just need part of

CAI-201007

(3.2) (b)

$$0 < \int_0^{2\pi} \frac{1}{\cosh(\varphi) + \cos(\theta)}\, d\theta = F(\varphi)$$

$\varphi > 0$ $\quad \cosh(\varphi) > 1$

$$(\underbrace{\cosh(\varphi) + \cos(\theta)}_{(>0)} \neq 0) \qquad F(\varphi) > 0$$

$$\cos(\theta) = \left(\frac{z + \frac{1}{z}}{2}\right) \quad \text{use } |z| = 1$$

$|z| = 1$: $\quad z = e^{i\theta} \qquad \left(\frac{z + \bar{z}}{2}\right) = \cos(\theta)$

$$\bar{z} = e^{-i\theta} = \frac{1}{z}$$

$$dz = i e^{i\theta} d\theta \qquad d\theta = \frac{dz}{i \cdot z}$$

$$F(\varphi) = \int_{|z|=1} \frac{1}{\frac{2 \cdot \cosh(\varphi) + (z + \frac{1}{z})}{2}} \cdot \frac{dz}{i \cdot z}$$

$$= \int_{|z|=1} \frac{dz}{z^2 + 2z\cosh(\varphi) + 1} \left(\frac{2}{i}\right)$$

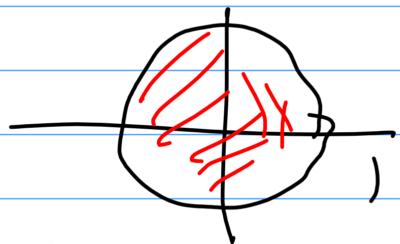
$\cos^2 + \sin^2 = 1 \qquad \cosh^2 - \sinh^2 = 1$

$$z^2 + z\cdot(2\cosh(\varphi)) + 1$$

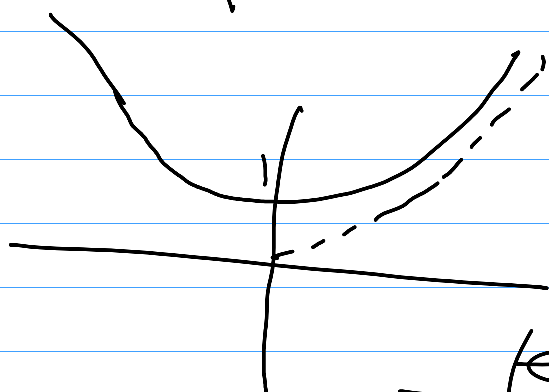
$$(z + \cosh(\varphi))^2 + \underbrace{(1 - \cosh^2\varphi)}$$

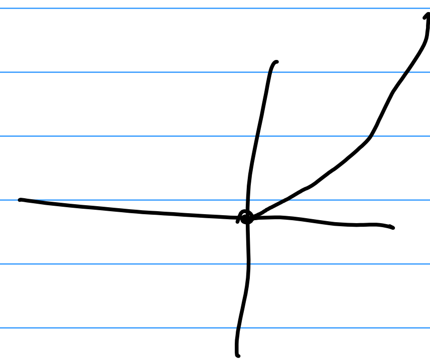
$$(z + \cosh(\varphi))^2 - \sinh^2(\varphi) = 0$$

$$z = \boxed{-\cosh(\varphi)} \pm \sinh(\varphi)$$



$|z| = 1$





e^x

$$-\left(\frac{e^{\varphi} + e^{-\varphi}}{2}\right) \pm \left(\frac{e^{\varphi} - e^{-\varphi}}{2}\right)$$

$$\frac{-e^{\varphi} - e^{-\varphi} \pm (e^{\varphi} - e^{-\varphi})}{2} =$$

$$-\frac{2e^{\varphi}}{2} = \boxed{-e^{\varphi}} \qquad \varphi > 0$$

$$\frac{-\cancel{e^{\varphi}} - e^{-\varphi} + \cancel{e^{\varphi}} - e^{-\varphi}}{2} = \boxed{-e^{-\varphi}}$$

$$\lim_{z \to -e^{-\varphi}} \frac{\cancel{(z+e^{-\varphi})}}{(z+e^{\varphi})\cancel{(z+e^{-\varphi})}} = \frac{1}{(-e^{-\varphi}+e^{\varphi})}$$

$$\left(\frac{2}{i}\right)(2\pi i)\frac{1}{(-e^{-\varphi}+e^{\varphi})} =$$

$$\frac{2\pi}{\sinh(\varphi)} \leftarrow (\neq 0) \qquad \varphi > 0$$

(2.9) (11)

