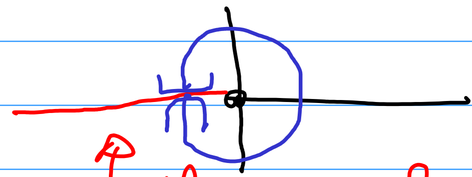
CAI : 201007

- Questions, I like to hear them.

$\sqrt{z}$



$-\pi < \varphi \leq \pi$

(branch cut) $(R > 0)$

$\sqrt{a^2 - z^2}$, $z \in \mathbb{R}$, when

$a^2 - z^2 > 0 \rightsquigarrow z^2 < a^2$

$-a < z < a$

(branch cut : $z < -a$, $z > a$)

(2.7) 10 a)

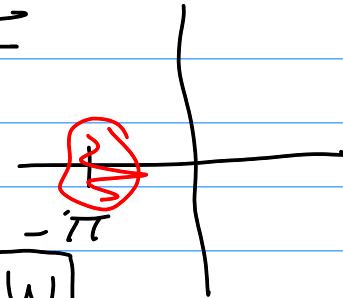
$z=0$ essent. singu

$$f(z) = \frac{z \cdot e^{1/z} \cdot \sin(z)}{(z-2)^2 (z+\pi)^2}$$

ε

$\underline{z=2}$: pole ~ $\underline{\text{order } 2}$

$\underline{z=-\pi}$: $z \approx -\pi$

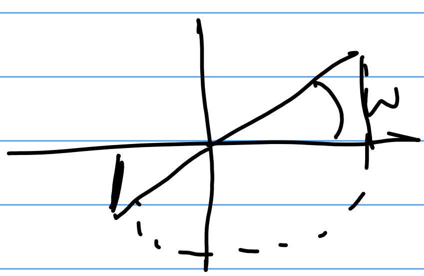


$$\boxed{z = -\pi + w}$$

$\underline{w \text{ is small.}}$

$$f(-\pi + w) = \frac{(-\pi+w)\, e^{\frac{1}{-\pi+w}}}{(-2-\pi+w)^2} \cdot \frac{\sin(-\pi+w)}{w^2}$$

$$\sin(-\pi + w) = -\sin(w)$$



$$-\frac{\sin(w)}{w^2} = -\left(\frac{\sin(w)}{w}\right) \cdot \frac{1}{w}$$

(12) $\underset{(z=a)}{\text{Res}}\ f(z) = ??$

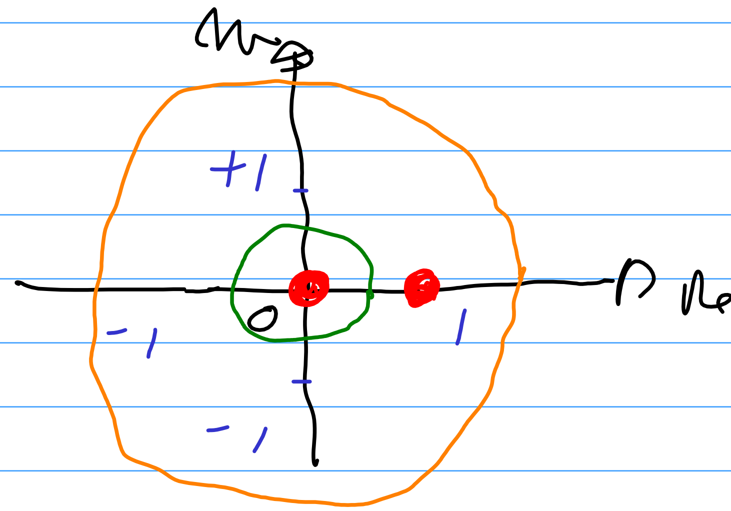
$$\boxed{\sum_{n=-\infty}^{\infty} c_n (z-a)^n} \qquad n=-1$$

Residue: coeff of $\frac{1}{(z-a)}$ term

$$\boxed{c_{-1}}$$

c) First:

$$\int_{|z|=R} \frac{e^{1/z}}{(1-z)^2}\, dz$$



* $1 < R$; → two singul., 2 residues
* $0 < R < 1$; → one singul. , 1 residue

$$\underset{z=0}{\text{Res}}\left(\frac{e^{1/z}}{(1-z)^2}\right)$$

$$\lim_{z\to 0}\left(z\cdot f(z)\right)$$

$$z\cdot f(z) = z\cdot\sum_{n=-\infty}^{+\infty} c_n\cdot z^n =$$

$$\sum_{n=-\infty}^{+\infty} c_n\cdot z^{n+1}$$

$n+1=-1$

$n=-2$

$$e^{\frac{1}{z}} = \sum_{n=0}^{\infty}\frac{\left(\frac{1}{z}\right)^n}{n!} =$$

$$\left(1+\frac{1}{z}+\frac{1}{2}\frac{1}{z^2}+\frac{1}{6}\cdot\frac{1}{z^3}\cdots\right)$$

$$\frac{1}{(1-z)^2} = 1+2z+3z^2+\cdots$$

$$\left(\frac{1}{1-z}\right)' = \left(\sum_{n=0}^{\infty} z^n\right)' = \sum_{n=1}^{\infty} n\cdot z^{(n-1)}$$

$$1+1+\frac{1}{2}+\frac{1}{3!}+\frac{1}{4!}$$

$$\frac{1}{z}+\frac{1}{z}+\frac{1}{2}\cdot\frac{1}{z}+\cdots = e$$

$$\frac{1}{z}\left(1+1+\frac{1}{2}+\frac{1}{6}\cdots\right)$$

$\frac{1}{z}$!!

$\to 0$

$$\cancel{\lim_{z\to 0}\left(z\cdot\frac{e^{\frac{1}{z}}}{(1-z)^2}\right)} =$$

$\exp(x) = 1 + x + \frac{x^2}{2} + \cdots$

$1 + 1 + \frac{1}{2} + \cdots$

$$\frac{e^{\frac{1}{z}}}{(1-z)^2}$$

$\boxed{z = 1}$

$$z = 1 + w$$

$$\frac{e^{\frac{1}{1+w}}}{w^2}$$

??

$$e^{\frac{1}{z}} = e^{\left(\frac{1}{1+(z-1)}\right)} =$$

$(z-1)$ small

$$1 - (z-1) + (z-1)^2 - (z-1)^3 - \cdots$$

$$e$$

$$= e \cdot \frac{e^{\left((z-1)+(z-1)^2 - \cdots\right)}}{(z-1)^2}$$

$$\frac{A}{z-a}$$

$$\frac{e^{\frac{1}{z}}}{(1-z)^2} \qquad z = 1+w$$

$$\frac{e^{\frac{1}{1+w}}}{(-w)^2} = \frac{e^{(1-w+w^2-w^3 - \cdots)}}{w^2}$$

$$= \frac{e \cdot e^{(-w+w^2-w^3 - \cdots)}}{w^2} =$$

!!

$$\frac{e \cdot \left(1 + (-w + w^2 - \cdots) + \frac{(-w+w^2 - \cdots)^2}{2} + \cdots\right)}{w^2}$$

$\boxed{\frac{1}{w}}$! (the only one)

$$\operatorname*{Res}_{z=1} \frac{e^{\frac{1}{z}}}{(1-z)^2} = e \cdot (-1) = -e$$

If $R>1$: $\int \cdots dz = 2\pi i(e-e) = 0$