CAI-200914: (cai-200914.pdf)

See: www.win.tue.nl/~rwhassel)

Hello! Questions ask them!

(I hear nothing ...)

$$f(z) = z \cdot \mathrm{Im}(z) - \mathrm{Re}(z)$$

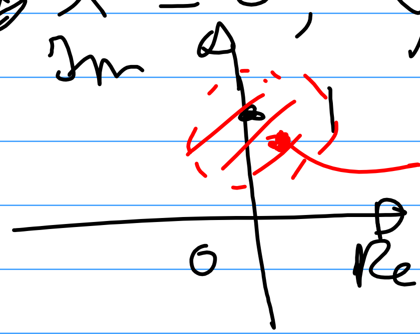
$\to z = x + iy$

$$\mathrm{Im}(z) = y, \quad \mathrm{Re}(z) = x$$

$$f(x,y) = (x+iy)y - x = \underbrace{(xy - x)}_{u(x,y)} + i\,\underbrace{y^2}_{v(x,y)}$$

$$\begin{cases} u_x = v_y \\ u_y = -v_x \end{cases} \qquad \boxed{\begin{aligned} (y-1) &= 2y \\ x &= 0 \end{aligned}}$$

$\Rightarrow x = 0, \; y = 1 \qquad z = i$!!



not diffbl $\to$

(not holomorphic)

Other questions ??

$u_x = y-1 \quad , \quad u_y = x$

$v_x = 0 \quad , \quad v_y = 2y$

$z = (1+i) \qquad x = 1 \; , \; y = 1$

(f ~~not diffbl~~ in $z = (1+i)$) $\Rightarrow$ $u_x = 0 \; , \; u_y = 1$

$v_x = 0 \; , \; v_y = 2$

(1, 6) (4, 6)

a) $x > 0$ ~~$z > 0$?~~

$$\boxed{\frac{|z-1|}{|z+1|} < 1}$$

$$|z-1| < |z+1|$$

$$Re(z) > 0.$$

$g(z) = \left(\frac{z-1}{z+1}\right)$, $Re(z) > 0$

g(z) is holomorphic.

$\leadsto g(x+iy) = u(x,y) + iv(x,y)$

$$h(w) = \sum_{n=1}^{\infty} \frac{1}{n} (w)^{n}$$ diff.

$$h'(w) = \sum_{n=1}^{\infty} w^{(n-1)} = \sum_{n=0}^{\infty} w^{n}$$

$$h'(w) = \frac{1}{1-w}$$

??

$$h(w) = -\ln(1-w) \; ??$$

series $h(w)$ $\sum_{n=1}^{\infty} \frac{1}{n} w^n \rightsquigarrow$

converge if $|w| < 1 \rightsquigarrow$

$h(w)$ is holomorphic

$h(\,g(z)) =$ holomorphic

$$f'(z) = \frac{1}{1 - \left(\frac{z-1}{z+1}\right)}$$

(1.4) (7) $\mathbb{C}$ - numbers ?

$|f(z)|$ $\leadsto$ C.R.

$$|f(z)|^2 = c^2$$

$$f(z) = f(x,y) = u(x,y) + i\,v(x,y)$$

$$|f(z)|^2 = u^2(x,y) + v^2(x,y) = c^2$$

$$\begin{cases} 2\cdot u\cdot u_x + 2\cdot v\cdot v_x = 0 \\ 2u\cdot u_y + 2v\cdot v_y = 0 \end{cases}$$

f holomorph $\leadsto$ C.R. $\quad u_x = v_y$, $\quad u_y = -v_x$

??

$\leadsto$ $u_x = 0,\ u_y = 0$
$v_x = 0,\ v_y = 0$ $\quad \not\!\emptyset$

$$|f(z)| = c \in \mathbb{R} \quad (c \geq 0)$$

$$\boxed{g(z)} \in \mathbb{R}$$

$$g(z) = u(x,y) + i \cdot 0$$

$$v = 0$$

$$\begin{cases} u_x = 0 \\ u_y = -0 \end{cases}$$

$$u = c$$

$(1,y)$ $(1, b)$

a) $$\lim_{z \to 1} \frac{(z-1)(z+1)}{(z-1)} = 2$$

$$l = 2 \qquad g(1) = 2$$

b) $$\boxed{\lim_{h \to 0} \frac{g(1+h) - g(1)}{h}}$$ $h \in \mathbb{C}$

$$\frac{f(1+h)-2}{h} =$$

$$\frac{\left(\frac{(1+h)^2-1}{h}-2\right)}{h} =$$

$$\frac{(1+h)^2-1-2h}{h^2} =$$

$$\frac{1+2h+h^2-1-2h}{h^2} = 1$$

$$\lim_{h\to 0}\left(\frac{h^2}{h^2}\right) = \lim_{h\to 0} 1 = 1$$

(9.2)(9)

$$\sum_{n=1}^{\infty}\left(\frac{3+2^n}{2^{n+2}}\right) =$$

$$\frac{3+2^n}{2^{(n+2)}} = \frac{3}{2^{n+2}} + \frac{2^n}{2^{n+2}} =$$

$$= \frac{3}{2^{n+2}} + \frac{1}{4} \longrightarrow \frac{1}{4} \neq 0$$

$$\sum_{n=1}^{\infty} \frac{1}{4} \quad \text{not conv}$$

$$\sum_{n=1}^{\infty} \frac{3}{2^{n+2}} = \frac{3}{4} \cdot \sum_{n=1}^{\infty} \frac{1}{2^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{4} = \frac{1}{4} + \frac{1}{4} \cdots\cdots ??$$

$$\sum c_n \quad ? \quad c_n \to 0 \quad ?$$

$$(1 - 1 + 1 - 1 \cdots\cdots ??)$$

(1.5) (1) $\frac{1}{z} \cdot \left(\frac{\bar{z}}{\bar{z}}\right)$

$$u = \frac{x}{x^2+y^2}, \quad v = \frac{-y}{x^2+y^2}$$

$$\begin{cases} u_{xx} + u_{yy} = 0 \\ v_{xx} + v_{yy} = 0 \end{cases}$$

$$\boxed{\begin{aligned} u_x &= v_y \\ u_y &= -v_x \end{aligned}} \quad CR$$

$$\frac{x}{x^2+y^2} = \frac{1}{c}$$

$$x^2+y^2 = c\cdot x$$

$$x^2 - cx + y^2 = 0$$

$$\boxed{\left(x - \frac{1}{2}c\right)^2 + y^2 = \frac{1}{4}c^2}$$

(2) $f(z)$??

$\rightsquigarrow u(x,y) = xy + e^{x}\cos(y)$

$v_x = -u_y$

$f(x,y) = u(x,y) + i\,v(x,y)$

$f(0) = 1 + i \longrightarrow c$

$z = 0$?? $\quad 0 + i0$

$x = 0 \quad y = 0$.

(4) $f(1)$ $\quad f(z)$ $\quad z = x + iy$

z $\quad \in \mathbb{C}$

$z = 1 \rightsquigarrow \begin{cases} x = 1 \\ y = 0 \end{cases}$

File will be saved at my homepage, see above.