Questions? (file: CAJ - 200909)

(See my website)

(...../~ wharsel)

1

$3+4x+5x^2+\cdots=$

$\sum_{n=0}^{\infty}(n+3)x^n$

Ratio: $|x|<1$

?

Sum $(n+3)x^n=$

$\rightsquigarrow x^{(n+3)}$?

$\frac{d}{dx}(x^{n+3})=(n+3)\cdot x^{(n+2)}$

$=\left(\frac{1}{x^2}\right)\sum_{n=0}^{\infty}(n+3)x^{(n+2)}$

$=\frac{1}{x^2}\cdot\sum_{n=0}^{\infty}\frac{d}{dx}\left(x^{(n+3)}\right)=$

2

$$= \frac{1}{x^2} \cdot \frac{d}{dx}\left(\sum_{n=0}^{\infty} x^{(n+3)} \right) =$$

$$\sum x^n$$

$$= \frac{1}{x^2} \cdot \frac{d}{dx}\left(x^3 \cdot \sum_{n=0}^{\infty} x^n \right)$$

$$= \frac{1}{x^2} \cdot \frac{d}{dx}\left(\frac{x^3}{(1-x)} \right)$$

$$\frac{1}{x^2} \cdot \frac{d}{dx}\left(x^3 \cdot \left(\frac{1-x^{n+1}}{1-x} \right) \right)$$

$(n \to \infty)$ $\quad |x^{n+1}| \to 0 \quad |x|<1$

$$\left(\sum_{n=1}^{\infty} \rightsquigarrow \left(\sum_{n=0}^{N} \right) \quad N \to \infty \right)$$

$\rightsquigarrow$ correction video - lect 4 hour 2

3

$$|z^2+2z+3i| \geq \cdots\cdots$$

$$|z_1+z_2| \leq |z_1|+|z_2|$$

2x

$$|z_1+z_2| \geq \big||z_1|-|z_2|\big|$$

$$|(z^2+2z)+(3i)| \geq$$

$$\big||z^2+2z|-|3i|\big| \geq$$

$$\big|(|z^2|-|2z|)-|3i|\big|$$

$$|z^2+(2z+3i)| \geq$$

See difference!!

