2DL60 II – 201022:

→ op kalender staat cancelled instruction. Waarom? geen idee.

A diagonaliseerbaar

i) $|A - \lambda I| = 0 \Rightarrow \lambda_i \ldots$

ii) E_{λ_i} eigenruimtes.

(eigenvectoren → e_i

$$(A - \lambda_i I)x = 0$$

iii) $\begin{pmatrix} | & & | \\ e_1 & \cdots & e_n \\ | & & | \end{pmatrix} = S$

→ $S^{-1} A S = D$

$$S^{-1} A (e_1 \ldots e_n) =$$

$$S^{-1} (\lambda_1 e_1 \ldots \lambda_n e_n) =$$

$$\begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$$ (0 allemaal nul)

$$D = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$S^{-1} = -\frac{1}{2}\begin{pmatrix} -1 & -1 \\ -1 & +1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$S^{-1} A S = D$$

$$A = S D S^{-1}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\frac{1}{2}\begin{pmatrix} 1 & +1 \\ 1 & -1 \end{pmatrix} =$$

$$= \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \qquad \boxed{?D?}$$

ii) $|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & -1 \\ -1 & -\lambda \end{vmatrix} = \lambda^2 - 1 = 0$$

$$\lambda_1 = +1, \quad \lambda_2 = -1$$

ii) $\underline{\lambda_1 = 1}$ $\quad (A - \lambda_1 I)\underline{x} = \underline{0}$

$$\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-x - y = 0 \Rightarrow y = -x$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -x \end{pmatrix} = x\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$E_1 = \left\langle \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\rangle$$

$\underline{\lambda_2 = -1}$ $\quad (A - \lambda_2 I)x = 0$

$$\left(\begin{array}{cc|c} 1 & -1 & 0 \\ -1 & 1 & 0 \end{array}\right)$$

$$x = y \qquad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ y \end{pmatrix} = y\underline{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

$$E_{-1} = \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\rangle$$

$$\underline{S = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}} \qquad \underline{S^{-1} = \frac{1}{2}\begin{pmatrix} +1 & -1 \\ 1 & +1 \end{pmatrix}}$$

δ

$$S^{-1} \cdot A \cdot S$$

$$\frac{1}{2}\begin{pmatrix} +1 & -1 \\ 1 & +1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

ew: (-1) (1)

$$\boxed{(A - \lambda_i I)\, x = 0}$$ van alle λ_i

$$\tilde{S} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad - \quad \Lambda = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

ew 1 ew: -1

opg. 2020 – (28 1)

$(1, 0, z_1)$ $(0, 1, z_2)$,

$(-1, 0, z_3)$, $(0, -1, z_4)$ onbekend

$\rightarrow (z = \alpha x + \beta y + \gamma)$

$z_1 = \alpha + \gamma$ $z_3 = -\alpha + \gamma$

$z_2 = \beta + \gamma$ $z_4 = -\beta + \gamma$

$$\boxed{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}}$$

kleinste kw. $\rightarrow$ <u>benaderde opl</u>

$$\widetilde{\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}} = (A^T A)^{-1} A^T \begin{pmatrix} z_1 \\ \vdots \\ z_4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}}_{(A^T A)}$$

$$(A^T A)^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} \end{pmatrix};$$

