2DME30-201012;

- Questions, let hear them!

(2.6) 5)

$$\int_\gamma \{\cdot\}\, dz \quad \frac{1}{z-a} \rightsquigarrow \text{value} \neq 0$$

residues: $\frac{1}{z}$

$$\frac{1}{(z-a)}$$

$$\rightsquigarrow 0$$

$f(z)$ $z = a$

$$\rightsquigarrow f(z) = \frac{1}{(z-a)^k} \cdot g(z) \qquad g(a) \neq 0.$$

$$f(z) = (z-a)^{-k} \cdot g(z)$$

$$f'(z) = -k(z-a)^{-k-1} \cdot g(z) + (z-a)^{-k} \cdot g'(z)$$

$$\frac{f'(z)}{f(z)} = \left(\frac{-k(z-a)^{-k-1} \cdot g(z) + (z-a)^{-k} \cdot g'(z)}{(z-a)^{-k} \cdot g(z)} \right)$$

$$-k \cdot \frac{(z-a)^k}{(z-a)^{k+1}} + \frac{\cancel{(z-a)^k} \cdot g'(z)}{\cancel{(z-a)^k} \cdot g(z)}$$

$g(a) \neq 0$

$$\frac{-k}{(z-a)^1}$$

$$\frac{1}{z-a}$$

$$\frac{2\pi i}{2\pi i} \cdot (-k)$$

(2.6)(b) ! L ! Liouville

a) $\lim_{z \to \infty} h(z) = 1$ → "h(z) = polynomial"

entire ?? $h(z) = 1$

b) $g(z) = (z \cdot h(z))$

$\lim_{z \to \infty} g(z) = 1 \to g(z) = 1$

$z \cdot h(z) = 1 \quad h(z) = \frac{1}{z} \quad (z \neq 0)$

c) $g(z) = z^2 \cdot h(z)$

$$\lim_{z \to \infty} g(z) = 0$$

$g(z) = 0$ $\qquad$ $\underline{\underline{h(z) = 0}}$

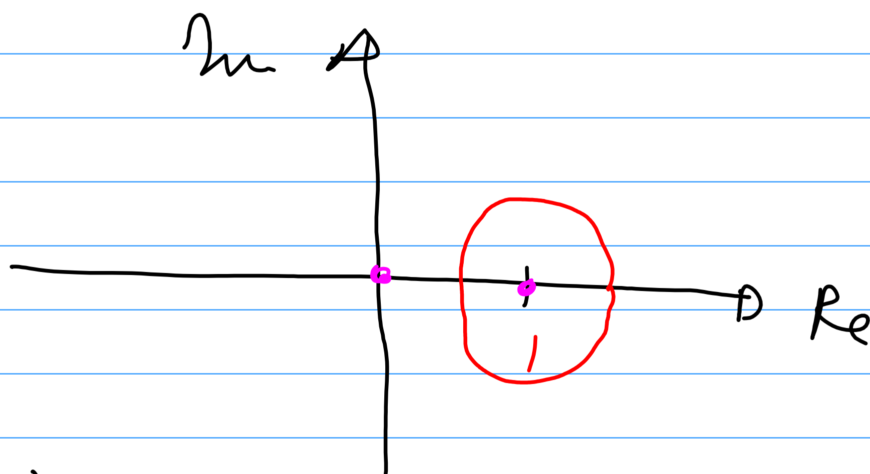
<u>if $g(z)$ entire</u> $\to$

$g(z)$ has taylor series in whole $\mathbb{C}$-plane

see (2.6.3) pg. 41.

<u>only $g(z) = 0 \ \forall z$</u>

(2.9) (6)



$\frac{1}{z}$, $\frac{1}{\log(z)}$

$z = 1 + w$ $\qquad$ $|w| < \frac{1}{2}$

$\log(1+w) = w + \mathcal{O}(w^2)$??

