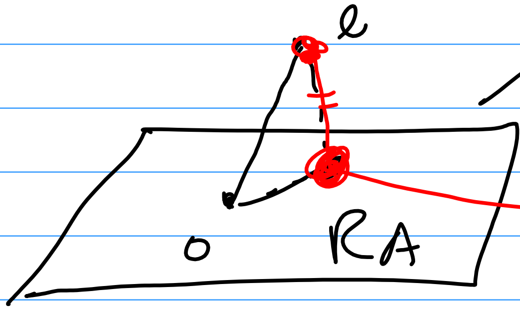
$m > n$ $R(A) = n$

$\begin{pmatrix}1\\2\\-1\end{pmatrix} x = \begin{pmatrix}3\\2\\1\end{pmatrix}$??

$R(A) = \left\langle \begin{pmatrix}1\\2\\-1\end{pmatrix} \right\rangle$ $b = \begin{pmatrix}3\\2\\1\end{pmatrix}$

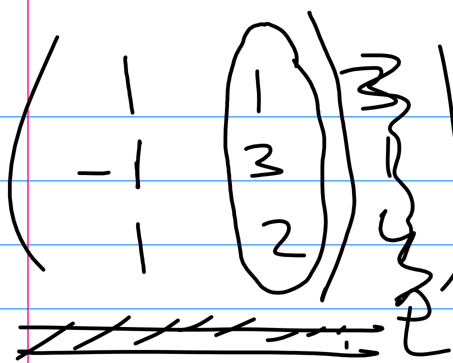
$\underline{b}$ projecteren op: $R(A)$

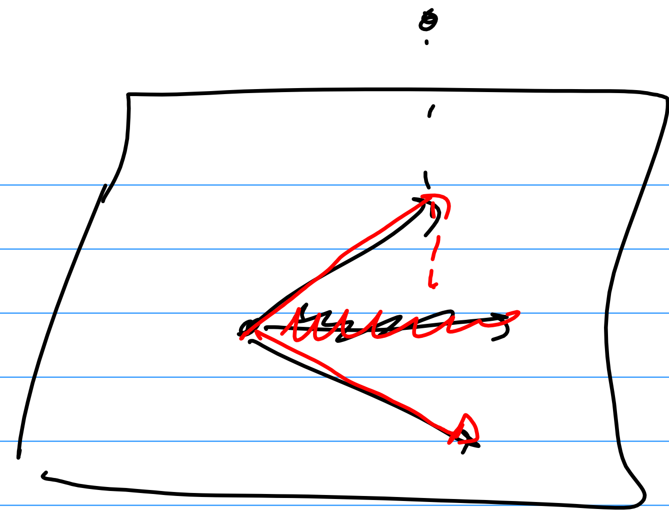
$Ax = b$ .... $\bar{x} = (A^TA)^{-1}A^Tb$



$A\tilde{x}$

$\left(A = \begin{pmatrix}1\\2\\-1\end{pmatrix}\right)$



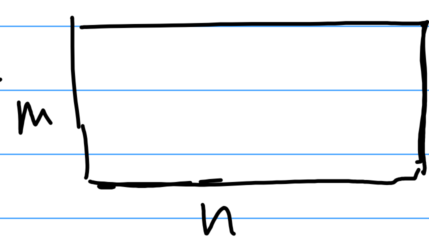


$\|Ax - b\|$ minimaal

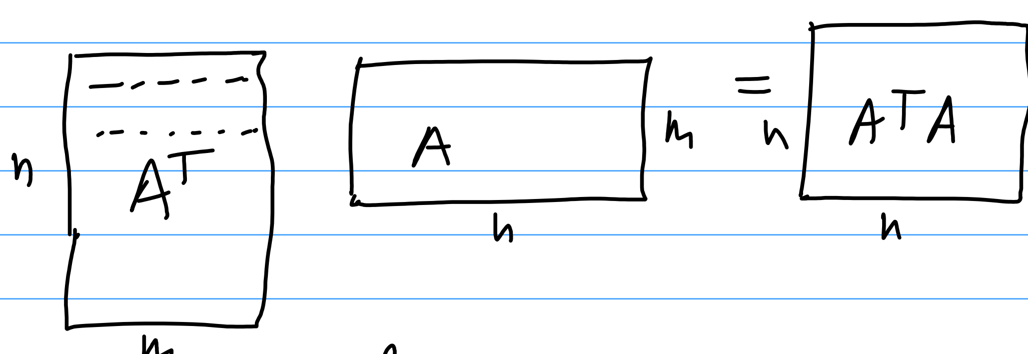
3) $(A^T A)^{-1}$ bestaat niet

4) Laat uit A de juiste kolom weg
een nieuwe $\tilde{A}$ en dan $\tilde{A}x = b$.

