CAI: 200916;

— (TUE 2DME30) "google"

(1.5) (2)

$z = x + iy$ $\quad f(x,y) =$

$$x = \frac{z + \bar{z}}{2}$$

$$y = \frac{z - \bar{z}}{2i}$$

$x = (z - iy)$

$$u(x,y) = xy + e^{x}\cos(y)$$

$$v(x,y) = e^{x}\sin(y) - \frac{1}{2}x^{2} + \frac{1}{2}y^{2} + 1$$

$$f(x,y) = u(x,y) + i\,v(x,y)$$

$$\left(-\frac{1}{2}x^{2} + \frac{1}{2}y^{2}\right)i + xy$$

$$e^{x}(\cos y + i\sin y) + 1 =$$

$$e^x(\cos(y)+i\sin y)=e^{iy}\cdot e^x$$

$$=e^{(x+iy)}=\underline{e^z}\qquad \frac{1}{i}\cdot\frac{i}{i}=-i$$

$$\frac{(x+iy)^2}{2i}=\frac{x^2-y^2+2i\cdot xy}{2i}$$

$$=x\cdot y+\frac{-i}{2}(x^2-y^2)$$

$$\frac{1}{2i}\cdot z^2=\frac{-i}{2}\cdot z^2$$

$$\left(-\frac{i}{2}\cdot z^2+e^z\right)$$

$$x^2\qquad x=z-iy$$

$$i\cdot(z-iy)^2=(z^2-2izy)i$$

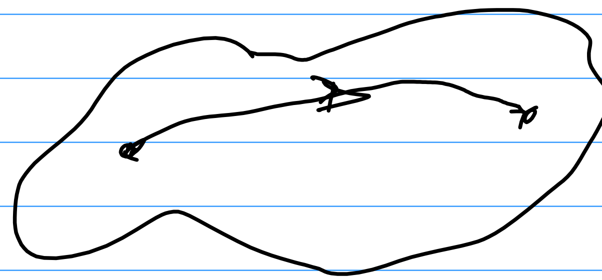
$$-y^2$$

$$xy=(z-iy)y=zy-iy^2$$

$$\left(f(x,y) \text{ "=" } f(z) \qquad x = \frac{z+\bar{z}}{2},\quad y = \frac{z-\bar{z}}{2i} \right)$$

(2.1) (4 C)

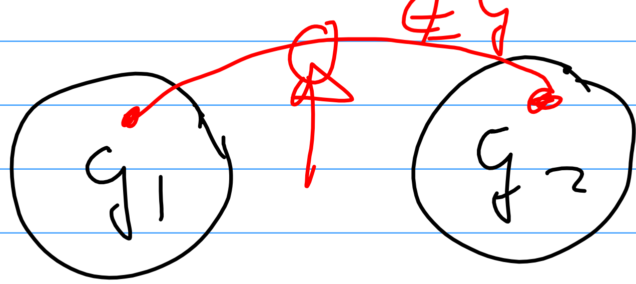
Connected:



G not connected:

G :

$\notin G$



not connected

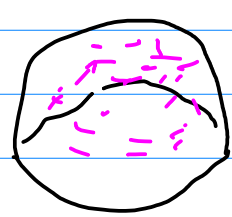
$f(z)$ not necessarily constant

$$f(z) = \begin{cases} 1 & z \in G_1 \\ 2 & z \in G_2 \end{cases}$$

$f(z)$ not constant

~~google~~: TUE 2DME30
another one:
the dictaten project, see
CASA Antieke Wiskunde
Dictaten → search:
functietheorie of Boersma
of Wiskunde 50.
The lecture notes you use
are based on those "dictaten".
Most of them are typed with a
typewriter, with
interchangeable heads.

Greek alphabet



→ math: $\int \ \cup \ \cap$
$\forall \ \exists$ .....