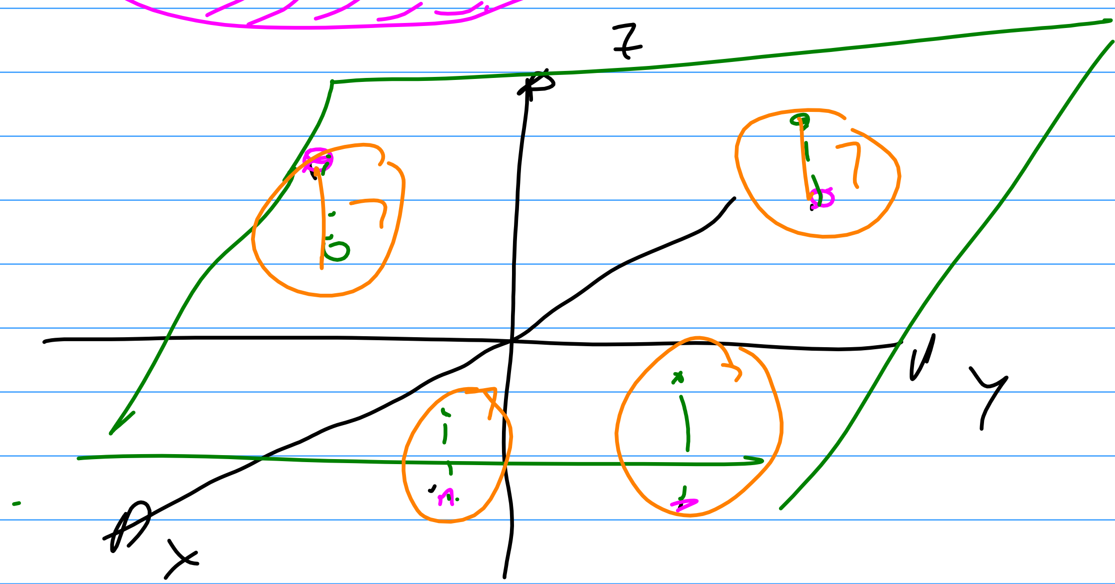
$$\underbrace{\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} z_1 \\ \vdots \\ z_4 \end{pmatrix}}_{A^T \begin{pmatrix} z_1 \\ \vdots \\ z_4 \end{pmatrix}} = \begin{pmatrix} z_1 - z_3 \\ z_2 - z_4 \\ z_1 + z_2 + z_3 + z_4 \end{pmatrix}$$

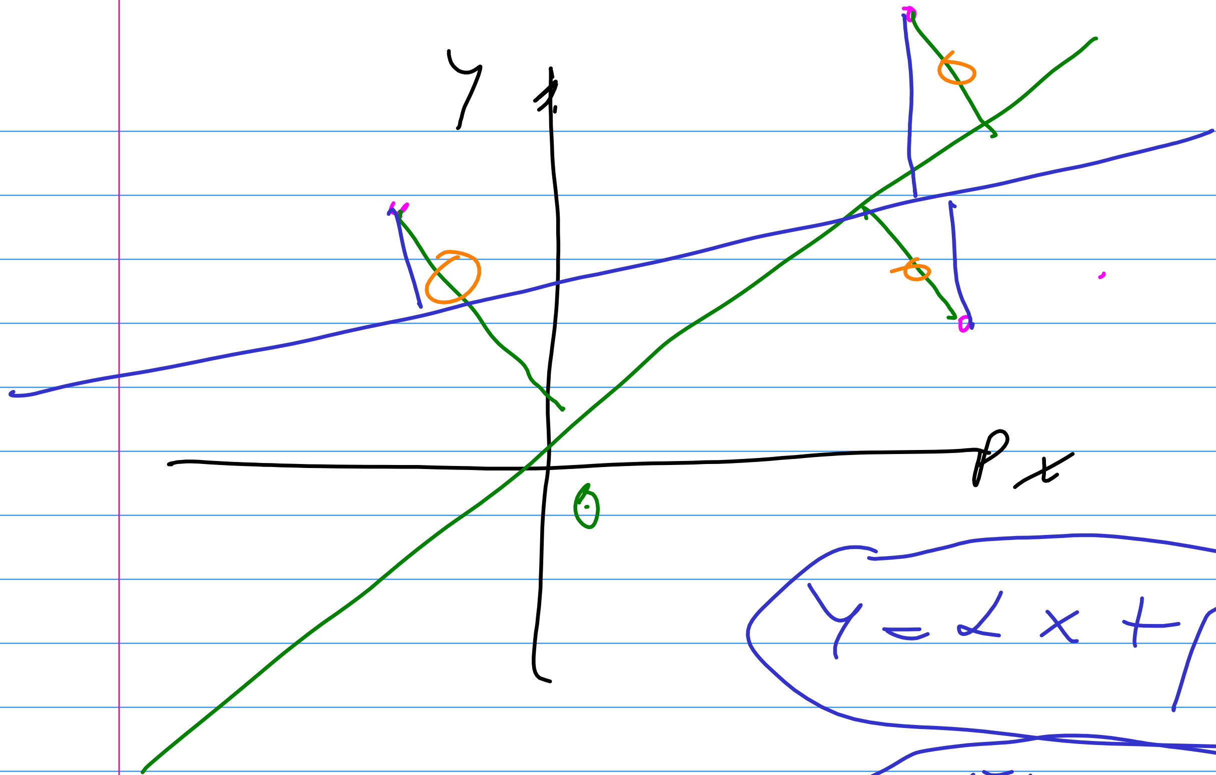
$$\begin{pmatrix}\tilde{\alpha}\\ \beta\\ \gamma\end{pmatrix} = \begin{pmatrix}\frac{1}{2} & 0 & 0\\ 0 & \frac{1}{2} & 0\\ 0 & 0 & \frac{1}{4}\end{pmatrix}\begin{pmatrix}z_1 - z_3\\ z_2 - z_4\\ z_1 + z_2 + z_3 + z_4\end{pmatrix}$$

