CAI-200921;

If questions, ask them!

(1.6) a) $\displaystyle\sum_{n=1}^{\infty} \frac{z^{2n}}{2^n} =$

$$\frac{z^2}{2} + \frac{z^4}{4} + \frac{z^6}{8} + \dots$$

g.m. ...

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\sum_{n=0}^{\infty} \left(\frac{z^2}{2}\right)^n - 1 =$$

$$\left(\frac{1}{1-\frac{z^2}{2}}\right) - 1 =$$

$$\frac{1}{\left(\frac{2-z^2}{2}\right)} - 1 = \frac{2}{2-z^2} - 1$$

c) $\displaystyle\sum_{n=0}^{\infty} \frac{(n+2)}{n!} (z-1)^n$??

$$\exp(z) = \sum_{n=0}^{\infty} \frac{1}{n!} z^n$$

$$\sum_{n=0}^{\infty} \left(\frac{n}{n!} + \frac{2}{n!} \right) (z-1)^n =$$

$(0! = 1)$

$$2 \cdot \sum_{n=0}^{\infty} \frac{(z-1)^n}{n!}$$

$(-1)!$

$$\underbrace{}_{2 \cdot \exp(z-1)}$$

$$\sum x^n = \frac{1}{1-x}$$

$$e^x = \sum \frac{x^n}{n!}$$

$$\sum_{n=1}^{\infty} \frac{1}{(n-1)!} (z-1)^n$$

$n = k+1 \quad (k = 0, 1, \cdots)$

$$(z-1) \underbrace{\sum_{k=0}^{\infty} \frac{1}{k!} (z-1)^k}_{} =$$

$$(z-1)\exp(z-1)$$

$$2\exp(z-1) + (z-1)\exp(z-1)$$

(5) $\displaystyle\int_{|z|=2} \frac{\bar{z}+1}{z^2+1}\,dz$ (2,4)

$\bar{z} = (x - iy)$

$u = x$

$v = -y$

Holom.?

C.R.

$u_x = 1,\ u_y = 0$

$v_x = 0,\ v_y = -1$

Ca..y R... n

$u_x = v_y$

$u_y = -v_x$

$\bar{z}$ not holomorphic

$\boxed{|z| = 2}$

$|z|^2 = z \cdot \bar{z} = 4$

$\bar{z} = \left(\frac{4}{z}\right)$

$\left[z = 0,\ z = +i,\ z = -i \right]$

$$I = \int_{|z|=2} \frac{\frac{4}{z}+1}{(z^2+1)}\,dz$$

the res.?

$$\frac{4+z}{z(z^2+1)} \left(= \frac{A}{z} + \frac{B}{z-i} + \frac{C}{z+i} \right)$$

(see next page)

or $\lim_{z \to 0} z \cdot \left(\frac{4+z}{z(z^2+1)} \right) = ?$

and so with:

$\lim_{z \to i} (z-i)\left(\cdots \right) = ?$

etc.

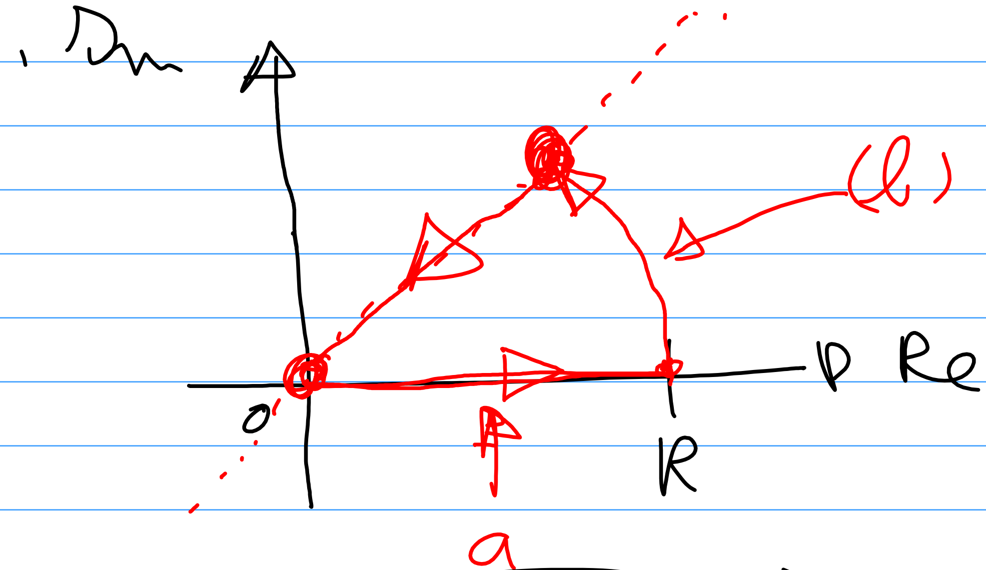
$A(z-i)(z+i) + B \cdot z(z+i) +$
$C \cdot z(z-i) = 4+z \quad \forall z \in \mathbb{C}$

