2018) 1g.

$A^2 - 15 \cdot A - 18 I$ ew

$(A^2 - 15 A - 18 I) - \lambda I$

$A: \left(0, \frac{1}{2}\left(15 \pm \sqrt{297}\right)\right)$

$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ 3 versch. ew → bijbeh

3 ev

$D = S^{-1} A S$

$A = S D S^{-1}$

$A^2 = S D S^{-1} S A S^{-1} = S D^2 S^{-1}$

$B = (A^2 - 15A - 18 I) =$

$S D^2 S^{-1} - 15 (S D S^{-1}) - 18\, S \cdot S^{-1}$

$= S\left((D^2 - 15 D - 18 I)\right) S^{-1}$

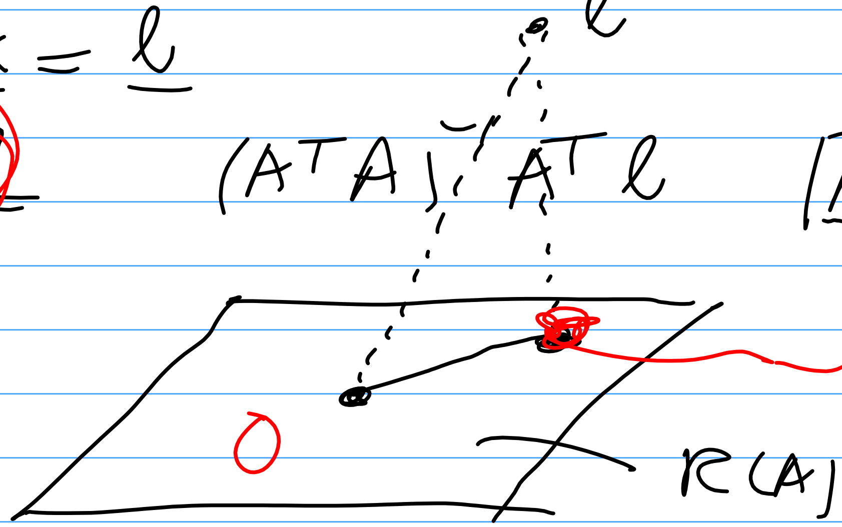
→ diag. gedaante van B

$D_B = S^{-1}(A^2 - 15A - 18 I) S$

Projectie op kolomruimte van A;

$A\underline{x} = \underline{b}$

$\underline{\tilde{x}} = (A^TA)^{-1}A^Tb$ $\qquad A\underline{\tilde{x}}$



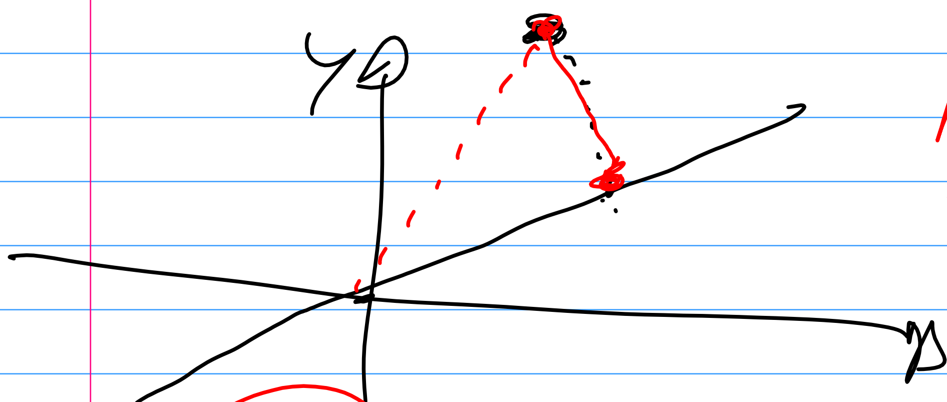
$A\tilde{x}$ projectie op kolom.

$$A(A^TA)^{-1}A^T$$

projectie matrix → op kolomruimte van A

$$P^2 = P$$

$$A(A^TA)^{-1}A^TA(A^TA)^{-1}A^T$$



$A = \begin{pmatrix} a \\ b \end{pmatrix}$

$A\underline{\tilde{x}}$ — proj van b op RA

$A\underbrace{(A^TA)^{-1}A^T b}_{x}$ — proj. matrix op $R(A)$

(1d) $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ $\quad b = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$