$$\big||z^2|-|2z+3i|\big| =$$

$$\big||2z+3i|-|z^2|\big| \geq$$

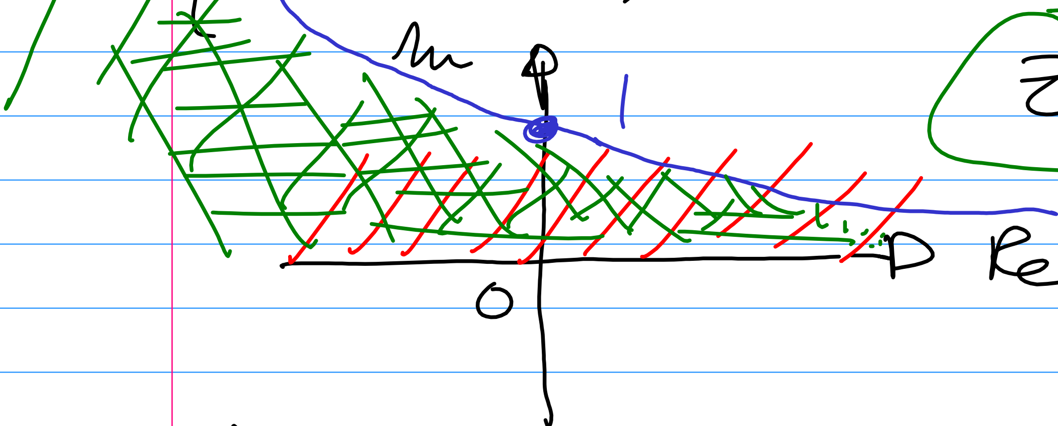
$$\big||2z|-|3i|-|z^2|\big| =$$

$$\big||z^2|-|2z|+|3i|\big| =$$

$$(R^2-2\cdot R+3)$$

$$\begin{cases} Im(z) \geq 0, \\ Im(z) \leq e^{(-Re(z))} \end{cases}$$

$$\boxed{z = x + iy}$$



$$Im(z) = y \geq 0$$

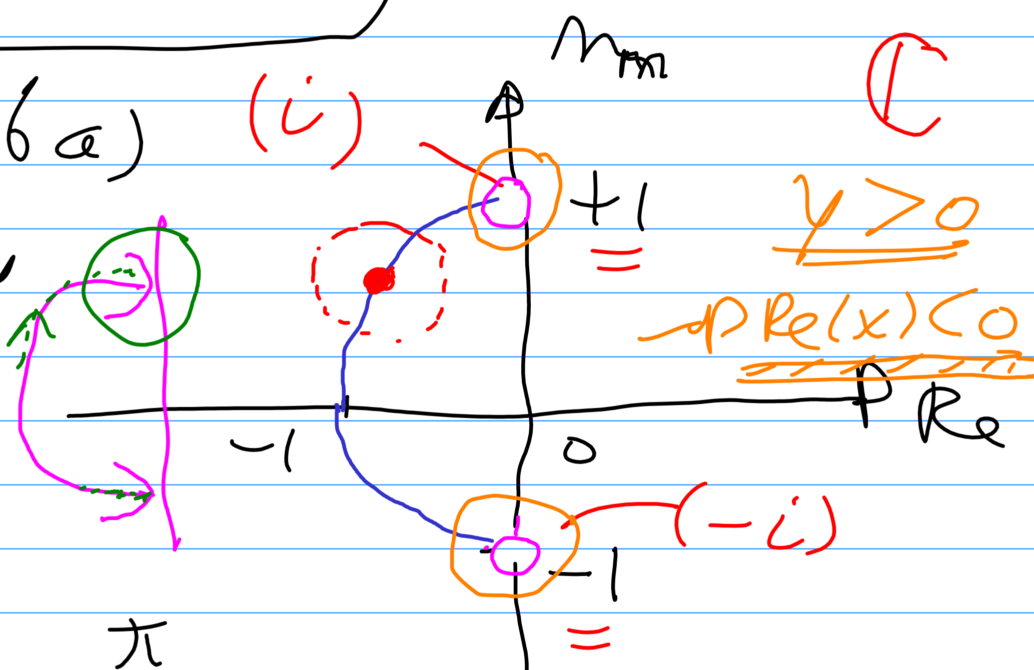
$$\boxed{y \leq e^{-(x)}} \qquad y = e^{-x}$$

(1.2) (6a)

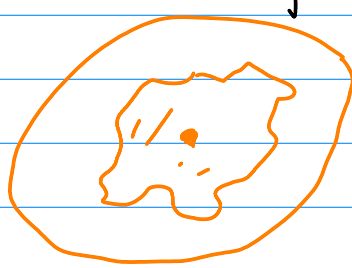
→ $z = x + iy$

$\underline{Im(z)} = y$

$y \in \mathbb{R}$



$$\arg\left(\frac{z - i}{z + i}\right) = \frac{\pi}{2}$$



(1,4) (g)

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

C.R. ?? Cauchy Riemann

$\rightsquigarrow f(z) = f(x+iy) = u(x,y) + i\,v(x,y)$

CR.

$$\boxed{\begin{aligned} u_x &= v_y \\ u_y &= -v_x \end{aligned}}$$

$-i$

!