Fill in: $z=0$, $z=i$, $z=-i$

[(2.4)(6) also a nice exerc.!]

always sketch the closed contour in $\mathbb{C}$, just to see where the singularities are lying!

(2.1) (1) a) c)



4l) $\boxed{|f(z)| = c}$

$(\Longrightarrow f = c)$??

$$f(z) = \underline{u(x,y)} + i\,\underline{v(x,y)}$$

$$|f(z)| = \sqrt{u^2 + v^2}$$

$$|f(z)|^2 = \boxed{c^2 = u^2 + v^2}$$

$$\begin{cases} 2 \cdot u \cdot u_x + 2v \cdot v_x = 0 \\ 2\, u \cdot u_y + 2v \cdot v_y = 0 \end{cases}$$

<u>f is holom</u>, $\quad u_x = v_y$

$\quad u_y = -v_x$

$$2 \cdot u \cdot v_y + 2 \cdot v \cdot u_x = 0$$

$$-2 \cdot u \cdot v_x + 2 \cdot v \cdot v_y = 0$$

$$u \cdot v_x = v \cdot v_y \qquad v_x = \frac{v}{u} v_y$$

$$2 \cdot u \cdot v_y + 2 \cdot v \cdot \frac{v}{u} \cdot v_y = 0$$

$$(2u^2 + 2v^2)\, v_y = 0$$

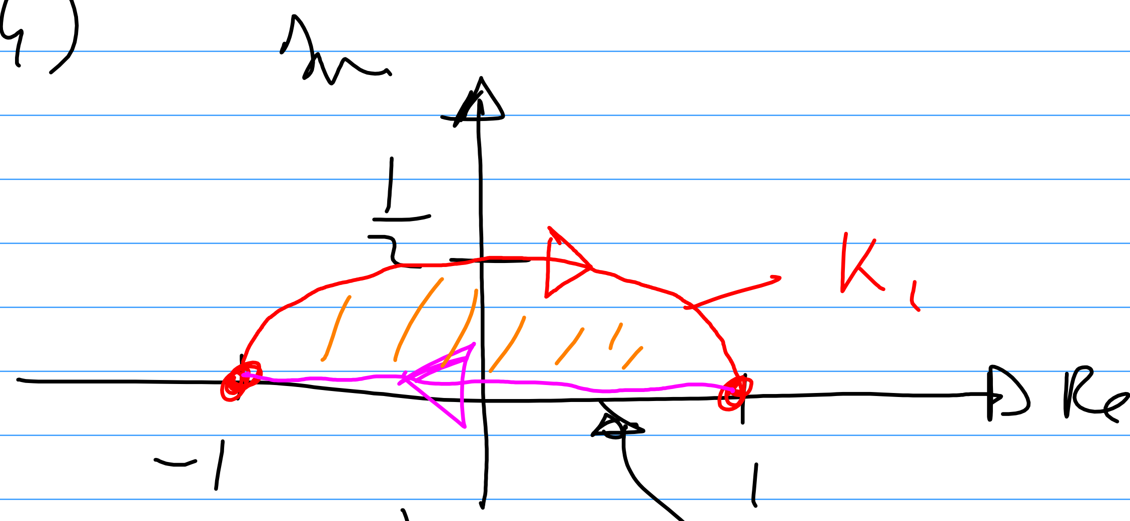
$$v_y = 0, \Rightarrow v_x = 0$$

$$u_x = 0, \quad u_y = 0.$$

$$u = c_1, \quad v = c_2$$

$$f = (c_1 + i c_2) = C$$

(2.4)



$$f(z) = \frac{1}{(z^2+1)}$$

(f is holomorf)

