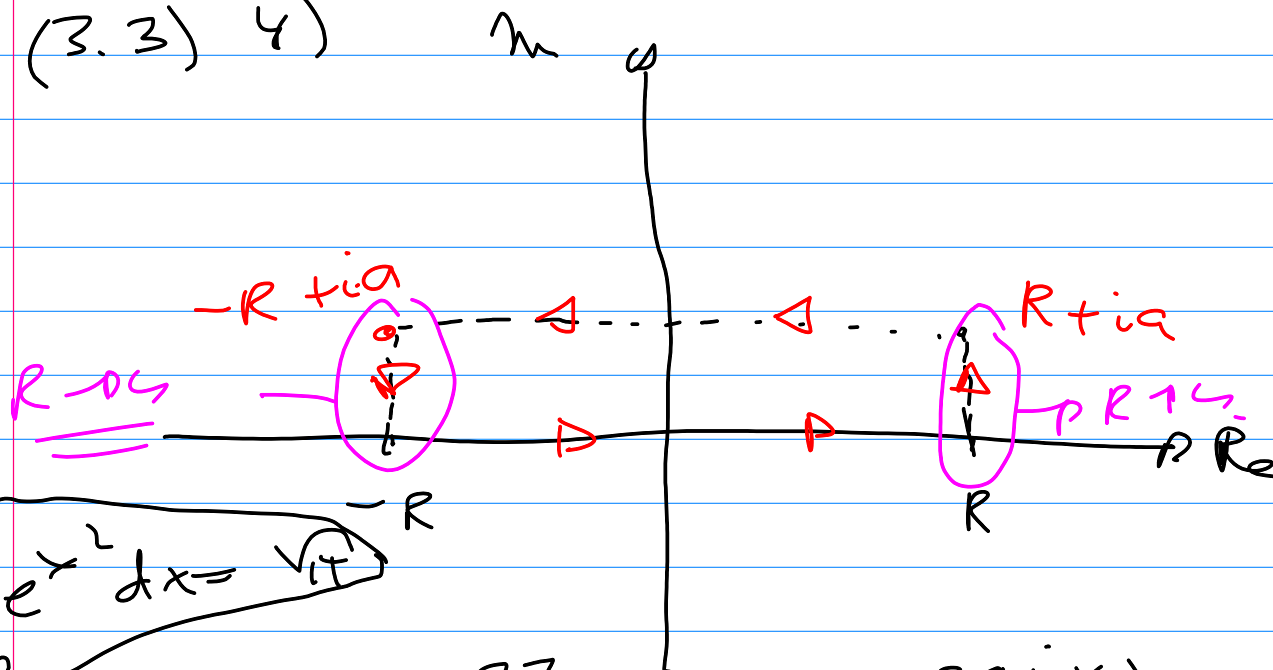
2CAI-201014:

- Questions? Just ask them!

(I started a little bit too early.)

(3.3) 4)



$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

$\cos(2a\cdot x)$?? $\qquad \mathrm{Re}\left(e^{2aix}\right)$

$$\int_{-R}^{R} f(x,a)\,dx = \mathrm{Re}\int_{-R}^{R} e^{-x^2+2aix}\,dx$$

$$\int_{-R+i\alpha}^{R+i\alpha} e^{-z^2}\,dx = \int_{-R}^{R} e^{-(t+i\alpha)^2}\,dt$$

$$= e^{\alpha^2}\cdot\int_{-R}^{R} e^{-t^2-2ti\alpha}\,dt =$$

$$\left(\int_{-R}^{R} - dt = 0 \right)$$

$$e^{-2ti\alpha} = \cos(2t\alpha) + \underbrace{i \sin(2 \cdot t\alpha)}_{\text{odd}}$$

$$e^{\alpha^2} \cdot \int_{-R}^{R} e^{-t^2} \cdot \cos(2t\alpha)\, dt$$

$$\int_{-R+i\alpha}^{R+i\alpha} e^{-z^2}\, dz = e^{\alpha^2} \cdot \int_{-R}^{R} e^{-t^2} \cdot \cos(2t\alpha)\, dt$$

$$\int_{-R}^{R} e^{-(t+i\alpha)^2}\, dt = \sqrt{\pi} \quad (\text{if } R \to \infty)$$

$$e^{\alpha^2} \cdot \int_{-R}^{R} e^{-t^2} \cos(2\alpha t)\, dt = \sqrt{\pi} \quad (R \to \infty)$$