$$\begin{pmatrix}\tilde{\alpha}\\ \beta\\ \gamma\end{pmatrix} = \begin{pmatrix}\frac{1}{2}(z_1 - z_3)\\ \frac{1}{2}(z_2 - z_4)\\ \frac{1}{4}(z_1 + z_2 + z_3 + z_4)\end{pmatrix}$$

$$\boxed{z = \tilde{\alpha} x + \tilde{\beta} y + \tilde{\gamma}}$$

$x = 0, y = 0 \rightarrow \bar{z} = \frac{1}{4}(z_1 + \cdots + z_4)$





$Y = \alpha x + \beta$

A^{-1}, U^{-1}

$A^{-1} = U^{-1}(U^{-1})^T$

veel te veel werk

(2) (2·02·1)

$A = (U^T U)$??

$A^{-1} = (U^{-1})(U^{-1})^T$

$A = U^T U$

$A^{-1} = (U^T U)^{-1} = U^{-1}\cdot(U^T)^{-1} = U^{-1}\cdot(U^{-1})^T =$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix}$$

$A \cdot A^{-1} = I$

$((U^T U)(U^{-1}(U^{-1})^T)) = I$

$I \quad (U^T)^{-1}$

$U^T U\, U^{-1}(U^{-1})^T = A^{-1}$

$U^T \cdot (U^{-1})^T = U^T (U^T)^{-1} = I$

$\boxed{(U^{-1})^T = (U^T)^{-1}}$

4 ew

4 x 4

ew: U : $\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 1, \lambda_4 = 1$

$|U - \lambda I| = 0$

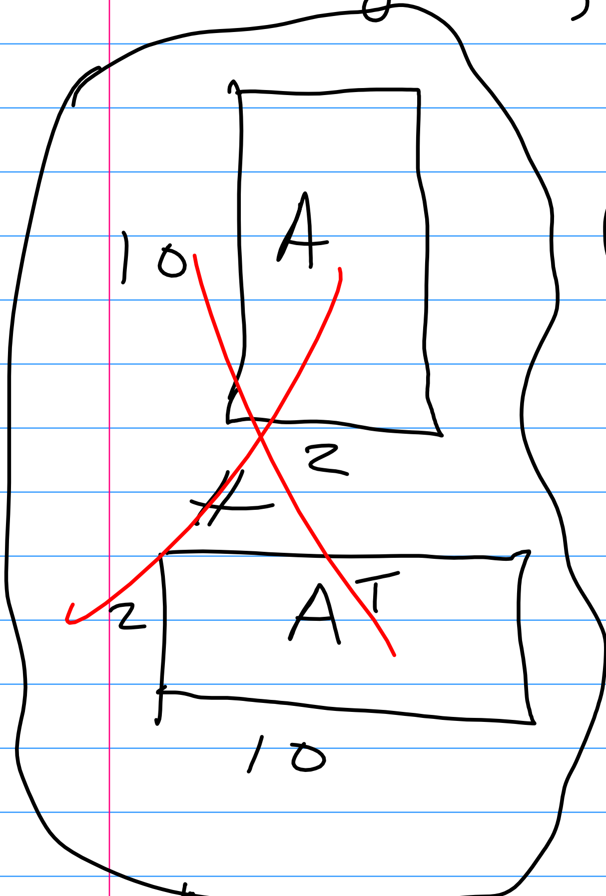
$\lambda_1 = 1$ (multipl 4)

$(1-\lambda)(1-\lambda)\cdots(1-\lambda) = 0$

A symm, $\boxed{A\ (n \times n)}$

$$\boxed{A^T = A}$$

symm



$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

(spiegel)

