(filename: CAI-200907.pdf)

This I also wanted to do the week before! I wanted to use: Xournal. File can be put at internet. Look at: www.win.tue.nl/~rwhassel and maybe of interest, the same server, but then $\boxed{\text{~sjoerdr}}$ ←

(— wikipedia convergence tests)

Always first try to do it yourself!!

I searched: TUE 2DM30

(1.3) a) $|z_1 + z_2| \leq |z_1| + |z_2|$

$\rightsquigarrow |z_1 - z_2| \geq \big||z_1| - |z_2|\big|$

$|z^2 + 2z + 3i| \geq \big||z^2| - |-2z - 3i|\big|$

$\big|(z^2 + 2z) - (-3i)\big| \geq$

$\big||z^2 + 2z| - |-3i|\big| \geq$

$$\big|\,|z^2| - |-2z| - 3\,\big| =$$

$|z| = R \qquad |z^2| = R^2$

$$|\underbrace{R^2 - 2R}_{>0 \text{ if } R>3} - 3| = \underline{R^2 - 2R - 3}$$

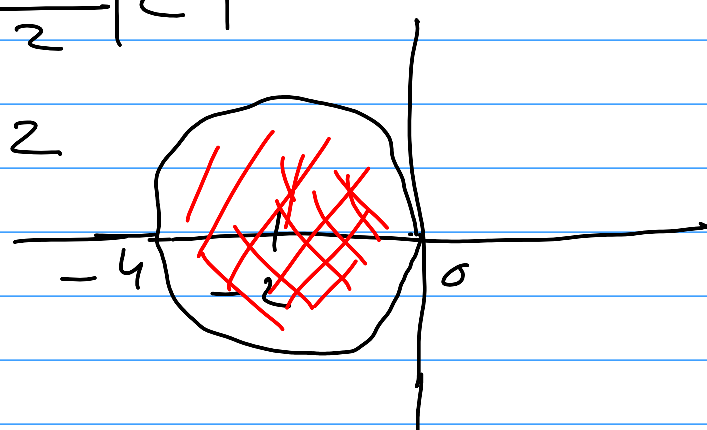
1 b) $\left\{ \begin{array}{l} |\ldots| \geq a \\ \dfrac{1}{|\ldots|} \leq \dfrac{1}{a} \end{array} \right.$?

(9.5) (3) adams

$$\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{x+2}{2} \right)^n \qquad (x \in \mathbb{C})$$

Ratio test: $\left| \dfrac{x+2}{2} \right| < 1$

$|x+2| < 2$



(9.4) (8) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{-n}{n^2+1} \right)$

converges ?? $-\sum_{n=0}^{\infty} \frac{n}{(n^2+1)} = -\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

$\frac{n}{n^2+1} \simeq \frac{1}{n}$ $\qquad \sum_{n=0}^{\infty} \frac{n}{n^2+1}$

$\frac{n}{n^2+1} > \boxed{\frac{1}{(n+1)}}$ $\qquad \cancel{n^2 > n^2+1}$

$n(n+1) > n^2+1$ & if $\underline{n \geq 1}$

$\sum \left(\frac{1}{n+1}\right)$ <u>diverges.</u> $\qquad \sum_{n=1}^{\infty} \frac{n}{n^2+1} > \sum_{n=1}^{\infty} \frac{1}{n+1}$

$\Rightarrow$ Taylor expansion around $x=0$:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$\rightsquigarrow f(0) + f'(0)\cdot x + \frac{f''(0)}{2}\cdot x^2 + \cdots + \frac{f^{(n)}(0)}{n!} x^n$

$\rightsquigarrow \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$

$\rightsquigarrow \begin{cases} \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \\ \cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \cdots \end{cases}$

(9.5) (3) adams

$$\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x+2}{2}\right)^n = f(x)$$

just example:

$$f'(x) = \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{x+2}{2}\right)^{n-1} =$$

$$\frac{1}{2}\cdot\sum_{n=0}^{\infty}\left(\frac{x+2}{2}\right)^n = \frac{1}{2}\cdot\frac{1}{1-\left(\frac{x+2}{2}\right)} =$$

$$\frac{1}{2}\cdot\frac{2}{-x} = -\frac{1}{x}$$

$$f(x) = -\ln|x| + C_1$$

$$f(-2) = 0 \Rightarrow C_1 = \ln(2)$$

$$f(x) = \ln\left(\frac{2}{|x|}\right)$$

you have not to do at this moment

$$\left(\sum_{n=1}^{\infty} a^{n} \right) \ell^{m}$$

$$\boxed{\sum_{n=1}^{\infty} c^{n} \cdot a^{n}}$$ convergent ??