$$\int_{-R}^{R} e^{-x^2} \cos(2\alpha x)\, dx \text{``}=\text{''} \int_{-R+i\alpha}^{R+i\alpha} e^{-z^2}\, dz$$

integrals along $(R - (R+i\alpha))$ and $(-R - (-R+i\alpha)) \to 0$

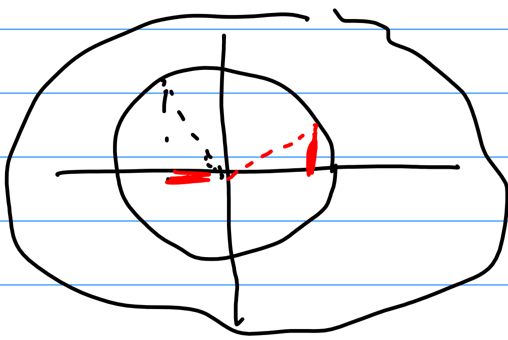
(2.7) (21)

$$\cos(z) = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$$

$z = \frac{\pi}{2}$

$z = \left(\frac{\pi}{2} + y\right)$ (y small)

$$\cos(z) = \cos\left(\frac{\pi}{2} + y\right) = \sin(y)$$



$$\sin y = y - \frac{y^3}{3!} + \cdots$$

$y = \left(z - \frac{\pi}{2}\right)$

$$\frac{1}{\sin y} = \frac{1}{\left(y - \frac{y^3}{3!} \cdots\right)} = \frac{1}{y} + \cdots$$

$$\frac{1}{y\left(1 - \frac{y^2}{3!} + \cdots\right)}$$

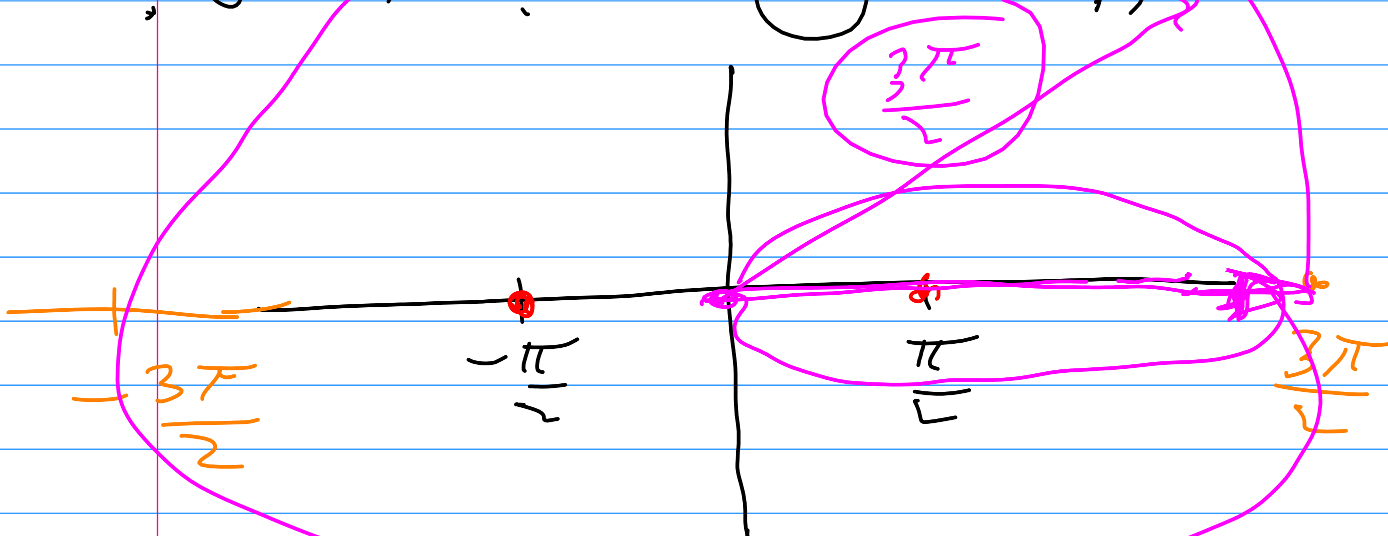
1

$$\frac{1}{z - \frac{\pi}{2}} + \frac{a}{(z - \pi/2)}$$ $(a = -1)$

$$z = -\frac{\pi}{2} + y$$

$$\cos(z) = \cos\left(-\frac{\pi}{2}+y\right) = -\sin(y)$$

?"$\ell = +1$"?