ANS

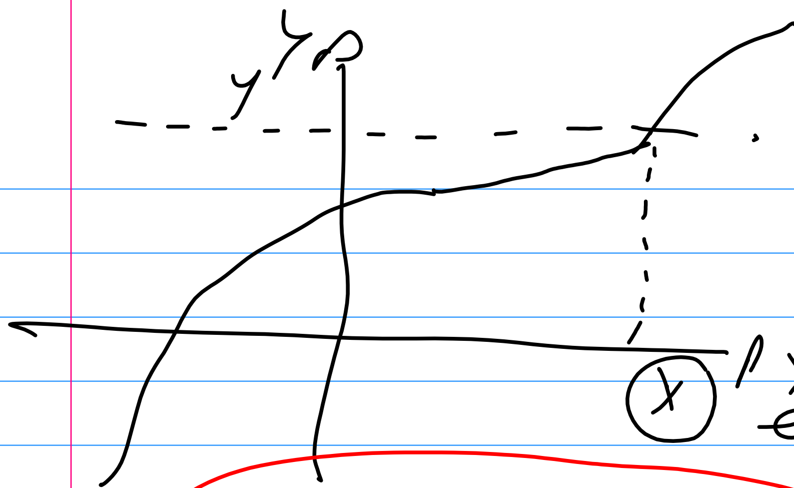
$Ax = Ay \Rightarrow x = y$

dan is A injectief

enige opl

$\boxed{Ax = 0} \Rightarrow$ enige opl. $x = 0$

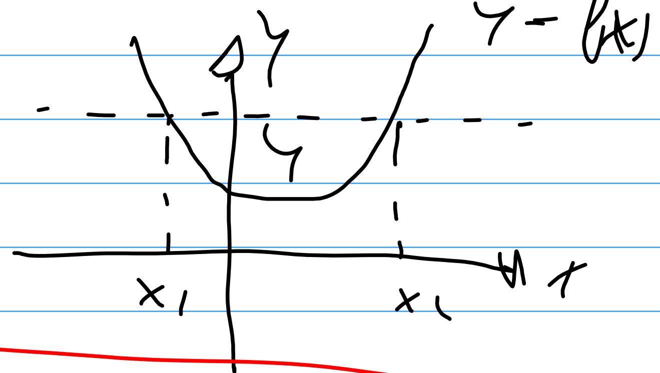
dan A injectief.



$y = f(x)$

wel inj.

$!x$ énige x met $f(x) = y$.



f niet inj.

defective matrix → (331)

A is defective $(n \times n)$

minder dan n lin onafh. eigenv

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$ (is defective)

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} = 0 \quad (1-\lambda)^2 = 0$$

$$\lambda_1 = 1, \quad \lambda_2 = 1$$

$$\boxed{\lambda_1 = 1 \ (\text{mult } 2)}$$

$\underline{\underline{E_1}}$?? $(A - 1 \cdot I)x = 0$

$$\left(\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad y = 0$$

$\underline{\underline{ev}}$: $\begin{pmatrix} x \\ 0 \end{pmatrix} = x \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\boxed{E_1 = \left\langle \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\rangle}$$

2x2

1 lin. onafh. ev

$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is defective

onvolkomen / $\boxed{\text{defecte}}$

niet op diagonaalgedaante te brengen

(6.3)(1 f.) ew.? en ev?

1) $|A - \lambda I| = 0$

$$-2 \qquad \begin{vmatrix} (1-\lambda) & 2 & -1 \\ 2 & (4-\lambda) & -2 \\ 3 & 6 & -3-\lambda \end{vmatrix} = 0$$

$\downarrow$

$$-\lambda(\lambda^2 - 2\lambda + 6) = 0$$

$\boxed{\lambda = 0}$ $\quad$ $\boxed{\lambda^2 - 2\lambda} + 6 = 0$

$$\boxed{(\lambda - 1)^2 - 1} + 6 = 0$$

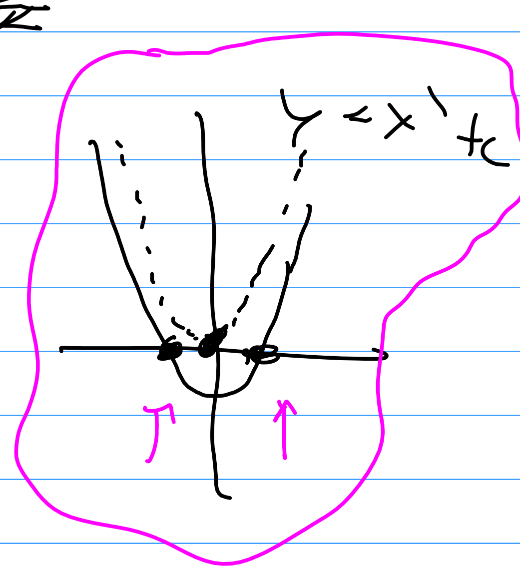
$$(\lambda - 1)^2 + 5 = 0$$

$$(\lambda - 1)^2 = -5$$

$\lambda^{10} = 0$ $\qquad$ $\lambda = 0$

10 keer

$$\boxed{\lambda^{10} = \frac{1}{10}}$$



$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{pmatrix}$$

-1

$2\times$

$A\underline{x}$ = lin comb. van kolom

$\underline{\lambda = 0}$; $A\underline{x} = \underline{0}$

$$A\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = 2\cdot\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - 1\begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} = \underline{0}$$