$A =$



$A^T A =$



rij × matrix = lin comb van de rijen van A

$x \in N(A^T A) \Rightarrow$

$(A^T A)x = 0 \Rightarrow$

idee $\Rightarrow$ $x^T((A^T A)x) = 0 \Rightarrow$

$(x^T A^T)(Ax) = 0 \Rightarrow$

$\|Ax\|^2 = 0 \Rightarrow Ax = 0$

$x \in N(A)$

(5, 2) 13 ℓ

$N(A^T A) \subseteq N(A)$

$x \in N(A) \Rightarrow Ax = 0 \Rightarrow A^T A x = 0$

$\Rightarrow x \in N(A^T A)$

$N(A) \subseteq N(A^T A)$

ℓ) $\dim(N(A^T A)) = \dim(N(A)) \Rightarrow$

$\mathrm{rank}(A^T A) = \mathrm{rank}(A)$

$A : \mathbb{R}^m \to \mathbb{R}^n$

rank(A)

$\dim(N(A)) + \dim(R(A)) = m$

$A : \mathbb{R}^m \to \mathbb{R}$

$(A^T A) : \mathbb{R}^m \to \cdots$

(5.3)(11)

$$P = A(A^TA)^{-1}A^T$$

$$P^2 = P \cdot P = A\underbrace{(A^TA)^{-1}A^T \cdot A}_{I}(A^TA)^{-1}A^T$$

$$= A(A^TA)^{-1}A^T = P$$

$k = 1; \quad P = P$

→ Stel $k = n$ dan $P^n \cdot P = P$

$k = (n+1) \quad P^{n+1} \cdot P = P(P^n \cdot P) = P \cdot P = P$

$$P^k \cdot P = (A(A^TA)^{-1}A^T) - - - - - (A(A^TA)^{-1}A^T)$$

$$= A(A^TA)^{-1}A^T = P.$$

P symm

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 2 \\ 3 & 2 & 4 \end{pmatrix}$$

$$P^T = P$$

$(A \cdot B)^T = B^T \cdot A^T$

$\left(A(A^TA)^{-1}A^T\right)^T =$

$(A^T)^T \cdot \left((A^TA)^{-1}\right)^T \cdot A^T =$

$A \cdot \left((A^TA)^{-1}\right)^T \cdot A^T$

$A \cdot \left(\left((A^TA)^T\right)^{-1}\right) A^T = A(A^TA)^{-1}A^T$

$(B^{-1})^T = (B^T)^{-1}$ $\qquad (A^TA)^T = A^TA$

$(A \cdot B)^T = B^T A^T$

$(B^{-1})^T = (B^T)^{-1}$

$\Leftarrow$ B !

(6.1) 3

A is singulier $\Longleftrightarrow$ $\lambda = 0$ is ew. van A

$\Leftarrow$

Stel $\lambda = 0$ is een ew. van A

$|A - 0 \cdot I| = 0 \Rightarrow |A| = 0$

A is singulier.

($\Leftarrow$) A is singulier $|A| = 0 \Rightarrow$

$$|A - 0.I| = 0 \Rightarrow$$

$\lambda = 0$ is eW. van. A.

$|A| = \lambda_1 \cdots \lambda_n$, λ_i ew.

$|A| = 0 \rightarrow \exists\ \lambda_i = 0$

(4)

$$|A| = \lambda_1 \cdots \lambda_n \qquad \lambda_i \neq 0$$

$$|A^{-1}| = \frac{1}{\lambda_1} \frac{1}{\lambda_2} \cdots \frac{1}{\lambda_n}$$

$$|A| \cdot |A^{-1}| = |A \cdot A^{-1}| = 1$$

$$|A \cdot B| = |A||B|$$

$A^{-1}(Ax = \lambda x)$ ($\frac{1}{\lambda}$ ew van A^{-1})??

$$\underbrace{A^{-1}A}_{I} x = A^{-1}(Ax) \qquad A(cx) = cAx$$

$$x = \lambda(A^{-1}x) \qquad A^{-1}x = \frac{1}{\lambda} \cdot x$$