$$\frac{1}{\log(z)} = \frac{1}{\log(1+w)} = \frac{1}{w + O(w^2)}$$

$$= \frac{1}{w} \cdot \frac{1}{(1 + O(w))}$$

$$\frac{1}{z} = \frac{1}{1+w} = (1 - w + O(w^2))$$

$$\frac{1}{z} \cdot \frac{1}{\log(z)} = \frac{1}{w} \cdot \frac{1}{(1+O(w))} \cdot$$

$$(1+\dots) \quad (1 - w + O(w^2))$$

$$\frac{1}{z} \cdot \frac{1}{\log(z)} = \frac{1}{w} + O(1)$$

$|w| < \frac{1}{2}$

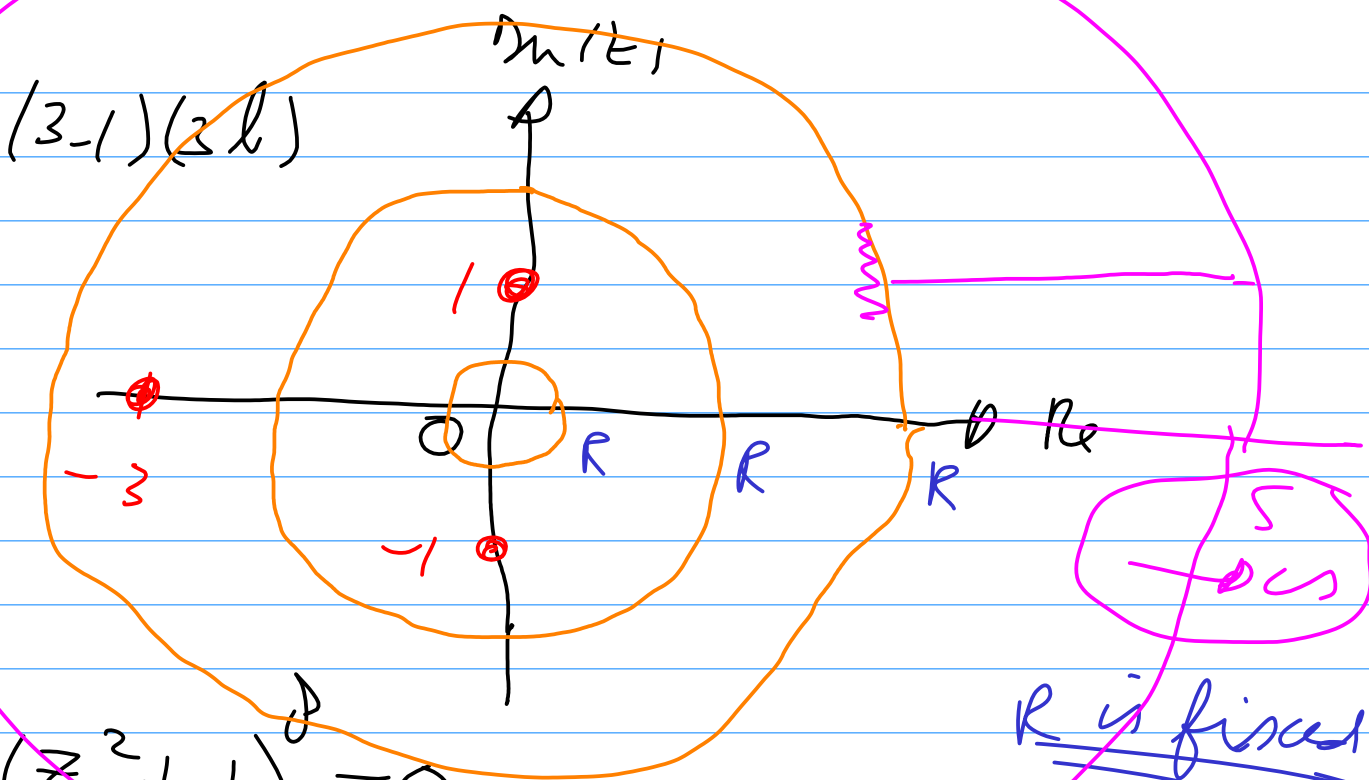
$$\int_{|w|=\frac{1}{2}} \frac{1}{w}\, dw = 2\pi i \cdot 1$$

