

2DL60I-201019:

Vragen? Stel ze maar!

pg. 145, 14)

$$A \cdot 0 = 0 \quad (N(A) \neq \{0\})$$

$$\begin{matrix} 4 \\ \left[\begin{array}{c} \square \\ \square \\ \square \\ \square \end{array} \right] A \end{matrix} \cdot \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} \quad N(A) = \{0\}$$

$\left(\begin{array}{c} \square \\ \square \\ \square \\ \square \end{array} \right) \begin{matrix} l_1 \\ \vdots \\ l_4 \end{matrix} \right) \rightarrow$

$$\left[\begin{array}{c|c} \begin{matrix} \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} \vdots \\ \vdots \\ \vdots \\ x \end{matrix} \end{array} \right] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{matrix} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{matrix}$$

$\left(\begin{array}{c|c} \begin{matrix} 1 & & & x \\ & 1 & & \vdots \\ & & & \vdots \\ 0 & 0 & 0 & x \end{matrix} \end{array} \right)$

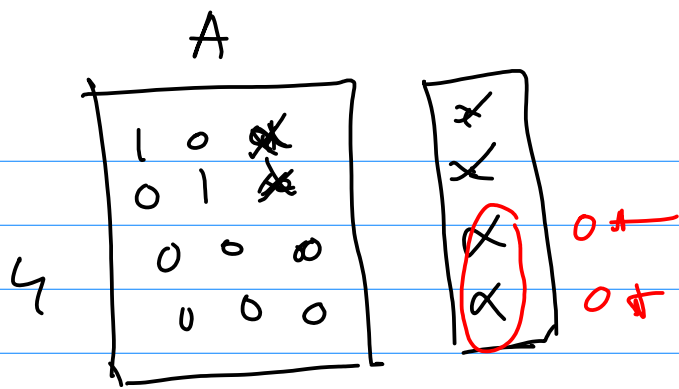
$\left\{ \begin{array}{l} \underline{l} \notin R(A) \quad \text{geen oplossing} \\ \underline{l} \in R(A) \quad \text{1 oplossing} \end{array} \right.$

$$\left(\begin{array}{c|c} \begin{matrix} \square & \vdots \\ \square & \vdots \\ \square & \vdots \\ \square & \vdots \end{matrix} & \begin{matrix} \vdots \\ \vdots \\ \vdots \\ \vdots \end{matrix} \end{array} \right)$$

$N(A) = \{0\}$

$$A \underline{x} = 0 \quad A (3 \times 3) \quad \underline{N(A) = \{0\}} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$N(A) \neq \{0\}$$



$$\underline{\underline{l \in R(A)}}$$

$$Ax = \underline{l}$$

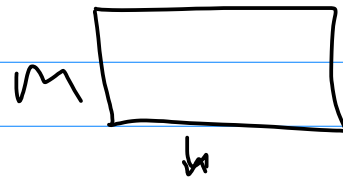
$$N(A) \neq \{0\}$$

$$Ay = \underline{0}$$

$$A(x+y) = Ax + Ay = \uparrow \quad \underline{l+0=l}$$

(5,2) (1,3)

$$A \quad (m \times n)$$



$$x \in N(ATA)$$

$$Ax = x_1 \cdot a_1 + \dots + x_n \cdot a_n \in \underline{\underline{R(A)}}$$

$$(A^T A)(x) = A^T (Ax)$$

$$y \in N(A^T) \quad \underline{\underline{A^T y = 0}}$$

$$\underline{\underline{x \in N(ATA)}}$$

yes or no \Rightarrow

$$\underline{(A^T A)x = 0}$$

$$\underline{A^T (Ax)} = 0$$

$$Ax \in N(A^T)$$

\uparrow

$$\underline{\underline{Ax \in N(A^T)}}$$

$$\underline{N(ATA) = N(A)?}$$

$$\left\{ \begin{array}{l} x \in N(ATA) \quad (? \cdot x \in N(A)?) \\ (A^T A)x = 0 \\ A^T(Ax) = 0 \end{array} \right.$$

$$x \in N(A) \Rightarrow Ax = 0 \Rightarrow$$

$$A^T(Ax) = A^T(0) = 0$$

$$(ATA)x = 0 \quad \underline{x \in N(ATA)}$$

$$\boxed{N(A) \subseteq N(ATA)} \quad \leftarrow ??$$

$$A = \begin{matrix} m \\ \boxed{} \end{matrix} \quad \boxed{= ?} \quad \leftarrow$$