$u \in N(A^T)$ $\qquad u \perp$ kolomruimte van A

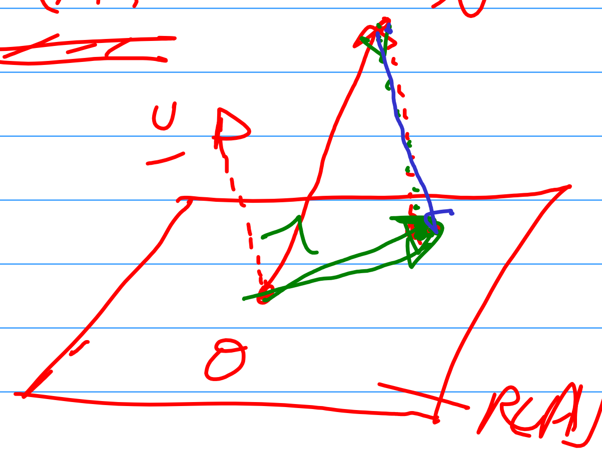
$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \end{pmatrix} = (1 \; 2) \begin{pmatrix} 3 \\ 4 \end{pmatrix} =$

$A^T = \begin{pmatrix} | & | & | \end{pmatrix}^T = \begin{pmatrix} \text{—} \\ \text{—} \\ \text{—} \end{pmatrix} \begin{pmatrix} u \end{pmatrix}$

$0 = A^T u$ $\qquad u \in N(A^T)$ — 1 dim

$u \perp R(A)$ $\qquad b$

$(b - \lambda u)$



$A(A^TA)^{-1}A^T$

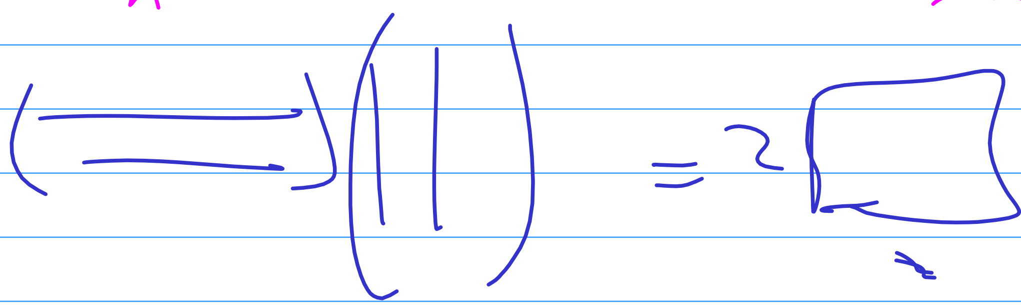
$A = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \left((1 \; -2 \; 1) \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right)^{-1} (1 \; -2 \; 1)$

$$A(A^TA)^{-1}A^T \ell$$

$R(A) = \begin{pmatrix}1\\-2\\1\end{pmatrix}$ → 1 dim

ℓ projecteren op lijn

$$\Big(\underbrace{(1\ \ -2\ \ 1)}_{A^T}\begin{pmatrix}1\\-2\\1\end{pmatrix}\Big)^{-1} = (6)^{-1} = \frac{1}{6}$$



A defective → (1 ew)

(λ ew van A
2 voudig)

$(A - \lambda I)\underline{x} = \underline{0}$ → ev

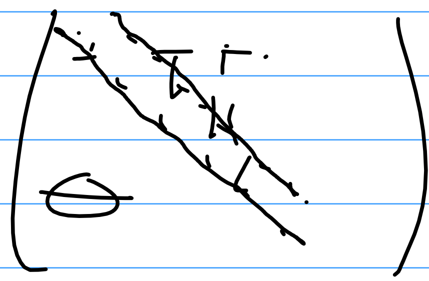
$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad |A - \lambda I| = 0 \quad (1-\lambda)^2 = 0$$

$\lambda_1 = 1, \ \lambda_2 = 1$

$$(A - 1 \cdot I) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \underline{x} = \underline{0}$$