$h \in \mathbb{C}$

$h = h_1 + h_2 \cdot i$

$$\lim_{h\to 0} \frac{f(-i+h)-f(i)}{h}$$

$z = -i + h_1 + ih_2$

$$f(-i+h) - f(-i)$$

$$((-i+h)\cdot(-1+h_2) - h_1)$$

$$-(-i\cdot(-1) - 0) =$$

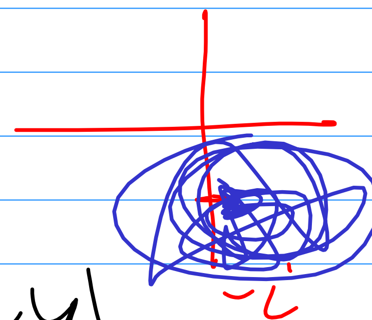
$$(-i + \underline{h_1} + i\underline{h_2})(-1+\underline{h_2}) - \underline{h_1})$$

$$f(z) = f(x+iy) =$$

$$(x+iy)\cdot y - x =$$

$$\underbrace{(x\cdot y - x)}_{u(x,y)} + i y^2$$

$v(x,y)$



$$u(x,y) = x\cdot y - x$$

$$v(x,y) = y^2$$

$$y - 1 = 2y$$

$$u_x = (y-1), \quad v_y = 2y \quad \boxed{y = -1}$$

$$u_y = -v_x$$

$$\underline{\underline{x = 0}}$$

$$(0, -1) \rightsquigarrow -i$$

$$|f(x)| \geq f(x)$$??

$f : \mathbb{C} \to \mathbb{C}$ $\quad$ $f : \mathbb{R} \to \mathbb{R}$

$$f(z) = i$$

$$|f(z)| = 1 \geq i \quad X$$

$$\cancel{1 < i}$$

(1.3) (1)

(a) $|z^2+2z-3i| \geq (R^2-2R-3)$

$$\frac{1}{|z^2+2z-3i|} \leq \frac{1}{(R^2-2R-3)}$$

b) $|z-4| \leq |z|+4 = R+4$

$$|z| = R$$

$$|x+y| \le |x| + |y|$$

$$|f(z)| \le \boxed{\frac{R+4}{(R^2 - 2R - 3)}} \to 0$$

$(|z| = R > 3)$

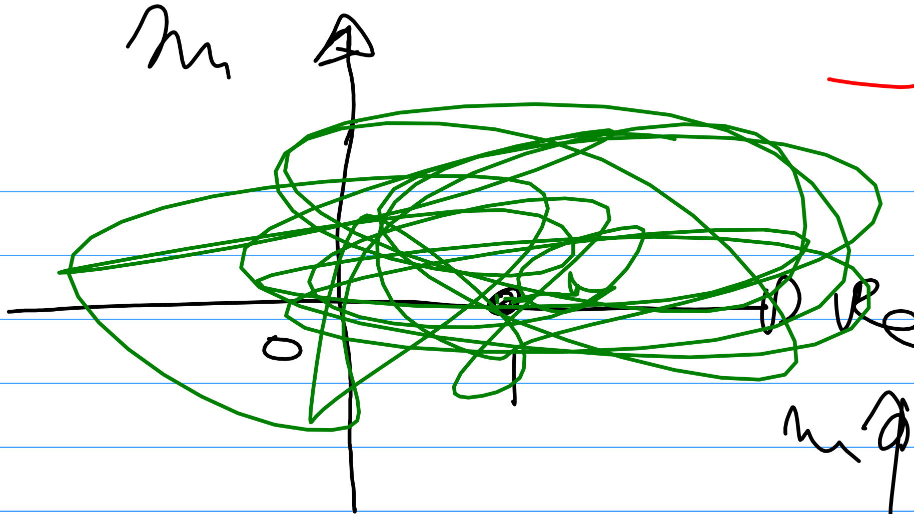
R ?? $|z| = R \to \infty$

(1.3) (b)