$$(x \in N(A) \Rightarrow Ax = 0 \quad A^T(Ax) = A^T(0) = 0)$$

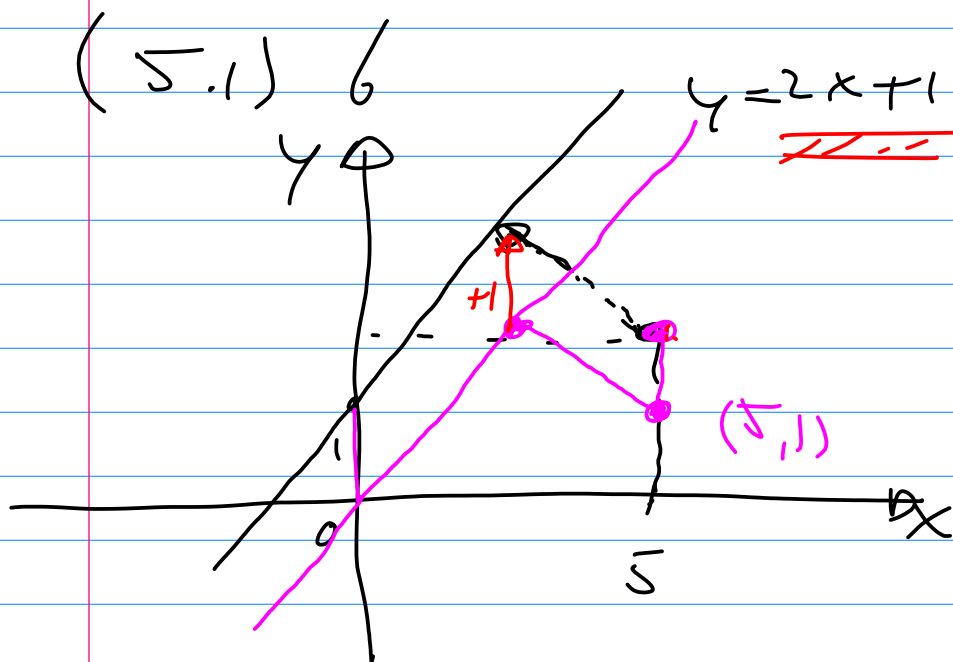
$$x \notin N(A) \quad A^T Ax \quad (Ax \neq 0)$$

$$\begin{array}{l} x \in N(ATA) \quad A^T(Ax) = 0 \\ Ax = y \quad y \in N(A^T) \quad \underline{A^T y = 0} \end{array}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad A^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \parallel N(A^T A) &= \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle \\ \parallel N(A) &= \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle \end{aligned}$$



(5.2) (13)

$$a) \quad \underline{\underline{X \in N(A^T A)}} \sim \begin{matrix} \textcircled{AX \in R(A)} \\ AX \in N(A^T) \end{matrix}$$

$$x \in N(A) \quad Ax = 0 \quad (ATA)x = A^T 0 = 0$$

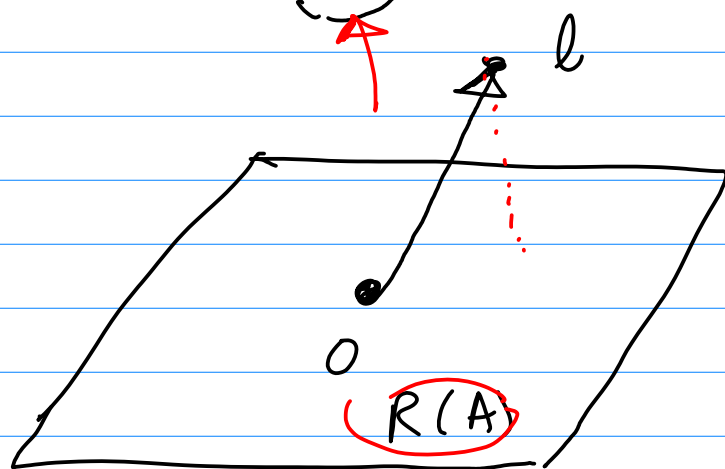
$$N(A) \subseteq N(ATA)$$

?? $N(ATA) \subseteq N(A)$??

(5.3) 4)

$$Ax = b$$

$$\tilde{x} = (ATA)^{-1} A^T b$$



$$Ax = b$$

$$\tilde{x} = (ATA)^{-1} A^T b$$

a)

