2CAI – 200930:

Questions ask them! Esc., Th. doesn't matter.

(1.5) (4)

$$f(x,y) = e^{ax} \cdot \cos(2\pi y) + i \cdot \frac{e^{ax}}{2\pi} \cdot \sin(2\pi y) + \frac{C(x)}{\cancel{A}}$$

$$u(x,y) = e^{ax} \cdot \cos(2\pi y)$$ $\sin' = \cos$, $\cos' = -\sin$

$$\rightsquigarrow u_x = a \cdot e^{ax} \cdot \cos(2\pi y) = v_y$$

$$v(x,y) = \frac{a}{2\pi} \cdot e^{ax} \sin(2\pi y) + C(x)$$

?

$$v_x = \frac{a^2}{2\pi} e^{ax} \cdot \sin(2\pi y) + C'(x)$$

$$= -u_y = -(-2\pi e^{ax} \cdot \sin(2\pi y))$$

$$= 2\pi \cdot e^{ax} \cdot \sin(2\pi y)$$

$v_x = -u_y$

$$\frac{a^2}{2\pi} = 2\pi$$

$a^2 = (2\pi)^2$

$a = 2\pi, \; a = -2\pi$

$$C'(x) = 0 \Rightarrow C(x) = C$$

$a = \pm 2\pi$

$$f(x,y) = e^{ax} \cdot \cos(2\pi y) + i\left(\frac{\pm 2\pi}{2\pi} \cdot e^{ax} \cdot \sin(2\pi y) + C\right)$$

$\mathrm{Re}(f(1)) < 1$ $\qquad a = \pm 2\pi$

$z = 1 \quad (x = 1,\ y = 0)$

$$\rightsquigarrow f(1,0) = e^{a} + iC \qquad C \in \mathbb{R}$$

$$\mathrm{Re}(f(1)) = e^{a} < 1$$

$a = \pm 2\pi \qquad a = -2\pi$

$$\mathrm{Im}(f(1)) = 2\pi = C$$

$$f(x,y) = e^{-2\pi x} \cdot \cos(2\pi y) + i\left(e^{-2\pi x} \cdot \sin(2\pi y) + 2\pi\right)$$

$f(z)$

$$f(x,y) = e^{-2\pi x} \cdot e^{i(2\pi y)} + i \cdot 2\pi$$

$$e^{-(\bar{z})}$$

$$e^{2\pi i y} \neq \underbrace{e^{-2\pi i y}}_{\cos(-2\pi y) + i \sin(-2\pi y)} \; ??$$

$$z = x + iy$$

$$(-x + iy)$$

$$-(x - iy) = -\bar{z}$$ not holom

$$x = z - iy \sim$$

$$iy = (z - x)$$

Other questions?

(2, 5) (4) $\underline{4}$ $|x+y| \leq |x| + |y|$

$$|1 - e^{-z}| < \frac{1}{2}$$

$$\left| \sum_{n=1}^{\infty} \frac{(1 - e^{-z})^n}{n} \right| \leq \underbrace{\sum_{n=1}^{\infty} \frac{|1 - e^{-z}|^n}{n}}$$

$$\leq \sum_{n=1}^{\infty} \frac{(\frac{1}{2})^n}{n}$$ is conv.

u.c. on U.

pg. 8 : use Weierstrass!

(2.6) 6)

entire?

a) $e^{\left(\frac{1}{z}\right)} = \sum_{n=0}^{\infty} \frac{z^{-n}}{n!}$

entire? see $z=0$

$h(z) = 1$

$|z| > R$

b) $\lim_{z \to \infty} z \cdot h(z) = 1$

$h(z) \simeq \frac{1}{z}$

$h(z)$ can not be entire

$h(z) \sim \frac{1}{z} = z^{-1}$

c) $h(z) = 0$

(1.5) (7) $f(z) = u(x,y) + i\,v(x,y)$

$$|f|^2 = (u^2 + v^2)$$

$$\frac{\partial}{\partial x}|f|^2 = 2\cdot u\cdot u_x + 2\cdot v\cdot v_x$$

$$\frac{\partial^2}{\partial x^2}|f|^2 = \frac{\partial}{\partial x}\left(2u\cdot u_x + 2v\cdot v_x\right)$$

$$= 2(u_x)^2 + 2\cdot u\cdot u_{xx} +$$
$$2(v_x)^2 + 2\cdot v\cdot v_{xx}$$

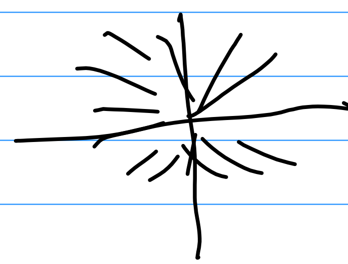
$= 0$

$u, v.$ $\quad u_{xx} + u_{yy} = 0$
$\quad v_{xx} + v_{yy} = 0.$

$= 0$

$$\frac{\partial^2 f}{\partial y^2} = 2(u_y)^2 + 2\cdot u\cdot u_{yy}$$
$$2(v_y)^2 + 2\, v\cdot v_{yy}$$

$$f'(x,y) = u_x + i v_x$$



$f(z)$ holomorphic,

? complex diff ? $\qquad \bar{z}$! diffble

$\bar{z} = x - iy \quad \frac{\partial \bar{z}}{\partial x} = 1 \quad \frac{\partial \bar{z}}{\partial y} = -1$ $(-i)$!