(1.b) (1b)

$$\sin(z) = \left(\frac{e^{iz} - e^{-iz}}{2i}\right)$$

$$\cos(z) = \left(\frac{e^{iz} + e^{-iz}}{2}\right)$$

$$\tan(z) = \left(\frac{2}{2i}\right) \cdot \left(\frac{e^{iz} - e^{-iz}}{e^{iz} + e^{-iz}}\right)$$

$$\tan(z) = \underline{\underline{a}} \; (\in \mathbb{C})$$

$$\frac{1}{i} \cdot \frac{e^{iz} - e^{-iz}}{e^{iz} + e^{-iz}} = a$$

what can it $\underline{\underline{\text{be}}}$?

$$e^{-iz} = \frac{1}{e^{iz}} \qquad \underline{\underline{e^{iz}}} \qquad \underline{\underline{e^{iz} = g(a)}}$$

~~See page 14!!~~

(1.5)(2)

$u(x,y)$ $\quad u_x = y + e^x \cdot \cos(y)$

$v(x,y)$

$$\begin{cases} u_x = v_y \\ u_y = -v_x \end{cases}$$

$\rightarrow u_y = x - e^x \cdot \sin(y) = -v_x$

$v_x = -x + e^x \cdot \sin(y)$

$v(x,y) = -\frac{1}{2}x^2 + e^x \sin(y)$

$+ g(y) \rightarrow ?$

$v_y = e^x \cdot \cos(y) + g'(y) =$

$y + e^x \cdot \cos(y) \; (= u_x)$

$g'(y) = y \quad g(y) = \frac{1}{2}y^2 + C_1$

$f(z) = (x \cdot y + e^x \cos(y)) +$

$i\left(-\frac{1}{2}x^2 + \frac{1}{2}y^2 + e^x \sin y + C_1\right)$

$$\beta(0) = 1 + i\cdot c = 1 + i$$

$$\boxed{c = 1}$$

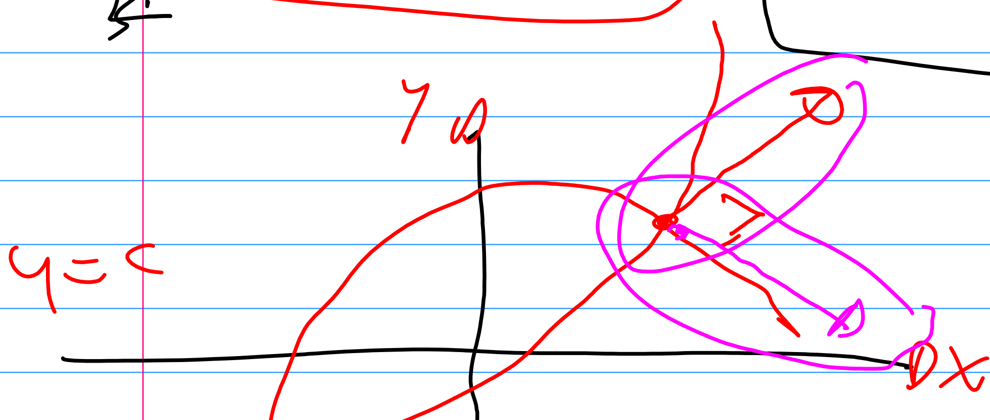
$$\boxed{\begin{aligned} &\text{i} \quad u_x = v_y \\ &\text{ii} \quad u_y = -v_x \end{aligned}}$$

start with i or ii

fill in result in other way

$$\begin{cases} u_x = v_y \\ u_y = -v_x \end{cases}$$

$$\boxed{\begin{aligned} u_{xx} + u_{yy} &= 0 \\ v_{xx} + v_{yy} &= 0 \end{aligned}}$$



$u(x,y)$

$v(x,y)$

v = arg

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \end{pmatrix} \cdot \begin{pmatrix} -u_y \\ u_x \end{pmatrix}$$

$$\Rightarrow = 0$$

$f(x,y) = u(x,y) + i\,v(x,y)$

$$\begin{cases} u_{xx} + u_{yy} = 0 \\ v_{xx} + v_{yy} = 0 \end{cases}$$

(f is holomorphic)

if not known ??

f holomorphic $\Rightarrow$

C.R. eqn's $\Rightarrow$ $\begin{cases} u_{xx} + u_{yy} = 0 \\ v_{xx} + v_{yy} = 0 \end{cases}$

only to read from left to right

the other way ??