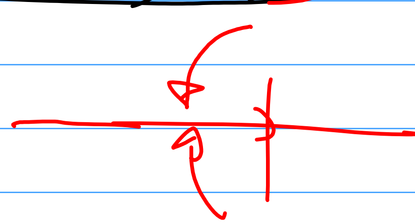
Theorems?? Liouville

$$\lim_{z \to c} \frac{(h(z))^2}{z} = 1$$

h entire $\quad h^2(z)$ entire

$$h^2(z) = \boxed{z + A} \qquad h(z) = \sqrt{z+A}$$

$\sqrt{\ }$



$g'(x)$

$$\frac{(h^2(z))'}{z^3} = \frac{Az^2+Bz+C}{z^3}$$

$$h^2(z) = Az^2 + Bz + C$$

$\boxed{h(z)}$ $\quad B=0,\ C=0 \quad h(z) = c \cdot z^2$

$A=0,\ B=0 \quad \boxed{h(z) = C}$

$$\frac{c \cdot z^4}{z^3}$$

$$h(z) = \sqrt{Az^2 + \cancel{Bz} + \cancel{C}}$$

$$h^2(z) = Az^2 + Bz + C$$

$$h(z) = \text{“}\sqrt{Az^2+Bz+C}\text{”}$$

$B=0,\ C=0 \quad h(z) = c \cdot z$

$$\frac{h'(z)}{z^3} = \frac{c' \cdot z^2}{z^3} \to 0$$

$A=0,\ B=0$

$h(z) = C$ $\quad \frac{c^2}{z^3} \to 0 \ z \to \infty.$

Wolfram alpha

residue(f(z), z, 0)

(3.1) $\dfrac{z^2+1}{z(2z-1)(z+1)}$ $2(z-\frac{1}{2})$

$\boxed{z=\frac{1}{2}}$ $\dfrac{\cancel{(z-\frac{1}{2})}}{2}$ $\dfrac{z^2+1}{z\cancel{(2z-1)}(z+1)}$

$\dfrac{(\frac{1}{2})^2+1}{\cancel{2\cdot\frac{1}{2}}\,(\frac{3}{2})} = \frac{5}{4}\cdot\frac{2}{3} = \frac{10}{12} = \frac{5}{6}$

(1.6)(1 d)

$\frac{1}{z}\sum_{n=2}^{\infty}\frac{n}{(2n+1)!}z^{2n+1}$ $|z|<\infty$

$\sinh(z) = \dfrac{e^{z}-e^{-z}}{2} =$

z^{2n+1}

$$\frac{d}{dz}\left(\frac{1}{2}\sum \frac{1}{(2n+1)!} z^{(2n)}\right) =$$

$$\frac{n}{\ } \quad \cancel{\frac{1}{2}} \sum \frac{(2n)}{(2n+1)!} z^{2n}$$

$$\frac{1}{2}\,\frac{1}{z}\sum_{n=2}^{\infty} \frac{1}{(2n+1)!} \cdot z^{2n+1}$$

$$\sum_{n=3}^{\infty} z^n = \left(\frac{1}{1-z} - 1 - z - z^2\right)$$

$\sinh(z)$. $\cosh(z)$??

$$\sum_{n=-\infty}$$

$$\sum_{n=2}^{\infty} \frac{n}{(2n+1)!} z^{2n} =$$

$$\frac{1}{2} \cdot \sum_{n=2}^{\infty} \frac{(2n)}{(2n+1)!} z^{2n} =$$

$$\frac{1}{2} \cdot \sum_{n=2}^{\infty} \frac{((2n+1)-1)}{(2n+1)!} \cdot z^{2n} =$$

$$\frac{1}{2} \cdot \left(\sum_{n=2}^{\infty} \frac{z^{2n}}{(2n)!} - \sum_{n=2}^{\infty} \frac{z^{2n}}{(2n+1)!} \right) =$$

$$\cosh(z) = \frac{e^{z} + e^{-z}}{2} =$$

$$\frac{\left(1 + \cancel{z} + \frac{z^2}{2} + \frac{z^3}{3!} \cdots \right) + \left(1 \cancel{- z} + \frac{z^2}{2} - \frac{z^3}{3} \cdots \right)}{2}$$

$$= \frac{2 + z^2 + 2 \cdot \frac{z^4}{4!} + 2 \cdot \frac{z^6}{6!} \cdots}{2} =$$

$$\left(1 + \frac{z^2}{2} + \frac{z^4}{4!} \cdots \right) = \sum_{n=0}^{\infty} \frac{(z^2)^n}{(2n)!}$$

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$$\sinh(z) = \frac{2z + 2\cdot\frac{z^3}{3!} + 2\cdot\frac{z^5}{5!} \cdots}{2}$$

$$= \sum_{n=0}^{\infty} \frac{z^{(2n+1)}}{(2n+1)!}$$

Combine these series and subtract the terms you don't need!