(a) $f(1+Re^{i\theta}) = 2e^{i\theta}$

$R \to 0$

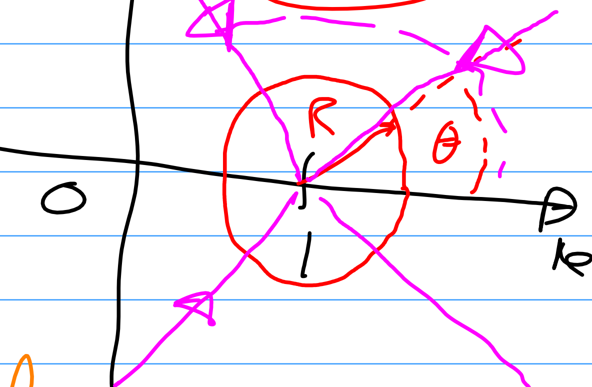
(b) $\lim_{z \to 1} f(z)$



$$\to \; Re^{i\theta}$$

$$|Re^{i\theta}| = R$$

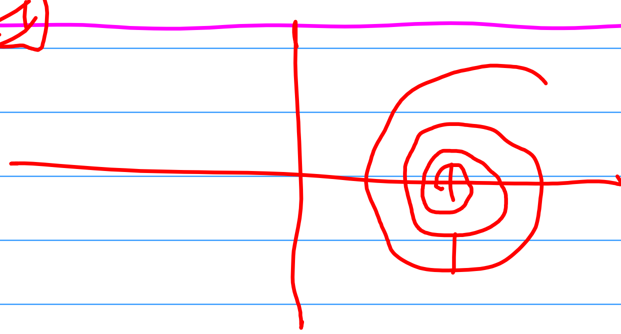
$$R \to 0$$

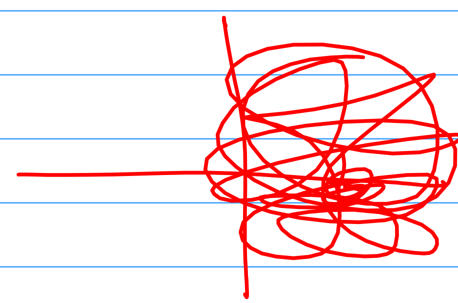


$$f(1+Re^{i\theta}) = 2e^{i\theta}$$
$R \to 0$

θ fixed, $R \to 0$

$\lim_{z \to 1} f(z)$ not excist

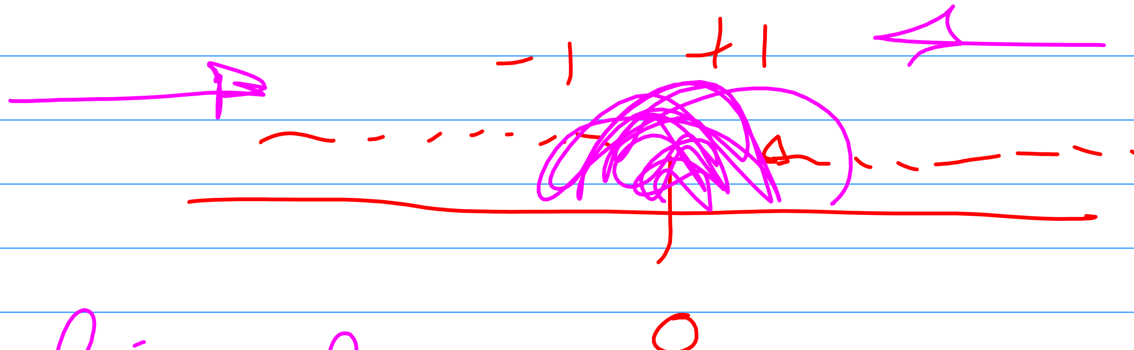




$$f(x) = \frac{x}{|x|} \quad \lim_{x \to 0} f(x) =$$

$x>0 \to f(x) = \frac{x}{x} = 1$

$x<0 \to f(x) = \frac{+x}{-x} = -1$



$\lim_{x \to 0} f(x)$ +, -1, 1, ...

