CAI-200923;

(1,6) (4l) (4c)

$$\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{z-1}{z+1}\right)^n$$ holomorphic on G

(w)

$$\sum_{n=1}^{\infty} \frac{1}{n} w^n = f(w)$$ holom

$$\sum_{n=0}^{\infty} w^n \to$$ holomorphic on conv domain

$$g(z) = \left(\frac{z-1}{z+1}\right)$$ holomorph

$$\left|\frac{z-1}{z+1}\right| < 1 \qquad \underline{f(g(z))} \qquad \sum z^n = \frac{1}{1-z}$$

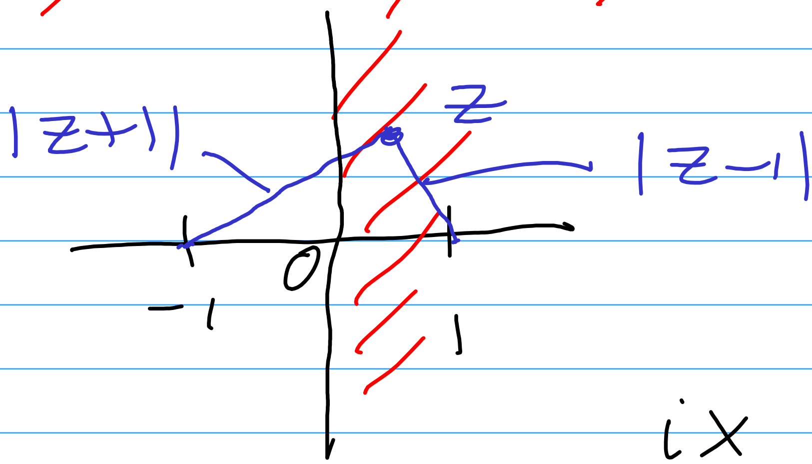
4c) $$\sum_{k=0}^{\infty} \left(\frac{z-1}{z+1}\right)^{k} \cdot \frac{d}{dz}\left(\frac{z-1}{z+1}\right)$$

$$\left|\frac{z-1}{z+1}\right| < 1 \qquad |z-1|^2 < |z+1|^2$$

$$(x-1)^2 + \cancel{y^2} <$$

$$(x+1)^2 + \cancel{y^2}$$

$$\cancel{x^2} - 2x + \cancel{1} < \cancel{x^2} + 2x + \cancel{1} \qquad \boxed{x>0}$$



