2 CA I – 201021;

Questions? Just ask them!

- From Luc I heared: at the escam no escercises about Fourier.

Escam: What can be asked??

⇝ certain power series ∼ Radius of conv. – sum of power series

⇝ use of equations of Cauchy?

$u_x = v_y$ and $u_y = -v_x$

⇝ integrals to calculate with residues

$f(z) = \frac{1}{p(z)} \sim p(z) = 0$ ⇝ singularities

$p(a) = 0$ ⇝ $f(z) = \sum_{n=-\infty}^{+\infty} (z-a)^n$

residue: coëff of $\frac{1}{(z-a)}$

$$\int_K f(z)\,dz = 2\pi i \,(\text{sum residues})$$

K = (Jordan curve)

⇝ those neg. powers of $(z-a)$ ⇝ Laurent series.

⇝ Liouville escercise (don't forget to write down that you use Liouville in such an escercise!)

⇝ What more? may be some $\int_{-\infty}^{+\infty} \cdots dx$?

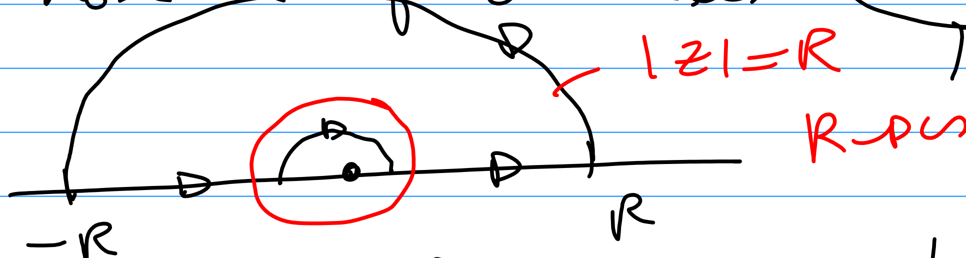
⇝ or something with $|z| = 1$?

$$\left.\begin{aligned} \cos(z) &= \left(z + \tfrac{1}{z}\right)/2 \\ \sin(z) &= \left(z - \tfrac{1}{z}\right)/2i \end{aligned}\right\} |z| = 1$$

$$z = e^{i\varphi} \qquad dz = i e^{i\varphi} d\varphi$$

$$\left(d\varphi = \frac{1}{iz} dz\right) \qquad dz = i \cdot z \cdot d\varphi$$

$\rightsquigarrow$ Those half residues $\qquad |f(z)| \to 0$



be careful with orientation !

I think above is given a survey of all what is done the last 7-8 weeks.

Forgotten something??

(Just put this survey at my website)

(3) $$f(z) = \frac{1}{(z-1)(3-z)}$$

$$= \frac{1}{(z-1)} \cdot \frac{1}{(3-z)}$$

$$= \frac{1}{-z^2 + 4z - 3}$$

$$= \ldots\ldots ?? \ldots\ldots$$

$$\boxed{\int \frac{1}{(x-1)(3-x)} dx}$$

$$\cancel{\int f \cdot g \ldots \ldots}$$

$$f(z) = \frac{A}{(z-1)} + \frac{B}{(3-z)}$$

$$= \frac{A(3-z) + B(z-1)}{(z-1)(3-z)}$$

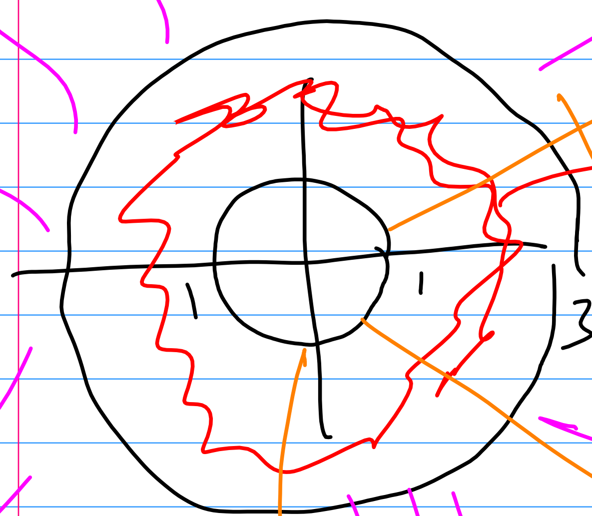
$$3A - B = 1 \qquad -A + B = 0$$

$$2A = 1 \qquad A = B$$

$$A = \frac{1}{2}, \; B = 1/2$$

$$f(z) = \frac{1}{2} \cdot \underbrace{\frac{1}{(z-1)}}_{\left(\left|\frac{1}{z}\right| < 1 \quad |z| > 1\right)} + \frac{1}{2}\underbrace{\left(\frac{1}{3-z}\right)}_{\frac{1}{3\left(1-\left(\frac{z}{3}\right)\right)}}$$

$$\left|\frac{z}{3}\right| < 1$$



$$A \cap B$$

$$A \cup B$$

$$\frac{1}{3} \cdot \sum_{n=0}^{\infty} \left(\frac{z}{3}\right)^n$$

$$\frac{1}{z-1} = \frac{1}{z}\left(\frac{1}{1-\frac{1}{z}}\right)$$

$$\frac{1}{z} \cdot \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)^n$$

b) Laurent around $z = 3$?