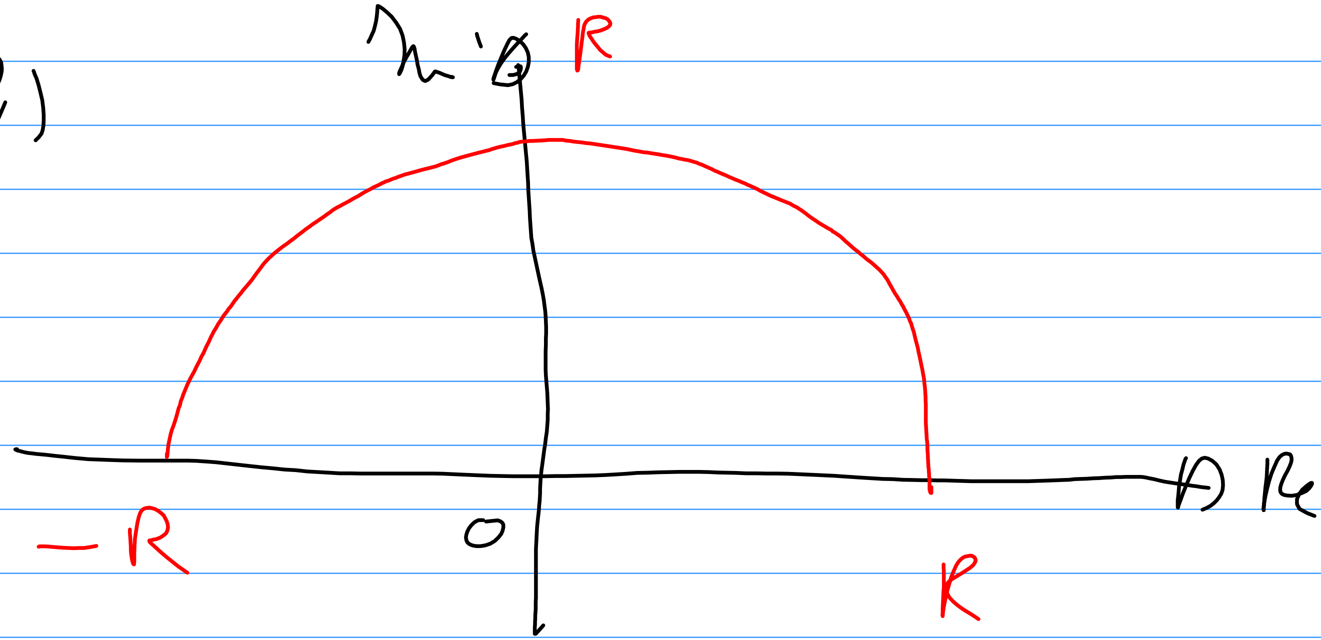
C. Int Th:

$$\int_{K_1+K_2} \frac{1}{1+z^2}\,dz = 0$$

$$\int_{K_1} \frac{1}{1+z^2}\,dz = - \int_{K_2} \frac{1}{1+z^2}\,dz$$

$$= \int_{-1}^{+1} \frac{1}{1+x^2}\,dx = \left[\arctan x\right]_{-1}^{1} = (\pi/2)$$

b)



$$\left| \int_{\substack{|z|=R \\ \mathrm{Im}(z)\geq 0}} \frac{1}{1+z^2}\, dz \right| \leq *$$