$\cos(z)=0$ $\quad \left(\frac{\pi}{2}+k\cdot\pi\right) \quad k\in\mathbb{Z}$

"To see if answer is correct:

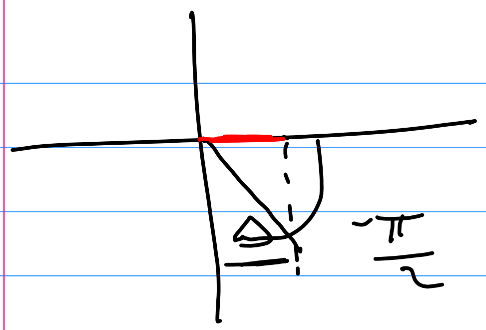
$$z = \frac{\pi}{2}+\frac{1}{10}$$

$$\cos\left(\frac{\pi}{2}+\frac{1}{10}\right) = -\sin\left(\frac{1}{10}\right) \approx -\frac{1}{10}$$

$$\frac{1}{\left(-\frac{1}{10}\right)} + \frac{a}{\left(\frac{1}{10}\right)} \rightsquigarrow \boxed{a=+1}$$

$$z = -\frac{\pi}{2}+\frac{1}{10} \qquad \cos\left(-\frac{\pi}{2}+\frac{1}{10}\right) = \sin\left(\frac{1}{10}\right)$$

(>0) $= +\frac{1}{10}$



$$\frac{1}{\left(\frac{1}{10}\right)} + \frac{b}{\left(\frac{1}{10}\right)}$$

$$\boxed{b = -1}$$

$$f(z) = \frac{1}{\cos(z)} + \frac{1}{z-\frac{\pi}{2}} - \frac{1}{z+\frac{\pi}{2}}$$

$$\frac{\left(z+\frac{\pi}{2}\right)-\left(z-\frac{\pi}{2}\right)}{\left(z^2-\frac{\pi^2}{4}\right)} = \frac{\pi}{z^2-\left(\frac{\pi}{2}\right)^2} =$$

$$\frac{-\pi}{\left(\frac{\pi}{2}\right)^2\left(1-\left(\frac{2}{\pi}z\right)^2\right)} =$$

geom. series

$$\frac{-\pi}{\left(\frac{\pi^2}{4}\right)}\cdot\left(1+\left(\frac{2}{\pi}z\right)^2+\left(\left(\frac{2}{\pi}z\right)^2\right)^2 + \dots\right)$$

$$\frac{1}{\cos(z)} = \frac{1}{\left(1-\frac{z^2}{2}+\frac{z^4}{4!}\dots\right)} =$$

$$\frac{1}{1-\left(\frac{z^2}{2}-\frac{z^4}{4!}+\cdots\right)} \overset{\text{geom. series}}{=}$$

$$1+\left(\frac{z^2}{2}-\frac{z^4}{4!}\cdots\right)+\left(\frac{z^2}{2}-\frac{z^4}{4!}\right)^2 + \cdots$$

$(2,2)(5c)$

$\underline{|z|=R}$

$$|z+1| \geq \big||z|-1\big| = |R-1|$$

$$=(R-1) \qquad \underline{\underline{(R>1)}}$$

$$|z+1| \geq (R-1)$$

$$\left(\frac{1}{|z+1|}\right)^2 \leq \left(\frac{1}{(R-1)}\right)^2$$

$$|z^2+2z+1| \geq \big||z^2|-\underbrace{|2z+1|}\big|$$

$$(|z^2|>|2z+1|) = |z^2|-|2z+1|$$

$$|2z+1| \leq 2R+1 \quad -|2z+1| \geq -2R-1$$

$$|z^2+2z+1| \geq R^2 - 2R - 1$$

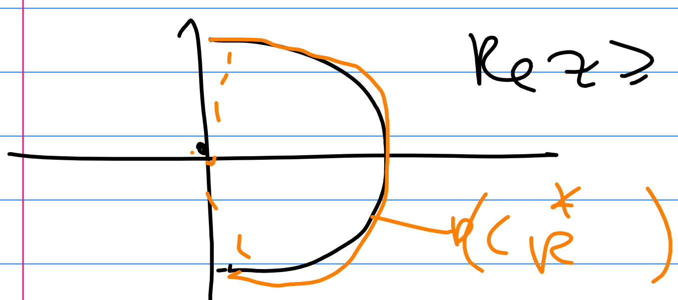
$$\frac{1}{|z^2+2z+1|} \leq \left(\frac{1}{R^2-2R-1}\right)$$