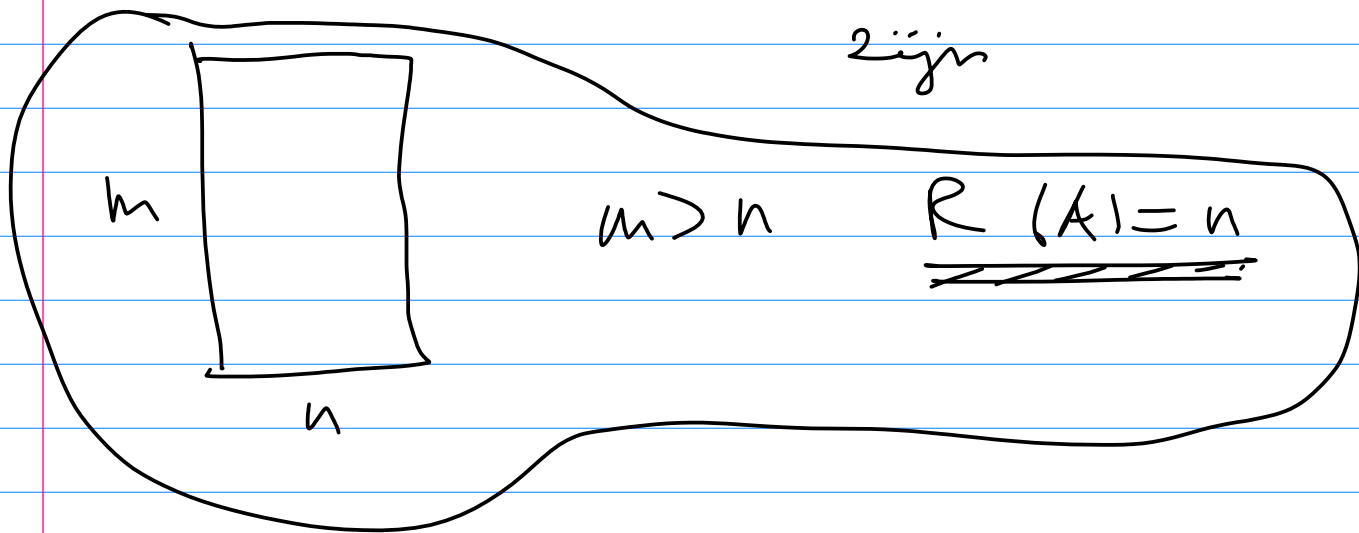
$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \end{pmatrix} \begin{pmatrix} 12 \\ 24 \\ -1-2 \end{pmatrix} = \begin{pmatrix} 6 & 12 \\ 12 & 24 \end{pmatrix}$$

$A^T \quad , \quad A$

$|ATA| = 0$ $A^T A$ nicht inv
 $(A^T A)^{-1}$ well inv !!

$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$$

kolommen van A
moeten lineairafh
zijn

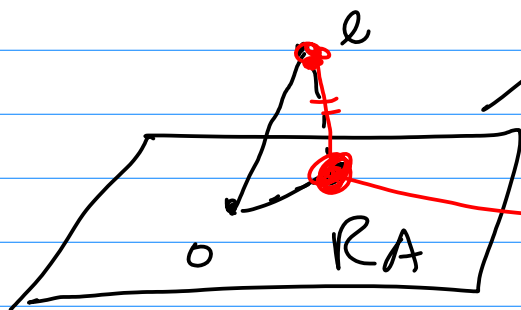


$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} x = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \quad ??$$

$$R(A) = \left\langle \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \right\rangle \quad \ell = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

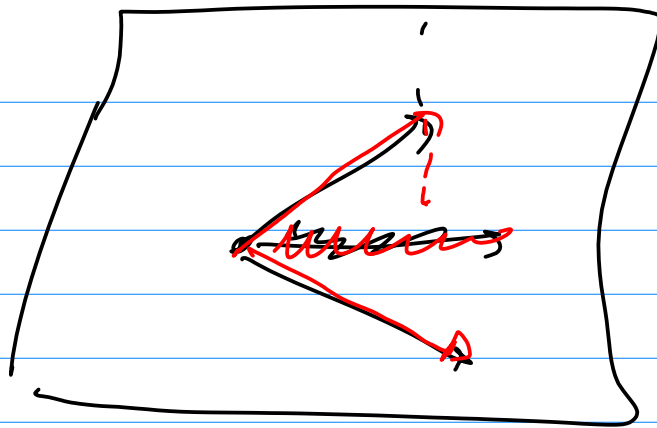
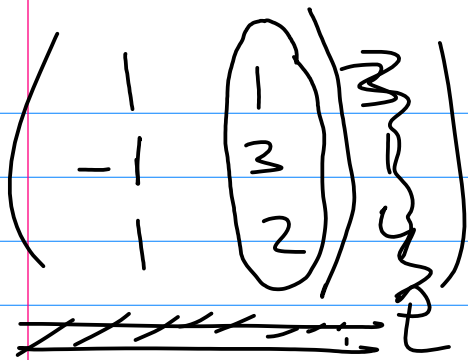
ℓ projecteren op $R(A)$

$$Ax = \ell \dots \quad \bar{x} = (A^T A)^{-1} A^T \ell$$



$$A \bar{x}$$

$$A = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$



$\|Ax - b\|$ minimaal

3) $(A^T A)^{-1}$ bevat niet

4) Laat uit A de juiste kolom weg
een nieuwe \tilde{A} endan $\tilde{A}x = b$.

