CAI; 201005:

- Questions, ask them!
- I hope you hear me.

(This is out of my home)

At my homepage I just put some old examexercises with their solutions.
They go about:

- Cauchy-Laurent,
- Entire,
- Holomorphic and
- Residues.

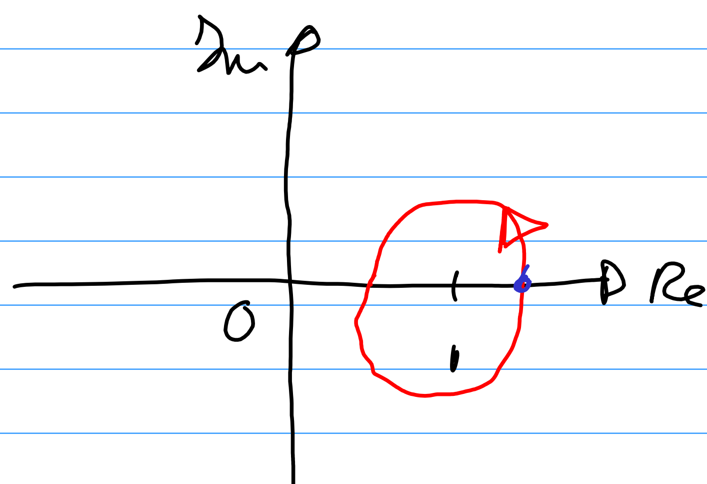
as always first try excercises before reading/looking to the solutions!

If Luc explains Fourier there will also be given some excercise at exam

(2.9) 6:

$\int$?

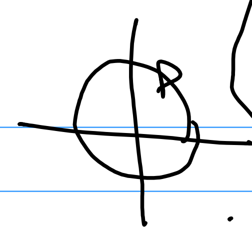
$$f(z) = \frac{1}{z \cdot \log(z)}$$



$z = 1$ difficult. $\log(1) = 0$

$|z-1| = 1/2$ parametrise ??

$$z = 1 + \frac{1}{2}\cdot e^{i\varphi}$$



$$e^{i\varphi}$$

$$0 \le \varphi \le 2\pi \qquad dz = \frac{1}{2} i e^{i\varphi}\, d\varphi$$

$$I = \int_0^{2\pi} \frac{i\,\frac{1}{2}\cdot e^{i\varphi}}{\left(1+\frac{1}{2}e^{i\varphi}\right)\log\left(1+\frac{1}{2}e^{i\varphi}\right)}\, d\varphi$$

$$\ln(z) = |z|\cdot e^{i\,\mathrm{Arg}(z)} \qquad z = 1+\frac{1}{2}e^{i\varphi}$$

$$\ln(z) \qquad (z-1)\ ?! \qquad = 1+x$$

$$\ln(1+x) = x - \frac{1}{2}x^2 \qquad \underline{\ln(1+x)}$$

$$z = 1 + \underbrace{\frac{1}{2}e^{i\varphi}}_{x} \qquad \frac{1}{1+x} = 1 - x + x^2 - \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} \dots$$

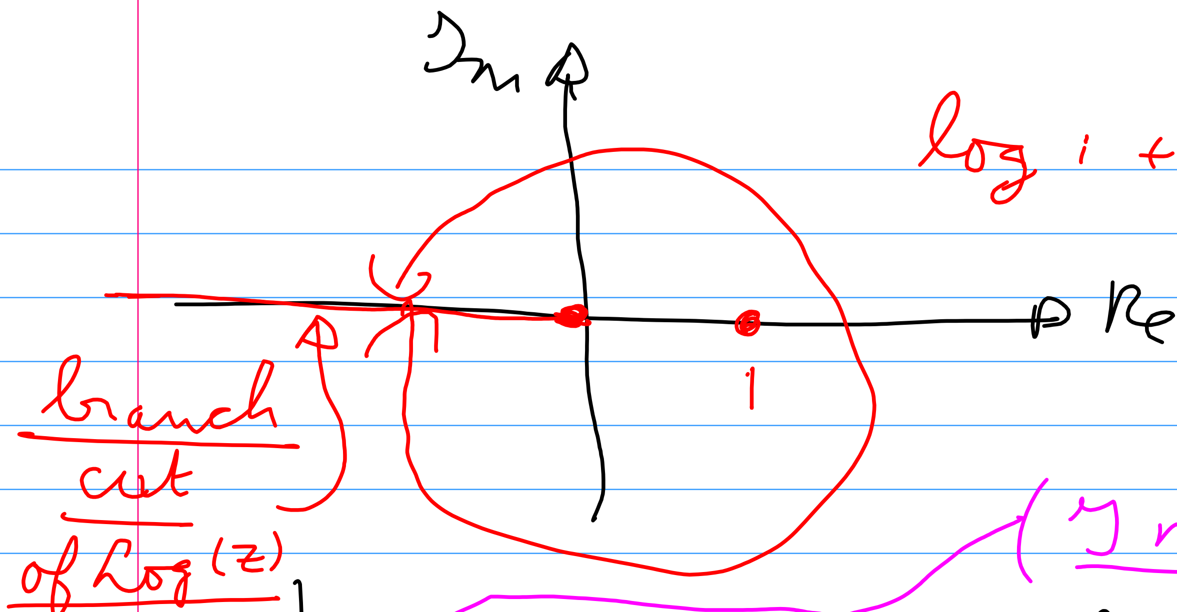
$$\ln(z) = (z-1) - \frac{1}{2}(z-1)^2 \dots$$

$$\frac{1}{z\cdot\ln(z)} = \frac{1}{z}\cdot\frac{1}{\underbrace{\left((z-1) - \frac{1}{2}(z-1)^2 \dots\right)}_{\left(\frac{1}{z-1}\right)}}$$