$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$$

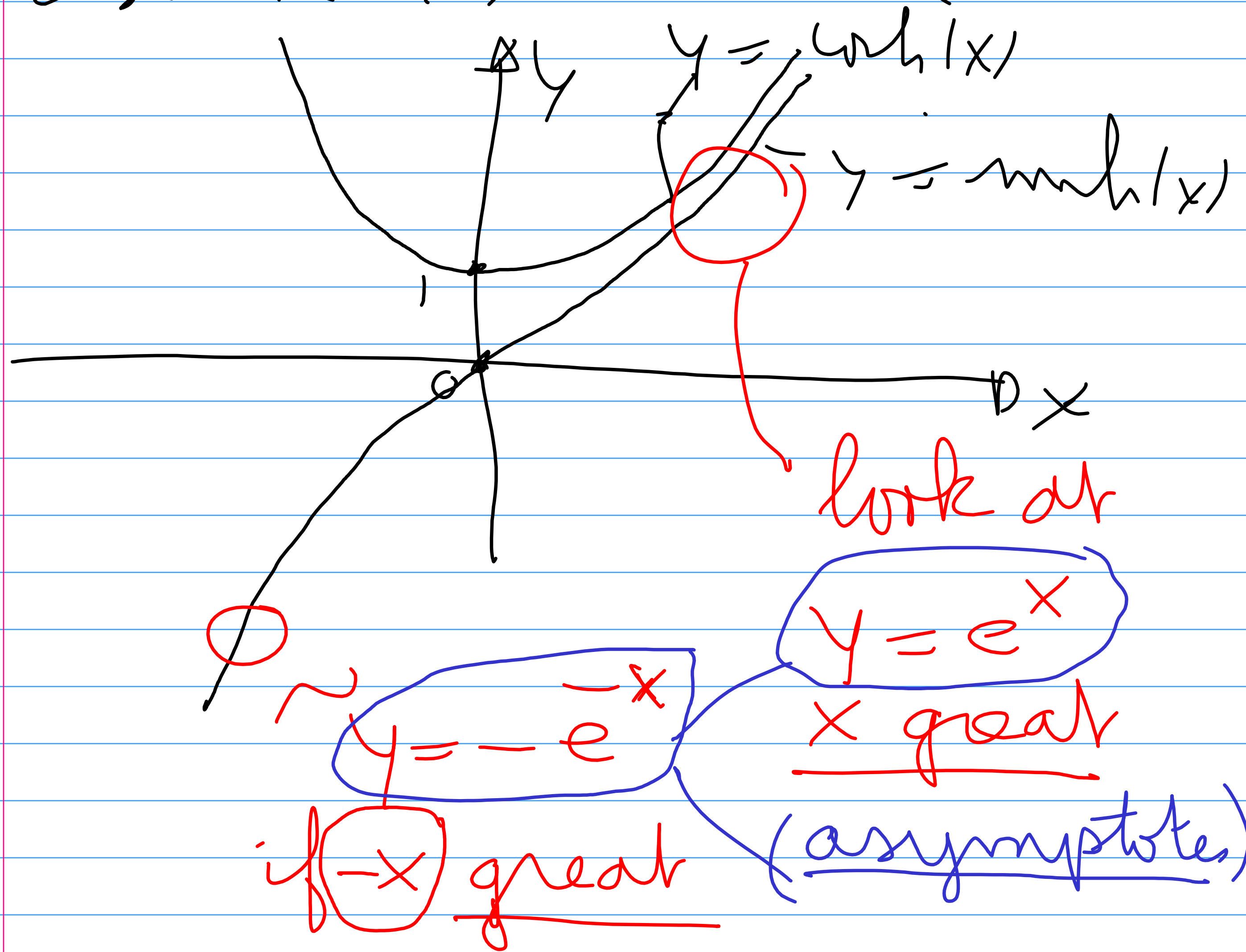
$$\cosh(x) = \frac{e^{x} + e^{-x}}{2}$$

$$(\cosh(ix) = \cos(x))$$

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\sinh(x) = \frac{e^{x} - e^{-x}}{2}$$

$$-i\sinh(ix) = \sin(x)$$



$$\cosh(z) = \frac{e^{z} + e^{-z}}{2}$$

→ <u>look at</u> Wolfram alpha!!
start and fill in $\cosh(z)$
even so: $\sinh(z)$, $\tanh(z)$
(Residue($f(z)$, $z=a$) also works!)

$(1,3)(1,6)$

$A > 2 \Rightarrow \frac{1}{A} < \frac{1}{2}$

$A \quad |\cdots| \geq R^2 - 2R - 3$

$$\frac{1}{|\cdots|} \leq \frac{1}{R^2 - 2R - 3}$$

$$|z-4| \leq R+4$$

$$\frac{|z-4|}{|\cdots|} \leq \frac{R+4}{R^2 - 2R - 3}$$

$$= \frac{R\left(1 + \frac{4}{R}\right)}{R^2\left(1 - \frac{2}{R} - \frac{3}{R^2}\right)} \to 0$$

$R \to \infty$

$$\lim_{|z| \to \infty} (\cdots) = 0$$

It's week 4, maybe we can already study the exams??

What is asked in the exercises

exercise 1: often C.R.?

exercise 3: Laurent series?