$$A = \begin{matrix} m \\ \boxed{} \\ n \end{matrix}$$

$$A^T A = \begin{matrix} \text{---} \\ \text{---} \\ \boxed{A^T} \\ \text{---} \\ n \end{matrix}$$

$$\begin{matrix} \boxed{A} \\ n \end{matrix} \begin{matrix} m \\ = \\ n \end{matrix} \begin{matrix} \boxed{A^T A} \\ n \end{matrix}$$

rij x matrix = lin comb van de rijen van A

$$\underline{x \in N(A^T A) \Rightarrow}$$

$$(A^T A)x = 0 \Rightarrow$$

idea $\Rightarrow x^T ((A^T A)x) = 0 \Rightarrow$

$$(x^T A^T)(Ax) = 0 \Rightarrow$$

$$\|Ax\|^2 = 0 \Rightarrow \underline{Ax = 0}$$

$$\underline{x \in N(A)}$$

$$(5, 2) | 3 \quad \text{Q}$$

$$N(A^T A) \subseteq N(A)$$

$$x \in N(A) \Rightarrow Ax = 0 \Rightarrow A^T Ax = 0$$

$$\Rightarrow x \in N(A^T A)$$

$$N(A) \subseteq N(A^T A)$$

Q

$$\left\{ \begin{array}{l} \dim(N(A^T A)) = \dim(N(A)) \Rightarrow \\ \text{rank}(A^T A) = \text{rank}(A) \end{array} \right.$$

$$A: \mathbb{R}^m \rightarrow \mathbb{R}^n$$

rank(A)

$$\dim(N(A)) + \dim(R(A)) = m$$

$$\begin{array}{l} A: \mathbb{R}^m \rightarrow \mathbb{R}^n \\ (A^T A): \mathbb{R}^m \rightarrow \dots \end{array}$$

(5.3)(11)

$$P = A(A^T A)^{-1} A^T$$

$$P^2 = P \cdot P = A \underbrace{(A^T A)^{-1} A^T A}_{I} \underbrace{(A^T A)^{-1} A^T}_{I} = A(A^T A)^{-1} A^T = P$$

$$k=1; P=P$$

→ Still $k=n$ da $P^n \cdot P = P$

$$k=(n+1) \quad P^{n+1} \cdot P = P(P^n \cdot P) = P \cdot P = P$$

$$P^k \cdot P = (A(A^T A)^{-1} \underbrace{A^T}_{(A^T A)}) \dots (A(A^T A)^{-1} A^T) = A(A^T A)^{-1} A^T = P$$

P symmetrisch

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 2 \\ 3 & 2 & 4 \end{pmatrix}$$

$$P^T = P$$

$$(A \cdot B)^T = B^T \cdot A^T$$

$$\underline{\underline{(A(A^T A)^{-1} A^T)^T =$$

$$(A^T)^T \cdot ((A^T A)^{-1})^T \cdot A^T =$$

$$A \cdot ((A^T A)^{-1})^T \cdot A^T$$

$$A \cdot ((A^T A)^T)^{-1} A^T = \underline{\underline{A(A^T A)^{-1} A^T}}$$

$$\underline{\underline{(B^{-1})^T = (B^T)^{-1}} \quad (A^T A)^T = A^T A$$

$$\underline{\underline{(A \cdot B)^T = B^T A^T}}$$

$$(B^{-1})^T = (B^T)^{-1}$$

(6.1) ③

A is singulier $\Leftrightarrow \lambda = 0$ is ew. van A

(\Leftarrow)

Stel $\lambda = 0$ is een ew. van A

$$|A - 0 \cdot I| = 0 \Rightarrow |A| = 0$$

A is singulier.

\Rightarrow A is singular $|A|=0 \Rightarrow$

$$\underline{|A - 0 \cdot I| = 0 \Rightarrow}$$

$\lambda = 0$ is ew. von A .

$$|A| = \lambda_1 \cdots \lambda_n, \quad \lambda_i \text{ ew.}$$

$$|A|=0 \rightsquigarrow \exists \underline{\lambda_i = 0}$$

(4)

$$|A| = \lambda_1 \cdots \lambda_n$$

$$\lambda_i \neq 0$$

$$|A^{-1}| = \frac{1}{\lambda_1} \frac{1}{\lambda_2} \cdots \frac{1}{\lambda_n}$$

$$\underline{|A| \cdot |A^{-1}| = |A \cdot A^{-1}| = 1}$$

$$|A \cdot B| = |A| |B|$$

$$A^{-1}(Ax = \lambda x)$$

$(\frac{1}{\lambda} \text{ ew. von } A^{-1})??$

$$\underbrace{A^{-1}A}_{I}x = A^{-1}(Ax) \quad A(cx) = cAx$$

$$x = \underline{\lambda}(A^{-1}x)$$

$$A^{-1}x = \frac{1}{\lambda} \cdot x$$