$z = 1 + w$

$dz = dw$

$|z - 1| = \frac{1}{2}$

$|w| = \frac{1}{2}$



5

0

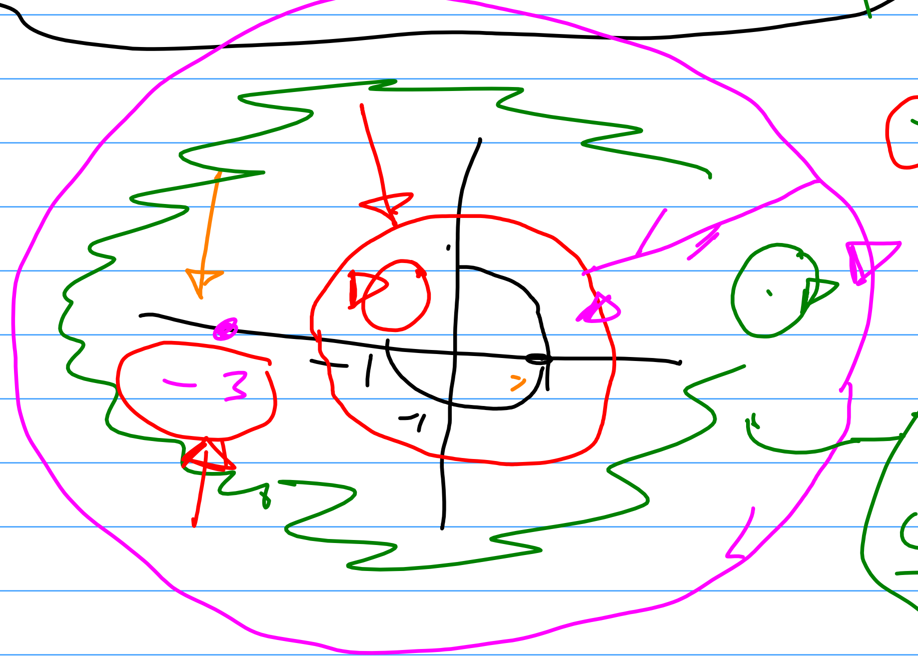
R is fixed

$$(z^2+1)^8 = 0$$

$z = \pm i$ $\qquad$ $z = -3$

$$\boxed{\frac{1}{((z-i)(z+i))^8} \cdot \frac{1}{(z+3)}}$$

??



$-z+i$

$|z| = R$

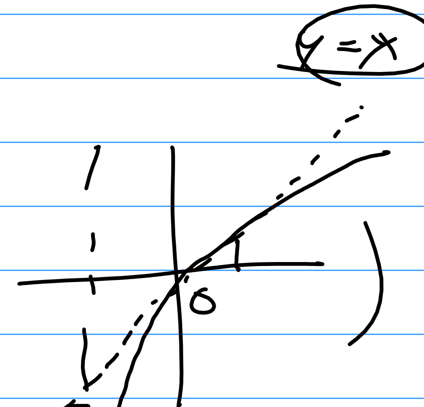
inside clockwise

(3-1)(5) (c)

$$\frac{1}{(\log(1+z))^2} =$$

$$\log(1+z) = z + O(z^2)$$

$$\left(\underline{\ln(1+x) = x + O(x^2)} \right)$$



$$(\log((1+z))^2 = z^2 + O(z^3)$$

$$\frac{1}{(\log(1+z))^2} = \frac{1}{z^2+O(z^3)} =$$

$$\frac{1}{z^2}\left(\frac{1}{1+O(z)}\right) \sim \frac{c}{z}$$

$$\frac{1}{1+(\frac{1}{z})} = \sum \left(\frac{-1}{z}\right)^n$$

gm $\left(\frac{1}{z}\right)$

$$\left|\frac{1}{z}\right| < 1 \quad |z| > 1$$

$$= \frac{z}{1+z}$$

$z=0$

(3.1) 5) (l) $\frac{1}{1+\frac{1}{z}} = \frac{z}{(1+z)}$

$z=0$ no singularity.

(3.2) (6) $\int_0^{2\pi} \cdots \, d\theta$

$z = e^{i\theta} \rightsquigarrow \cos(\theta) = \left(\frac{z+\bar{z}}{2}\right) = \left(\frac{z+\frac{1}{z}}{2}\right)$

$\bar{z} = e^{-i\theta} = \frac{1}{z}$

$\boxed{|z|=1}$

$\int_0^{2\pi} \cdots d\theta = \int_{|z|=1}$

$z = e^{i\theta} \quad dz = i\underbrace{e^{i\theta}}_{=z} d\theta$

$d\theta = \left(\frac{1}{iz} \cdot dz\right)$

$$\overline{\left(1+a^2+2a\cdot\left(\frac{z+\frac{1}{z}}{2}\right)\right)}\cdot\frac{dz}{i\cdot z}$$