$$\left[\left(\frac{\pi R}{R^2 \ldots} \right) \to 0 \text{ if } R \to \infty \right]$$

$$|z+1| \geq ||z|-1|$$

$$x+iy \quad \subset C_R^+ \qquad (y>0)$$

$$|e^{-z}| = |e^{-(x+iy)}| = |e^{-x}| < 1$$



$\mathrm{Re}\, z \geq 0$ $\quad x \geq 0$ $\quad |e^{-x}| \leq 1$

$$|z^2+2z+1| \geq ||z^2+2z|-1||$$

$$(|z^2+2z| > 1 \; ?!)$$

know

$$|z+1| \geq ||z|-1| \qquad |z| = R$$

$$|z+1| \geq |R-1| = (R-1)$$

$$(R>1)$$

$$|z_1| - |z_2|$$

$$|\underbrace{z^2+2z}_{z_1}+\underbrace{1}_{z_2}| \geq \left| \underbrace{|z^2+2z|}_{?\, |z^2+2z|>1\, ?} - 1 \right|$$

$$(z+1)^2 \geq \cdots$$

$$|z^2+2z| \geq \left| \underbrace{|z^2| - |2z|} \right| =$$

$$(|z^2| > |2z|) \qquad = |z^2| - |2z| =$$

$$= R^2 - 2R$$

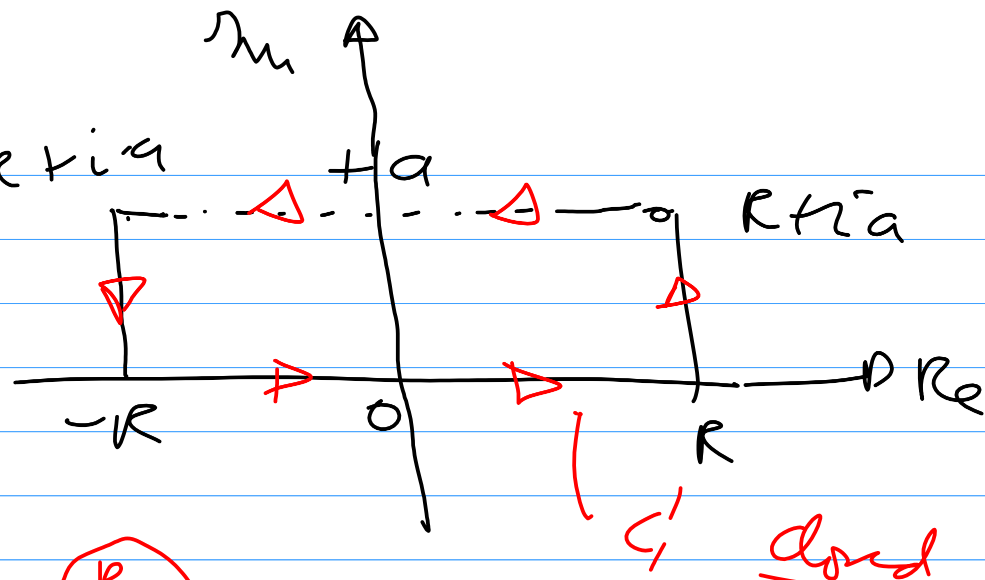
$$|(z+1)^2| \geq R^2-2R-1$$

$$\left(R^2-2R-1 = ((R-1)^2-2)\right)$$

$$|\cdots| \leq \frac{\pi R}{R^2-2R-1} \to 0$$

$$R > 3$$

(3.3) 4)



$(a>0)$

$$\int_{-\infty}^{+\infty} \rightsquigarrow \int_{-R}^{R} \text{ real axis}$$

$(R \to \infty)$

$$\int_{-\infty}^{+\infty} e^{-x^2} \cdot e^{2iax}\, dx \quad \int_{-R}^{R} \sin(2a\cdot x)\, dx = 0$$