not exist

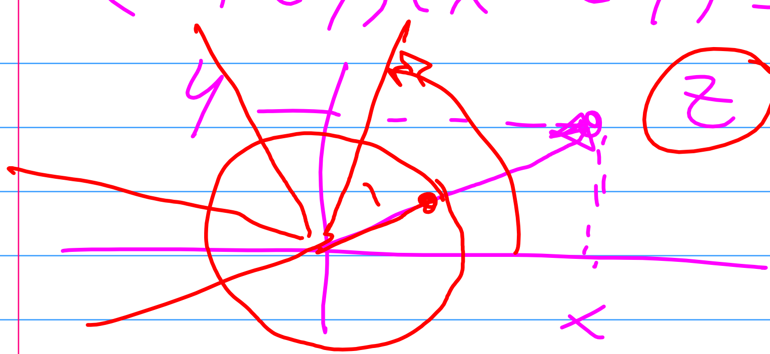
$\boxed{\left| \frac{z}{|z|} \right| = 1}$

unit circle

$f(z) = \frac{z}{|z|} = \frac{z}{\sqrt{z \cdot \bar{z}}}$

$z \cdot \bar{z} = |z|$

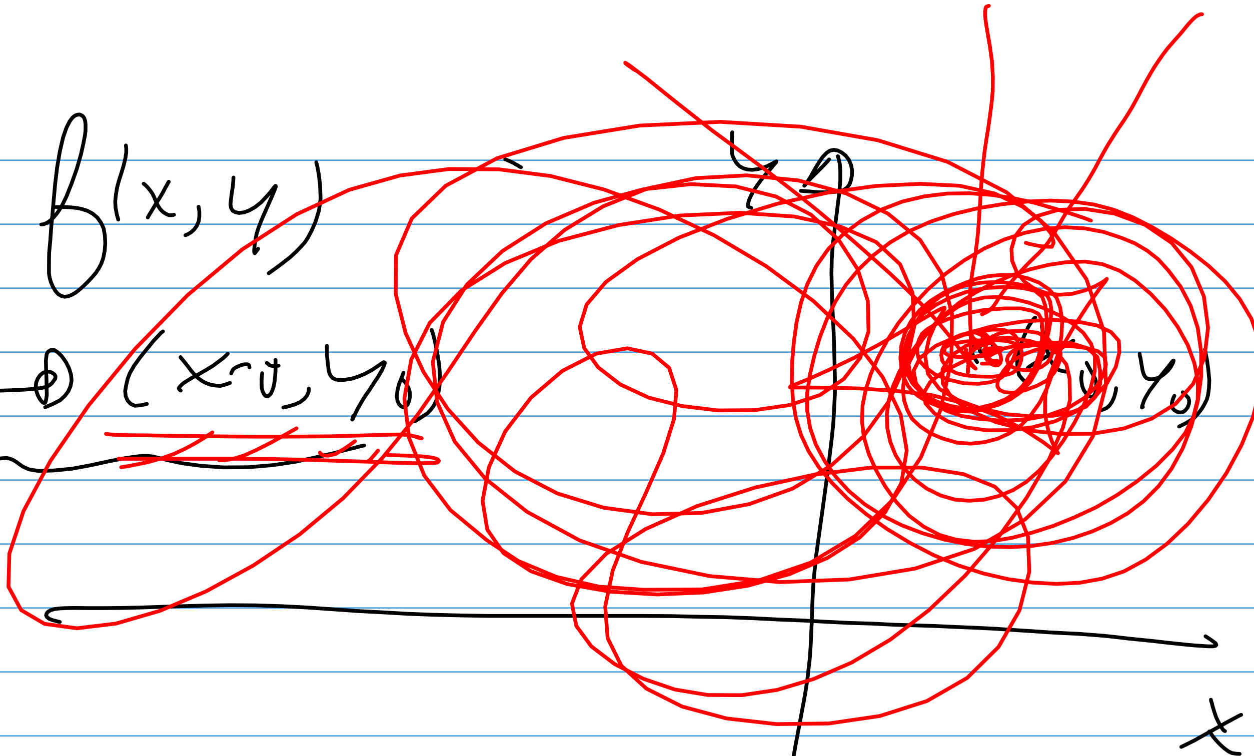
$(x+iy)(x-iy) = x^2+y^2 = |z|^2$



z

$|z|$

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y)$$



~~path-independent~~

$(3a)\ f(1 + R\cdot e^{i\theta})$

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