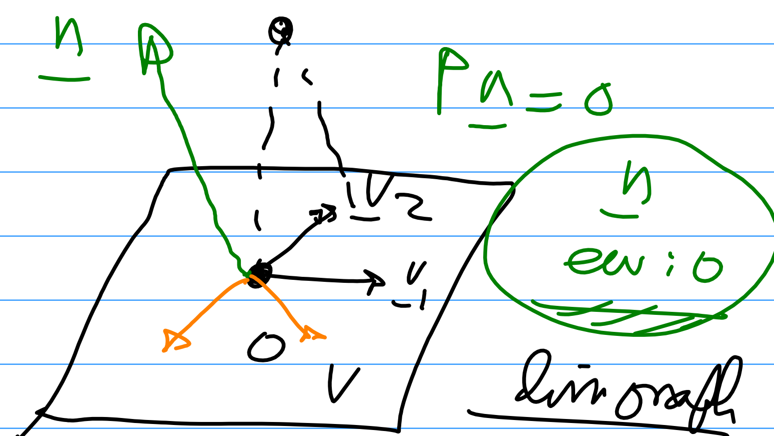
$x_1, \ x_2 = 0$

ev $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad A \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$



Jordan normalform

Projektio



$P\underline{v}_1 = \underline{v}_1$

$P(\underline{v}_2) = \underline{v}_1 \quad \lambda = 1 \ (m: 2) \rightsquigarrow 2$ ev

lin. riipp.

$$\lambda^2 - 15\lambda - 18I$$

$$|A^2 - 15A - 18\lambda I| = 0$$

$$(A^2 - 5A - 8I)\,\underline{v} =$$

$$(\lambda^2 - 5\lambda - 8$$

$$A(A^TA)^{-1}A^T \to \underline{R(A)}$$

$$\underline{v} \perp R(A) \quad (\underline{b - \lambda v}, \underline{v})$$

$$\underline{v} \perp R(A)$$

$$(\underline{b} - \lambda \cdot \underline{v}) \cdot \underline{v} = 0$$

$$\underline{b} \cdot \underline{v} - \lambda\, \underline{v} \cdot \underline{v} = 0$$

$$\lambda = \frac{b \cdot v}{(\underline{v} \cdot \underline{v})}$$

$$\left(b - \frac{b \cdot v}{v \cdot v} \cdot \underline{v}\right)$$

2018 $B = A^2 - 15A - 18I$

$v \in N(A) \Rightarrow$

$A\underline{v} = 0 \Rightarrow A^2 v = A(Av) = 0$

$$\boxed{B(\underline{v}) = -18 \cdot v}$$

$E_B(-18) = \langle \underline{v} \rangle$

$N(A) = \left\langle \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\rangle$ ✓

$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ ✓

$\frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ 2 & -1 & -2 \end{pmatrix} \frac{1}{3}\begin{pmatrix} 1 & 2 \\ -2 & -1 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

I $\quad$ $R(A)$

$P = Q(Q^T \cdot Q)^{-1} Q^T$ $\quad$ on $R(A)$

$P = Q \cdot Q^T$

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column space: $B = A^2 - 15A - 18I$??

$\underline{v} \in N(A)$

$B\underline{v} = -18v$; v ev met ew -18

N(A) ??

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 0 \\ 0 & -6 & -12 & 0 \end{array}\right)$$

$$\left((A^T A) \stackrel{?}{=} I \right)$$

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 8 \\ 8 & 14 \end{pmatrix}$$

$(A^T A) = I \rightarrow$

kolommen $\perp$ op elkaar en lengte 1

Q orthogonale matrices

$B = (A^2 - 15A - 18I)$

$\frac{2018}{19}$

$A\underline{w} = \mu \underline{w}$

$B\underline{w} = (\mu^2 - 15\mu - 18)\underline{w}$ $= 0$

A een eigenv. met eW. μ

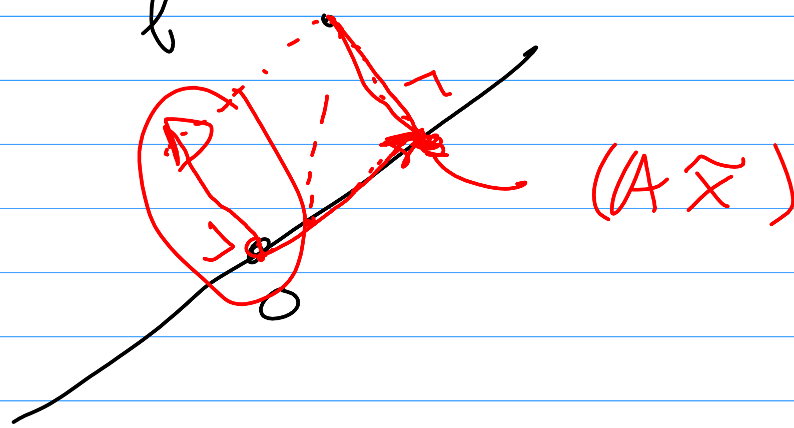
$A\underline{w} = \mu\underline{w} \Rightarrow$