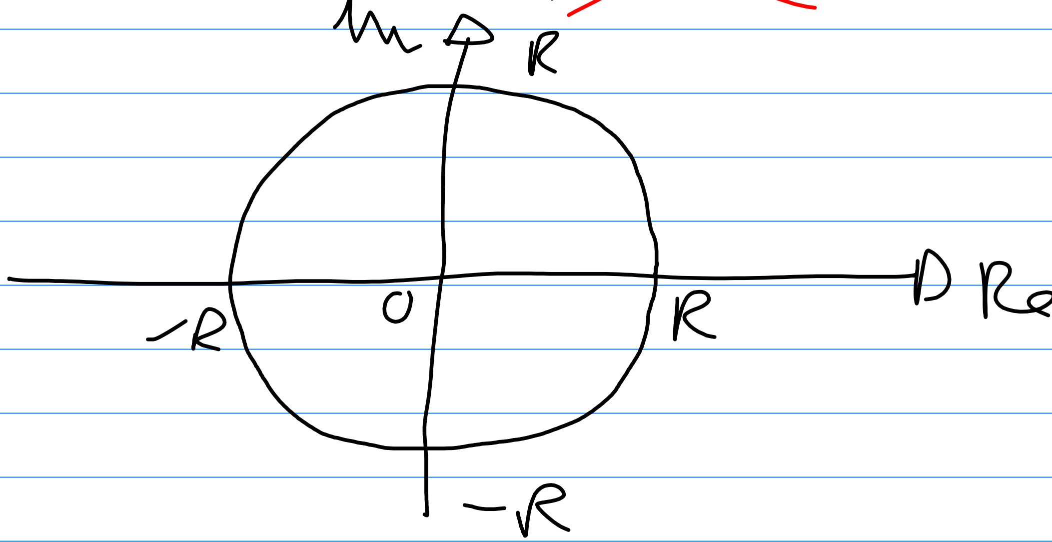
exercise 2: residues?

exc. 4 and 5: this moment: "too far from my bedshow"

exercise 6: Also something with residues?

(2,3) (3 b)

→ (C. → ~~param~~)



a) $|z^{10}-1| \geq (R^{10}-1)$

$$\frac{R^{9} \cdot (2\pi R)}{(R^{10}-1)}$$

$$\int_{|z|=R} \frac{z^{9}}{(z^{10}-1)}\, dz =$$

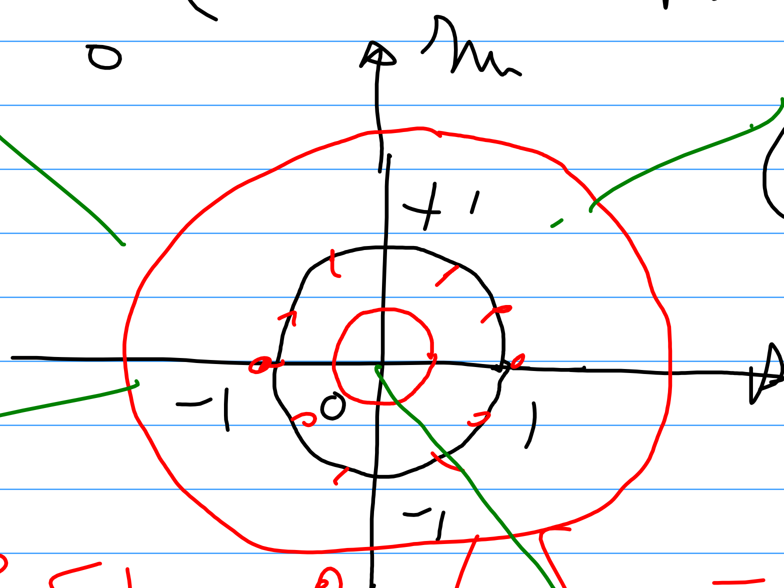
$dz = \underline{i} \cdot R e^{it}\, dt$

$z = R \cdot e^{it}, \quad 0 \leq t \leq 2\pi$

$$\int_0^{2\pi} \frac{R^9 \cdot e^{it\cdot 9} \cdot iRe^{it}}{R^{10}\cdot e^{it\cdot 10}-1}\,dt$$

$$= i\int_0^{2\pi} \frac{R^{10} e^{i10t}-1+1}{R^{10}\cdot e^{i10t}-1}\,dt$$

$$i\int_0^{2\pi}\left(1+\frac{1}{R^{10}e^{i10t}-1}\right)dt$$



$\boxed{z^{10}-1=0}$

$z^{10}=1$

$0<R<1 \to (\ ) = 0)$

$R=1$

fixed $R>1$

$S \to 2\pi$

S

$$i\int_0^{2\pi} 1 + \left(\frac{1}{R^{10}e^{i10t}-1}\right) dt$$

?