c is known, a is unknown

$$c^{n} \cdot a^{n} = (c \cdot a)^{n}$$

$$\boxed{\sum x^{n} \text{ converge}}$$ $|x| < 1$

$$x^{2} \cdot x^{2} = x^{4} \qquad x^{2} \cdot x^{3} = x^{5}$$

$$c^{5} \cdot x^{5} = c \cdot c \cdot c \cdot c \cdot c \cdot x \cdot x \cdot x \cdot x \cdot x$$

$$(cx) \cdot (cx) \cdots (cx) = (cx)^{5}$$

$$|ca| < 1 \quad \Rightarrow \quad \boxed{|a| < \frac{1}{|c|}}$$

(9.4)(8) $\displaystyle\sum_{n=1}^{\infty} \frac{(4x-1)^{n}}{n^{n}}$

$\cancel{\dfrac{(n+1)^{n+1}}{n^n}}$ $\displaystyle\lim_{n\to\infty} \left(\left|\frac{4x-1}{n}\right|^{n}\right)^{\frac{1}{n}} =$

$\displaystyle\lim_{n\to\infty} \frac{|4x-1|}{n} = 0 < 1$

always

$\displaystyle\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{n} \sim e$

$\displaystyle\left(\lim_{n\to\infty}\right) e^{\ln\left(\left(1+\frac{1}{n}\right)^{n}\right)} = e^{n\cdot\underbrace{\ln\left(1+\frac{1}{n}\right)}_{\sim\frac{1}{n}\ (n\to\infty)}} = e^{1}$

$\displaystyle\frac{(n+1)^{n+1}}{n^{n}} = \underline{\underline{(n+1)}}\cdot\underbrace{\left(1+\frac{1}{n}\right)^{n}}_{e} \to \underline{\underline{\infty}}$

$\displaystyle\left(\frac{n+1}{n}\right) = \left(1+\frac{1}{n}\right)$

Ratio test: $\displaystyle |4x-1| \left(\frac{n}{n+1}\right)^{n} \frac{1}{n+1} \to 0$

$\displaystyle\left(\frac{n}{n+1}\right)^{n} \to \left(\frac{1}{e}\right)\ \underline{\underline{n\to\infty}}$

always converges.

(9.5) (12)

$$\frac{1}{1-x} = 1 + x + x^2 \cdots = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{2-x} = \frac{1}{2} \frac{1}{\left(1 - \left(\frac{x}{2}\right)\right)} =$$

$$\frac{1}{2} \cdot \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n \qquad x^n$$

$$\left|\frac{x}{2}\right| < 1 \qquad |x| < 2$$

$$\frac{1}{2-x} \qquad x = 4 \text{ goes well} \qquad x^{-n}$$

$$\frac{1}{2-x} = \frac{1}{x} \cdot \frac{1}{\left(\frac{2}{x} - 1\right)} =$$

$$-\frac{1}{x} \cdot \frac{1}{1 - \left(\frac{2}{x}\right)} = -\frac{1}{x} \cdot \sum_{n=0}^{\infty} \left(\frac{2}{x}\right)^n \; ?$$

$$\left|\frac{2}{x}\right| < 1 \Rightarrow |x| > 2$$

$$\frac{1}{(2-x)^2} \xleftarrow{\frac{d}{dx}} \frac{1}{(2-x)}$$

$$\frac{1}{1-x}$$

$$\ln(2-x) \leftarrow \int \cdots dx$$

$$\frac{1}{2-x} = \frac{1}{2} \cdot \sum \left(\frac{x}{2}\right)^n$$

$$\ln(2-x) \overset{??}{=} -\frac{1}{2} \cdot \sum_{n=0}^{\infty} \frac{1}{n+1} \left(\frac{x}{2}\right)^{n+1} + C ??$$

$$\frac{-1}{2-x}$$

$$\rightarrow \ln(1) = 0 \quad x = 1$$

$$w = (a + ib)$$
$$z = (x + iy)$$

(1.1) (g):

$$\rightarrow z \cdot \overline{w} - \overline{z} \cdot w = 0 \qquad z = c \cdot w$$

$$z = \left(\frac{z}{\overline{w}}\right) \cdot \overline{w}$$

$\in \mathbb{R}$??