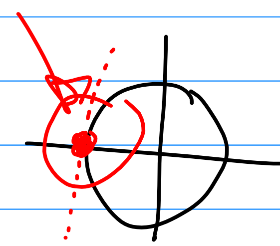
$$\int_{|z|=1}\frac{1}{i}\,\frac{dz}{(1+a^2)z+a\cdot z^2+a}$$

$$\left(a\cdot z^2+(1+a^2)z+a\right)=0$$

$$(a\cdot z+1)(z+a)=0$$

$$a\cdot z=-1\,,\quad z=-a$$

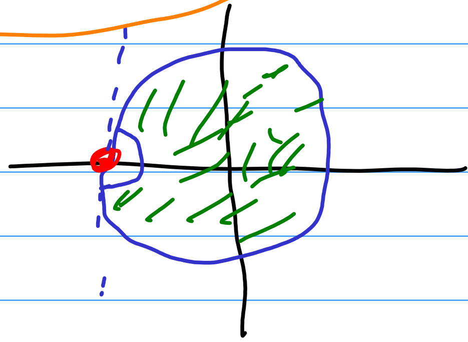
$\underline{a>0}$ $\quad z=-\frac{1}{a}\,,\quad z=-a.$



$\underline{a>1}$: $\quad z=-\frac{1}{a}$

$\underline{0<a<1}$: $\quad z=-a$ $\quad\frac{1}{(z+1)^2}$

$\leadsto$ $\underline{a=1}$: $\quad(z+1)^2$



$$\left(\frac{1}{2}\cdot\operatorname{Res}_{z=-1}\right)$$

b) φ is given, fixed

$$f(\varphi) = \int_0^{2\pi} \frac{1}{\left(\cosh(\varphi) + \cos\theta\right)} d\theta$$

$$\left(\frac{e^{\varphi} + e^{-\varphi}}{2} + \left(\frac{z + \frac{1}{z}}{2}\right)\right) iz \qquad \frac{1}{} dz$$

$(z + e^{-\varphi})(z + e^{\varphi})$??

$\varphi > 0$ $\qquad$ $(e^{\varphi} > 1)$

$$z^2 + 1 + z(e^{\varphi} + e^{-\varphi}) =$$

$$(z + e^{\varphi})(z + e^{-\varphi})$$

$$(e^{\varphi} \cdot e^{-\varphi} = 1)$$

$$\frac{2\pi i}{i} \cdot \operatorname{Res}_{z = -e^{-\varphi}} \left(\frac{1}{(z + e^{\varphi})(z + e^{-\varphi})} \right) =$$

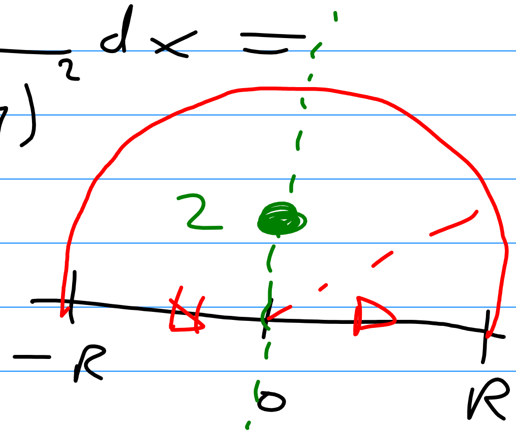
$$2\pi \cdot \frac{1}{(-e^{-\varphi} + e^{\varphi})} = \frac{\pi}{\sinh(\varphi)} = f(\varphi)$$

!! $\varphi > 0$ $\qquad$ $\sinh(0) = 0$

→ Notes put on my website.

(3.3) b) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{(x^2+4)^2}\,dx =$

$\displaystyle\lim_{R\to\infty} \int_{-R}^{R} \dots dx$



$|x| = R$ $\quad \boxed{|x^2+4| \geq \big|\,|x^2| - 4\,\big|} =$

$= (R^2 - 4) \qquad (R > 2)$

$$\frac{1}{|x^2+4|^2} \leq \frac{1}{(R^2-4)^2}$$

$\boxed{|a+b| \geq \big|\,|a| - |b|\,\big|}$