$\boxed{\text{use Ex. 2 and Ex. 3}}$

$$u(x,y) = x - \frac{x}{(x^2+y^2)},$$

take $v(x,y) = \operatorname{Im}(f(z))$

with f, the solution of Esc 2.

Construct:

$f = u + i v$

$u_x \neq v_y$

$u_y \neq - v_x$

CR not satisfied

f is not holomorphic

ada. (9.3) (38)

$$\sum_{n=1}^{\infty} \left(\frac{2^{n+1}}{n^n}\right)$$

$$\lim_{n\to\infty} \left(\frac{2^{n+1}}{n^n}\right)^{\frac{1}{n}} =$$

$$\lim_{n\to\infty} \left(\frac{2^{1+\frac{1}{n}}}{n}\right) = 0 < 1$$

$$\frac{(n+1)^{n+1}}{n^n} = (n+1)\left(\frac{n+1}{n}\right)^n$$

$n \to \infty$

$$\exp \ln \left(\left(\frac{n+1}{n}\right)^n\right) = \left(1+\frac{1}{n}\right)^n$$

$$\exp\left(n \cdot \ln\left(1+\frac{1}{n}\right)\right) =$$

$(n+1)$ $A(n)$ $\left(1+\frac{1}{n}\right)^n = e^1$

$(n \to \infty)$

$n \cdot \ln\left(1+\frac{1}{n}\right) \approx \frac{n}{n} = 1$ if $n \to \infty$

$$\frac{2^{\cancel{(n+1)}+1}}{(n+1)^{n+1}} \cdot \frac{n^n}{2^{\cancel{n+1}}} =$$

Ratio

$$2 \cdot \left(\frac{n^n}{(n+1)^{n+1}}\right) \to \frac{2}{\infty} = 0$$

root test easier

(1.5) (3)

$f(z)$

ϕ

$$u(x,y) = x - \frac{x}{|x^2+y^2|}$$

Guessing!

$$v(x,y) = y \pm \frac{y}{x^2+y^2}$$

$$u(x,y) + i \cdot v(x,y) =$$

$$z - \frac{z}{z \cdot \bar{z}}$$ head

good or not !!

$$u(x,y) = x - \frac{x}{x^2+y^2}$$

f holomorf.

$$u_y(x,y) = \frac{+x \cdot 2y}{(x^2+y^2)^2} = -v_x$$

$$v_x = -\frac{2 \cdot x \cdot y}{(x^2+y^2)^2}$$

$$v(x,y) = \frac{y}{(x^2+y^2)} + g(y)$$

$$v_y = u_x$$

$u_{xx} + u_{yy} = 0$

u is harmonic

$u(x,y) \in \mathbb{R}$

$f = u + iv \rightsquigarrow$ holomorphic

$\Rightarrow \quad u_{xx} + u_{yy} = 0$

$v_{xx} + v_{yy} = 0$

$$\boxed{f(z) = u(x,y) + i \cdot 0}$$

u is harmonic

? f holomorf ?

$u_x = 0$

$u_y = 0$

$f = c$

$$f(z) = u + iv$$

u, v harmonic

$\Rightarrow f$ holom.

take: $u = \mathrm{Re}(f \text{ ex.} 3)$

$v = \mathrm{Im}(f \text{ ex.} 2)$

$u_x = v_y$

$u_y = -v_x$

$$e^{2iz} = \frac{a-i}{-a-i}$$

$\left(\cancel{a = i \Rightarrow e^{2iz} \neq 0} \quad \cancel{a = -i \Rightarrow e^{2iz} \neq \infty} \right)$

$e^{2iz} \neq 0 \Rightarrow a \neq i$

$e^{2iz} \neq \infty \Rightarrow a \neq -i$