$$w\left(\frac{\bar z}{\bar w}\right)\cdot \bar w - \overline{\left(\frac{\bar z}{\bar w}w\right)}\cdot w = 0$$

$$\left(\frac{\bar z}{\bar w}\right)\bar w\cdot w - \overline{\left(\frac{\bar z}{\bar w}\right)}\cdot \bar w\cdot w = 0$$

$z \neq 0,\ w \neq 0$

$$\left(\frac{\bar z}{\bar w}\right) = \overline{\left(\frac{\bar z}{\bar w}\right)} \qquad \frac{\bar z}{\bar w} \in \mathbb{R}$$

$$\bar z = z \rightarrow z \in \mathbb{R}$$

$$z\cdot \bar w - \bar z\cdot w = 0$$

$$z\cdot \bar w = \bar z\cdot w$$

$$\frac{z}{w} = \frac{\bar z}{\bar w} = \overline{\left(\frac{z}{w}\right)}$$

$$\frac{z}{w} \in \mathbb{R} \qquad \frac{\bar z}{\bar w} \qquad z = \frac{\bar z}{\bar w}\cdot w$$

c

(1.) (b) $(z-1)^4 = 0$

$z = 1 \quad (4x)$

$1 \cdot z^4 + 2 z^2 + 4 = 0$ $\quad \bar{z}$ also solution

$w = z^2 \qquad w^2 + 2w + 4 = 0$

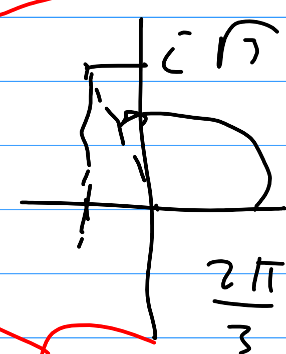
$z = \sqrt{2} \cdot e^{i\frac{\pi}{3}}$

$(w+1)^2 = -3$

$w + 1 = \pm i\sqrt{3}$

$w = -1 + i\sqrt{3} = z^2$

$w = -1 - i\sqrt{3} = z^2$ $\quad 2$



$z = R \cdot e^{i\varphi}$ $\quad 2$

$|z^2| = |-1 - i\sqrt{3}| = 2$

$\arg(-1 + i\sqrt{3}) = 2 \cdot \arg(z)$
$+ k\, 2\pi \qquad k \in \mathbb{Z} \qquad k = 0,\ k = 1$

$1 z^4 + 2z^2 + 4 = \bar{0} \qquad (\bar{z})^4 + 2(\bar{z})^2 + 4 = 0$

(9.5) (1)

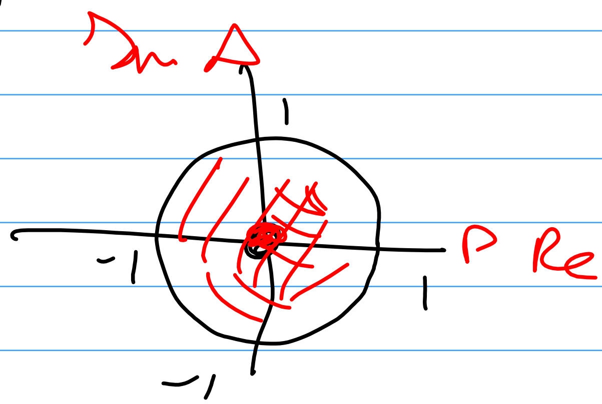
$$\sum_{n=0}^{\infty} \frac{x^{2n}}{\sqrt{n+1}}$$ Ratio

$|x^2| < 1$

$|x - 0| < 1$ $\quad |x| < 1$

center radius

$$\sum_{n=0}^{\infty} \frac{z^{2n}}{\sqrt{n+1}}$$ $|z| < 1$



(17) $\dfrac{1}{x^2}$ in powers of $(x+2)$

$\dfrac{1}{x}$ in powers $(x+2) = y$

$x = (-2+y)$

$$\frac{1}{x} = \frac{1}{(-2+y)} = \frac{-1}{(2-y)}$$

$$\rightarrow (y = (x+2))$$

$$\frac{1}{x^2} = \frac{1}{(-2+y)^2} = \frac{1}{(2-y)^2}$$