lin onafh

$$\underline{\lambda = 0} \quad A\begin{pmatrix} 1 \\ 0 \\ +1 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + 1\begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix} = \underline{0}$$

$\boxed{\underline{x} \neq 0}$ $\quad A\underline{x} = \underline{0} = 0\cdot\underline{x}$

$N(A) = \{x \mid Ax = 0\}$

$\underline{Ax = 0\cdot x = 0}$

(eigenruimte met $\lambda = 0$.)

(1f) $\boxed{\lambda^2(\lambda \ldots\ldots) = 0}$ $\quad \boxed{2\lambda^2 - \lambda^3 = 0}$

$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1^2 & 0 \\ 0 & 2^2 \end{pmatrix}$$

pg (330) onderaan:

$$D = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$D^3 = \begin{pmatrix} 1 & 0 \\ 0 & 8 \end{pmatrix}$$

?

$$X D^k X^{-1} = X D X^{-1}$$

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

opg 7

$$A^6 = X D X^{-1} \text{ ??}$$

$$A = X D X^{-1}$$

$$A^2 = (X D \underbrace{X^{-1})(X}_{I} D X^{-1}) =$$

$$X D I D X^{-1} = X D^2 X^{-1}$$

$$\underline{A^6 = X D^6 X^{-1}}$$

③ $A = X \Lambda X^{-1}$

$A^{-1} = (X \Lambda X^{-1})^{-1} =$

$(A \cdot B)^{-1} = B^{-1} A^{-1}$ $\quad (X^{-1})^{-1} \cdot \Lambda^{-1} \cdot X^{-1} =$

$(AB)^T = B^T A^T$

$\boxed{A^{-1} = X \Lambda^{-1} X^{-1}}$

$$\begin{pmatrix} \lambda_1 & & \Theta \\ & \ddots & \\ \Theta & & \lambda_n \end{pmatrix}$$

ew

A heeft geen ew $= 0$

$\lambda_i \neq 0$ allemaal

$|\Lambda| = 0 \sim$ is een $\lambda_i = 0$

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tegenvoorbeeld

$(\cdot \; - \; - -) \not\Rightarrow (- \; -)$

niet waar

Bewijs $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}^n = I \quad \forall n \geq 0$

$n=1$ $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I \neq I$

gevraagde is onwaar

(6.1)
(33)

A, B $n \times n$ matrices

$10^{10} \times 10^{10}$

a) $\lambda \neq 0$, λ ew van AB

$\Rightarrow \lambda$ is ook een ew van (BA)

$|A \cdot B - \lambda I| = 0$

weet niet hoe?

?? $|BA - \lambda I|$

is het waar ??

b) $\lambda = 0$ is een ew van AB

$|B||A| = |A||B| = |AB| = 0$

$\quad = |BA|$

$|BA| = 0 \Rightarrow$ er is een ew $\lambda = 0$

(AB) $\qquad$ $(\lambda_i \neq 0)$

$$|AB| = |D| = \lambda_1 \cdots \lambda_n$$

$$\shortparallel$$

$$|BA|$$

$$\boxed{AB\underline{x} = \lambda \underline{x} \quad (\lambda \neq 0)}$$

?? BA

$$\boxed{AB \neq BA}$$

?

* A

Ga na ofad ?
woorx is

$$D = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$$

$$S^{-1} A S = D$$

$$A = \boxed{S D S^{-1}}$$

[illegible]

$\lambda \neq 0$, ew van AB dan

$\exists x$ met $(AB)x = \lambda x \Rightarrow$

$A(Bx) = \lambda x$

<u>Neem</u>: $y = Bx$ dan

$A(y) = A(Bx) = (AB)x = \lambda x \neq 0$

$(BA)(y) = B(A(Bx)) =$

$B((AB)x) = B(\lambda x) = \lambda(Bx)$

$(BA)((Bx)) = \boxed{\lambda}(Bx)$

$\Rightarrow \lambda$ ew van BA