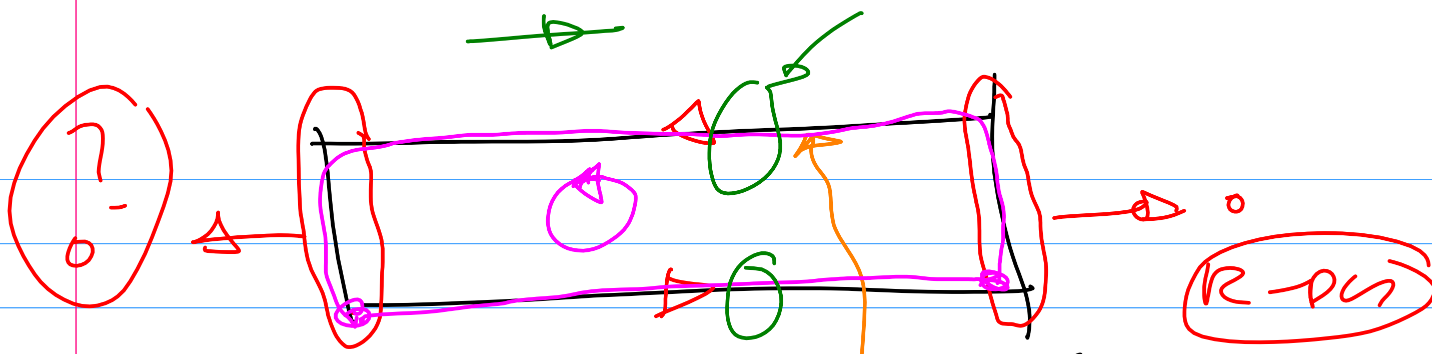
$\cos(2a\cdot x) = \mathrm{Re}\left(e^{i2a\cdot x}\right)$

$$\| $$

$$\int_{-\infty}^{+\infty} e^{-(x+ia)^2} \cdot e^{-a^2}\, dx$$

$$e^{-a^2} \cdot \int_{-R}^{R} e^{-(x+ia)^2}\, dx$$

$$\oint_{C_1} e^{-(z+ia)^2}\, dz = 0$$



$$\int_{\to} + \int_{\leftarrow} = 0$$

$$\int_{-R}^{R} \cdots dx = -\int_{\leftarrow} \cdots = \int_{-R+ia}^{R+ia}$$

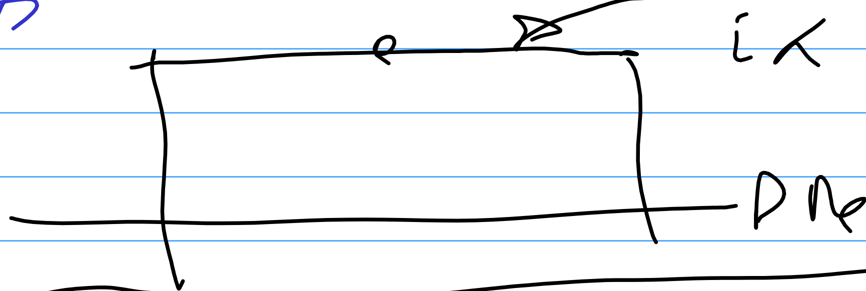
$$e^{-a^2} \int_{-R+ia}^{R+ia} e^{-(z+ia)^2} dz \to \sqrt{\pi}$$

$$\cancel{|\cos(z)| \le 1}$$

$$\int_{-R}^{R} e^{-x^2+2iax} dx = e^{-a^2} \int_{-R}^{R} e^{-(x+ia)^2} dx \to \sqrt{\pi}$$

$$\int_{-R+i\alpha}^{R+i\alpha} e^{-z^2}\,dz = \int_{-R}^{R} e^{-(x+i\alpha)^2}\,dx$$

$\sqrt{\pi}$



$$z = (x+i\alpha) \qquad -R \le x \le R$$

$$e^{\alpha^2}\cdot\int_{-R}^{R} e^{-x^2}\cdot\cos(2ax)\,dx$$

$$\int e^{-t^2}\,dx = \sqrt{\pi}$$

$$e^{2iax} = \cos(-\,-) + i\sin$$

$$\int_{R}^{R+i\alpha} \qquad |e^{-z^2}| = |e^{-(R+iy)^2}| =$$

$(0 \le y \le \alpha)$ $\quad R$ [illegible]

$$\cos(ix) = \cosh(x) \ge 1$$

$$e^{-R^2}\cdot \max_{0\le y\le a} |e^{-2Riy+y^2}| \le e^{-R^2} \max_{0\le y\le a} e^{y^2} \le e^{a^2} e^{-R^2}$$

$$|e^{-z}| = |e^{-(x+iy)}| = e^{-x} \le 1$$

$z = x + iy \quad (x \geq 0)$

$$\rightsquigarrow |e^{-z}| \le 1$$

$z \in C_R^*$

(3.3) (4) I will write out and put at my website