R^{10}

$\boxed{?\ R<1\ ?}$??

$R>1 \to i2\pi$

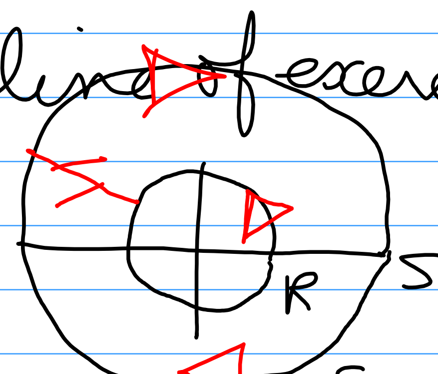
Read exercise first line

$R>1$

So: $$\int_{|z|=R} \frac{z^n}{z^{10}-1}\,dz = I(n,R)$$

$R>1$; see first line of exerc

R is fixed



(a) $0 \le n \le 8$

$$|I(n,R)| = |I(n,S)|$$

$S\to\infty$

$$|I(n,S)| \le \frac{S^n (2\pi \cdot S)}{S^{10}-1}$$

$$\longrightarrow 0 \text{ if } S \to \infty$$

So $I(n,R) = 0$, $R > 1$; if $0 \le n \le 8$

$n = 9$; $z = R \cdot e^{it}$, $0 \le t \le 2\pi$

$dz = iR \cdot e^{it}$

$$I(9,R) = i \int_0^{2\pi} \frac{(R \cdot e^{it}) \cdot R^9 \cdot e^{i9t}}{(R^{10} e^{i10t} - 1)} dt$$

$$= i \cdot \int_0^{2\pi} \left(1 + \frac{1}{R^{10} e^{i10t} - 1}\right) dt \; ?$$

$$I(9,R) = \oint_{|z|=R} \frac{z^9}{(z^{10}-1)} dz$$

$$= I(9,S) \quad (S > R)$$

be careful oriéntation

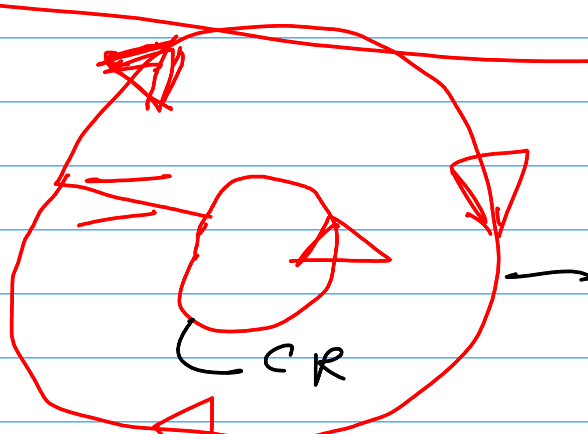
$$\int_0^{2\pi} \frac{1}{(S^{10}e^{i10t}-1)}\,dt \to 0$$

if $S \to \infty$

$I(g,R) = 2\pi i$, if $R > 1$

f holomorf on C_1 and on the inner region of C_1, C_1 a closed contour (Jordan curve)

$$\int_{C_1} f(z)\,dz = 0$$



R fixed

$S > R$

$(S \to \infty)$

Let's Wolfram alpha calculate the residues!

$$\text{residue}\left(\frac{z^9}{z^{10}-1},\ e^{i \cdot k \cdot \frac{2\pi}{10}}\right)$$

do it for $k=0$, $k=1, \dots, 9$.

$\Rightarrow \left(\frac{1}{10}\right)$ <u>10 times</u>!

$$\int \dots \, dz = 2\pi i \,(\text{sum residues})$$

$$= 2\pi i \cdot \left(10 \cdot \frac{1}{10}\right)$$

$$= \underline{\underline{2\pi i}}$$