$$|f'|^2 = \boxed{(u_x)^2 + (v_x)^2}$$

$$\underline{(v_x)^2 + (v_y)^2} = (-u_y)^2 + (u_x)^2$$

(C.R.) + (harmonic:
$u_{xx} + u_{yy} = 0$
$v_{xx} + v_{yy} = 0$)

$$f' = u_x + i v_x \; (= v_y - i u_y)$$

$f(z) = \bar{z}$ not holomorf

$$\frac{f(iy) - f(0)}{iy - 0} = \frac{-iy - 0}{iy} = -1$$

$$\frac{f(x) - f(0)}{x - 0} = \frac{x}{x} = 1$$

$$f = u + iv \quad f' = u_x + i v_x$$

$$= v_y - i u_y$$

C.R $\boxed{\begin{aligned} u_x &= v_y \\ u_y &= -v_x \end{aligned}}$

(1.4) — (g):

$$f(z) = z \cdot \operatorname{Im}(z) - \operatorname{Re}(z)$$

diff in z, direction indep.

C.R. $\begin{cases} u_x = v_y \\ u_y = -v_x \end{cases}$

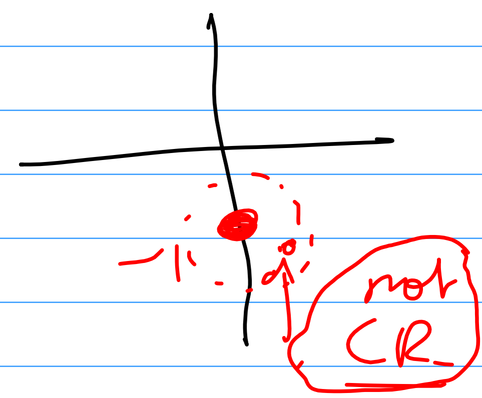
$$f(z) = (x+iy)\cdot y - x$$
$$= (x \cdot y - x) + i y^2$$

$$u = x(y-1)$$
$$v = y^2$$

$\boxed{y = -1}$

CR $\begin{cases} u_x = (y-1) = 2y = v_y \\ u_y = \boxed{x = 0} \end{cases}$ $(0,-1)$

in $(0,-1)$ diffb



(2.5) Pd

$$f(z) = \frac{1}{(1-z)\cdot(1+z^2)}, \quad a = 0$$

Taylor series:

$$f(z) = \frac{\frac{1}{2}}{(1-z)} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right)}{(i-z)} + \frac{\left(\frac{1}{4} + \frac{i}{4}\right)}{(i+z)}$$

$$(z-i)(z+i)$$

$$= \frac{1}{2} \cdot \sum_{n=0}^{\infty} z^n$$

$$\frac{1}{i-z} = \left(\frac{-i}{1+iz}\right) = -i \cdot \sum_{n=0}^{\infty} (-iz)^n$$

$$f(z) = \frac{1}{(1-z)} + \frac{Az+B}{(1+z^2)}$$

$$\frac{1}{1+z^2} = \sum_{n=0}^{\infty} (-z^2)^n$$

$$f(z) = \sum_{n=0}^{\infty} (\text{----}) \, z^n$$

$$\sum (z^2)^n = \sum z^{2n} = \sum (z^n)^2$$

* ((1.4) 7.):

f is holomorf,

$$|f(z)|^2 = c^2$$

$f = u + iv$

$\rightarrow u^2 + v^2 = c^2$ CR

$$\boxed{\begin{aligned} \frac{d}{dx}&: 2u \cdot u_x + 2v \cdot v_x = 0 \\ \frac{d}{dy}&: 2u \cdot u_y + 2v \cdot v_y = 0 \end{aligned}}$$

f is constant $\rightarrow$ how to proof.

$$\frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = 0$$

$\rightarrow u_x + i v_x = 0, \ \forall z,$ $u_x = 0$ $v_x = 0$

$$\boxed{\begin{aligned} u \cdot \ & 2u \cdot u_x + 2v(-u_y) = 0 \\ v \cdot \ & 2u \cdot u_y + 2 \cdot v(u_x) = 0 \end{aligned}}$$

$+$

$$2 \cdot u^2 \cdot u_x + 2 \cdot v^2 \cdot u_x = 0$$

$$u^2 + v^2 \neq 0$$

$$\boxed{u_x = 0} \qquad \boxed{v_y = 0}$$

$$\Rightarrow \boxed{u_y = 0 \quad v_x = 0}$$

$$u = c_1 \, , \quad v = c_2$$

$$f = \underline{\underline{c_1 + i c_2}}$$