$(A - \lambda I)\underline{w} = \underline{0} \Rightarrow$

$|A - \lambda I| = 0 \Rightarrow$

$\mu(\mu^2 - 15\mu - 18) = 0$

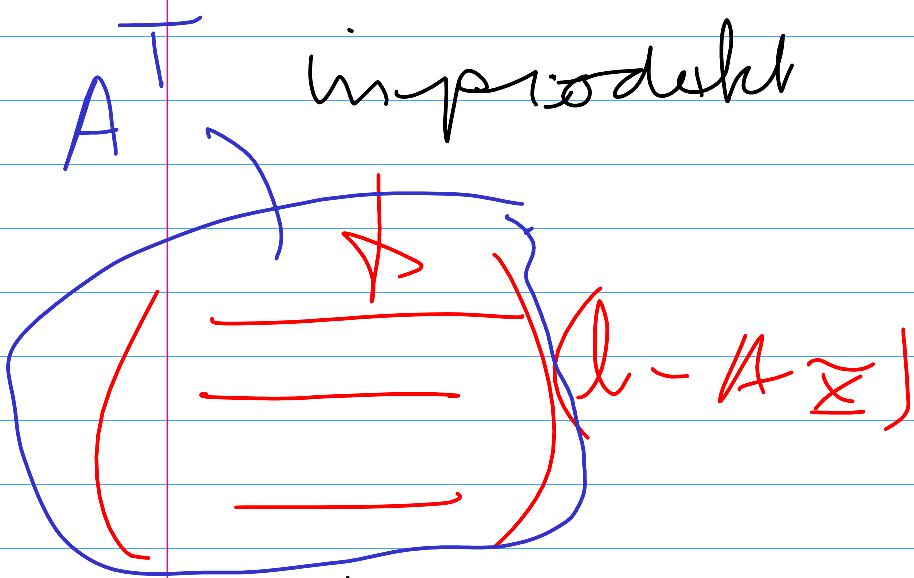
$$\frac{(l - A\bar{x}) \perp R(A)}{l}$$



$A = (\;|\;|\;|\;|\;) \rightarrow R(A)$

$(l - A\tilde{x}) \perp$ alle kolomme van A

inproduct $x \cdot y = x^T \cdot y$



$A^T(l - A(A^TA)^{-1}A^Tl) =$

$$A^T l - (A^T A)(A^T A)^{-1} \cdot A^T l = 0$$

I

$$l - A\tilde{x} \perp R(A)$$

$l - A\tilde{x}$

vector

$$\|x\|^2 = x^T \cdot x$$

$$\|l - A\tilde{x}\|^2 \leq \|l - Ax\|^2$$

$$(l - A\tilde{x})^T (l - A\tilde{x}) =$$

$$l^T l - l^T A \tilde{x}$$

$$-\tilde{x}^T \cdot A^T + \tilde{x} A^T A \tilde{x}$$

$$\tilde{x} = (A^T A)^{-1} A^T l$$

$$\tilde{x}^T = l^T \cdot A \cdot ((A^T A)^{-1})^T$$

$$(A\tilde{x})^T = \tilde{x} \cdot A^T = l^T A((A^TA)^{-1})^T \cdot A^T$$

$$(l^T - l^T A((A^TA)^{-1})^T A^T)(l - A(A^TA)^{-1}A^T l)$$

$$l^T l - l^T A(A^TA)^{-1}A^T l$$

$$- l^T A((A^TA)^{-1})^T A^T l$$

$$+ l^T A((A^TA)^{-1})^T A^T A (A^TA)^{-1} A^T l =$$

$$l^T (I - A(A^TA)^{-1}A^T) l$$

$$(l - Ax)^T (l - Ax) =$$

$$l^T l - l^T A x - x^T A^T l$$

$$+ x^T A^T A x$$

2019

$$P = Q (Q^TQ)^{-1} Q^T \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$Q^TQ = \frac{1}{9}\begin{pmatrix} 1 & -2 & 2 \\ 2 & -1 & -2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -2 & -1 \\ 2 & -2 \end{pmatrix}$$