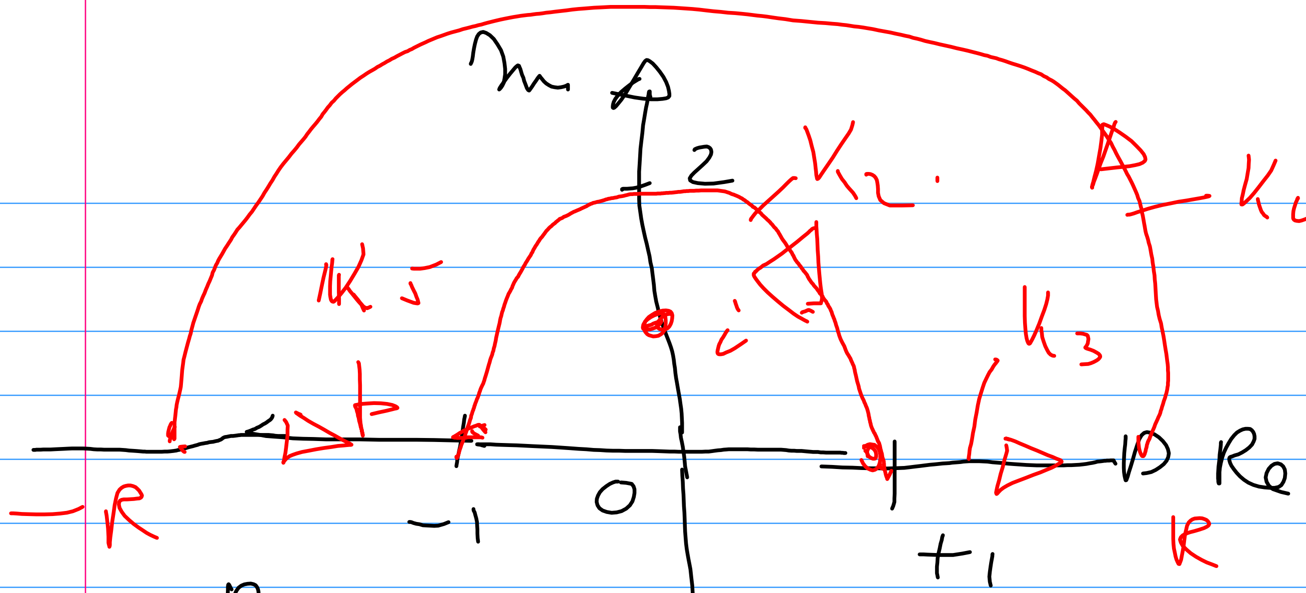
$\leftarrow$ length: $\pi \cdot R$

$$* \leq \frac{\pi R}{(R^2-1)} \to 0 \text{ if } R \to \infty$$

$$\left| \frac{1}{1+z^2} \right| \leq \frac{1}{(R^2-1)}$$

$$|1+z^2| \geq |1-|z^2||$$

c)



$$\int_1^R \frac{1}{1+x^2}\,dx = \arctan R - \arctan(1) =$$

$(R \to \infty)$

$$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\int_{-R}^{-1} \frac{1}{1+x^2}\,dx = \Big[\arctan(x)\Big]_{-R}^{-1} =$$

$$\underbrace{\arctan(-1)}_{(R \to \infty)} - \arctan(-R) = -\frac{\pi}{4} - \left(-\frac{\pi}{2}\right) = +\frac{\pi}{4}$$

$$\int_{K_2} f(z)\,dz + \left(\frac{\pi}{4}\right) + \left(\frac{\pi}{4}\right) = 0$$

$$\int_{K_2} \frac{1}{1+z^2}\,dz = -\frac{\pi}{2}$$

Be careful

$$\int_{-R}^{-1} \frac{1}{1+x^2}\,dx = \int_{R}^{+1} \frac{1}{1+(-y)^2}\cdot d(-y)$$

$x = -y$

$$= -\int_{R}^{1} \frac{1}{1+y^2}\,dy = \int_{1}^{R} \frac{1}{1+y^2}\,dy.$$