$$\operatorname*{Res}_{z=1} \frac{1}{z\cdot\ln z} = 1$$

$$\int_{|z-1|=1/2} \frac{1}{z\ln(z)}\, dz = 2\pi i\cdot 1$$

log : $+\pi < \varphi < -\pi$



(I mean log)

$$\frac{1}{z \cdot \ln(z)}$$ analytic, but not

in $z = 1$

$$|z-1| = \frac{1}{2} \to z = 1 + \frac{1}{2}e^{i\varphi}$$

$$0 \le \varphi < 2\pi$$

(z)

$$\ln\Big(\underbrace{1 + \frac{1}{2}e^{i\varphi}}_{x}\Big) = x - \frac{x^2}{2} + \dots$$

$$\ln(z) = (z-1) - \frac{(z-1)^2}{2} - \dots$$

$$\frac{1}{z\ln(z)} = \frac{1}{z}\cdot\frac{1}{(z-1)}\cdot\frac{1}{\left(1 - \frac{(z-1)}{2} + \dots\right)}$$

$z = 0$ outside contour.

$z = 1$ inside contour

$$\operatorname{Res}_{z=1}\left(\frac{1}{z\ln z}\right) = 1 \Rightarrow \oint \dots dz = (2\pi i)\cdot 1$$

(2.6) 5)

$$f(z) = (z-a)^k + O((z-a)^{k+1})$$

$$f'(z) = k\cdot(z-a)^{k-1} + O((z-a)^k)$$

$$\frac{f'(z)}{f(z)} = \frac{k\cdot(z-a)^{k-1} + O((z-a)^k)}{(z-a)^k + O((z-a)^{k+1})}$$

$$\approx \frac{k}{(z-a)} + \underline{O((z-a)^n)} \quad (n=0)$$

$$\frac{1}{2\pi i}\int \dots dz = \frac{1}{2\pi i}\cdot 2\pi i\cdot k = \underline{\underline{k}}.$$

important for the integral (all other terms give: 0.)

(1.6) 1c) $\sum_{n=0}^{\infty} \frac{(n+2)}{n!}(z-1)^n$

exp(...) $(-1)^n \dots$

$$\sum_{n=0}^{\infty} \frac{(n+2)}{n!}(z-1)^n$$

$$\frac{n}{n!} + \frac{2}{n!}$$

$$\frac{n}{n!} = \frac{1}{(n-1)!} \qquad n=0 \quad \frac{n}{n!} = 0$$

$$\sum_{n=0}^{\infty} \frac{2}{n!}(z-1)^n$$

$$2 \cdot \sum_{n=0}^{\infty} \frac{(z-1)^n}{n!} = 2 \cdot \exp(z-1)$$

$$\sum_{n=1}^{\infty} \frac{n}{n!}(z-1)^n =$$

$$\sum_{n=1}^{\infty} \frac{1}{(n-1)!}(z-1)^{((n-1)+1)} =$$

$+$

$$(z-1)^{(n-1)+1} = (z-1)\cdot(z-1)^{n-1}$$

$$(z-1)\cdot \underbrace{\sum_{n=1}^{\infty} \frac{(z-1)^{n-1}}{(n-1)!}}_{n-1=k} = (z-1)\underbrace{\sum_{k=0}^{\infty} \frac{(z-1)^k}{k!}}$$

$$= (z-1)\exp(z-1)$$

f holom. $z = a$

$$f(z) = f(a) + f'(a)(z-a) - - \cdot$$

$$= c_0 + c_1(z-a) - - - \cdots$$

$$\sum_{n=0}^{\infty} c_n \cdot (z-a)^n$$

$$f'(z) = \sum_{n=1}^{\infty} c_n \cdot n \cdot (z-a)^{n-1}$$

$z = a$ $\qquad 0$ $\qquad (0^0 = 1)$

$(0{,}01)^{(0{,}01)}$

$$\lim_{h \to 0} e^{\underbrace{h \cdot \log(h)}} = 1 \qquad \left(0^0 = e^{\ln(0^0)}\right)$$

(2.7) 3 b)

$$f(z) = e^{z} + \frac{1}{(z-1)^2}$$ Laurent

around $z=1$, (conv in $z=0$)

$z = 1 + w$ $w \approx$ small around 0

$$e^{z} = e^{1+w} = e \cdot e^{w} =$$

$$e \cdot \sum_{n=0}^{\infty} \frac{w^n}{n!} = e \cdot \sum_{n=0}^{\infty} \frac{(z-1)^n}{n!}$$

(2.7) 3 a)

centro $z=0$: $e^{z} = \sum_{n=0}^{\infty} \left(\frac{z^n}{n!}\right)$

$\frac{1}{(z-1)^2}$ this around $z=0$?

$$\left(\frac{1}{1-z}\right)' = \left(\sum_{n=0}^{\infty} z^n\right) = \sum_{n=1}^{\infty} n \cdot z^{n-1}$$

$$\frac{-1\cdot(-1)}{(1-z)^2} = \frac{1}{(z-1)^2}$$

$$\frac{1}{(z-1)^2} = \sum_{n=1}^{\infty} n \cdot z^{(n-1)}$$

this is around $z=0$.

~~I have done wrong!~~

My result if $|z|<1$,

so not in $z=2$

exp(z) doesn't matter

$$\frac{1}{(z-1)} = \frac{1}{z\left(1-\frac{1}{z}\right)} =$$

$$\frac{1}{z} \cdot \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)^n \quad \left|\frac{1}{z}\right| < 1 \Rightarrow$$

$|z| > 1$ all goes well