$$f(z) = \frac{1}{(z-1)(3-z)} =$$

$$\frac{1}{2}\cdot\left(\frac{1}{(z-1)}\right) + \frac{1}{2}\left(\frac{1}{(3-z)}\right)$$

$$(z-3)^n$$

z in neighbourhood around 3

$z = 3 + w$ (w small)

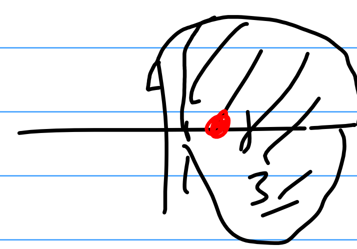
$\sum \cdots w^n$

$w = (z-3)$

$$\frac{1}{(z-1)} =$$

$$\frac{1}{((3+w)-1)} = \frac{1}{2+w}$$

$$\frac{1}{2\left(1+\frac{w}{2}\right)} = \frac{1}{2}\cdot\sum_{n=0}^{\infty}\left(\frac{w}{2}\right)^n$$



$z = 3 + w$

$$\left|\frac{w}{2}\right| < 1 \qquad |w| < 2$$

$$f(z) = \frac{1}{(3-z)} \cdot \left(\frac{1}{(z-1)} \right)$$

$$\hookrightarrow \left(\sum (z-3)^n \right)$$

$$\frac{1}{(z-1)} = \frac{1}{2+w} = \frac{1}{2} \cdot \frac{1}{\left(1+\frac{w}{2}\right)}$$

$z = 3 + w$

$$\frac{1}{2} \cdot \sum_{n=0}^{\infty} \left(\frac{w}{2}\right)^n = \frac{1}{2} \cdot \sum_{n=0}^{\infty} \frac{(z-3)^n}{2^n}$$

$*$

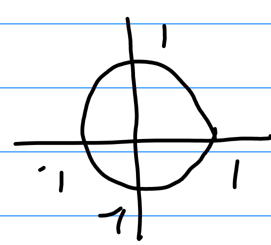
$$\left| \int_{|z|=1} \frac{f(z)}{z} \, dz \right| \le \int |f(z)| \cdot \left|\frac{1}{z}\right| dz$$

$= 1$

$$= \int_{|z|=1} |f(z)| \, dz$$

(2019)(5a)

$$\text{"} e^{(e^{i\varphi})} \text{"} \Big\{ \qquad 0 \le \varphi \le 2\pi \qquad e^{i\varphi}$$



$$\int_{|z|=1}$$

$$e^{-in\varphi} = \left(e^{+i\varphi}\right)^{(-n)} = \left(z^{-n}\right)$$

$$e^{\left(e^{i\varphi}\right)} = e^{z} \qquad \int_{|z|=1} \frac{e^{z}}{z^{n}} \cdot \frac{1}{i\,z}\,dz$$

$$z = e^{i\varphi} \sim dz = i\,\underline{\underline{e^{i\varphi}}}\,d\varphi$$

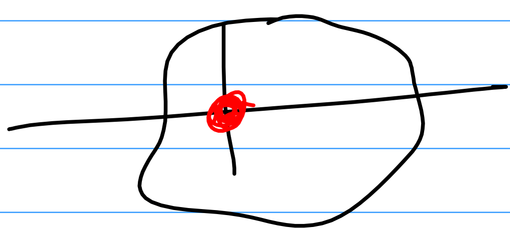
$$d\varphi = \left(\frac{1}{i\,z}\right) dz \qquad z^{n+1}$$

$z = 0$ singu. $\frac{c}{z}$

$$\frac{e^{z}}{z^{n+1}} = \sum_{k=0}^{\infty} \frac{z^{k}}{\underline{\underline{k!}}} \qquad k = n$$

$$\frac{1}{n!} \qquad \frac{z^{n}}{n!} \cdot \frac{1}{z^{n+1}}$$

$$\int_{K} \frac{f(z)}{z^{n}}\,dz \overset{??}{=} \left(\frac{f^{(n-1)}(0)}{n!}\right) 2\pi i$$



$$f(z) = f(0) + f'(0)\,z + \dots \frac{f^{(k)}(0)\,z^{k}}{k!} + \dots$$

$$\frac{1}{n!} \quad \frac{(e^{z})}{z^{n+1}}$$

(3) $f(z) = \frac{e^{z}}{z} + \frac{1}{(z-1)^{2}}$

$$\left(\frac{1}{(z-1)} = \cdots\cdots\right) \qquad \frac{d}{dz} \;--$$

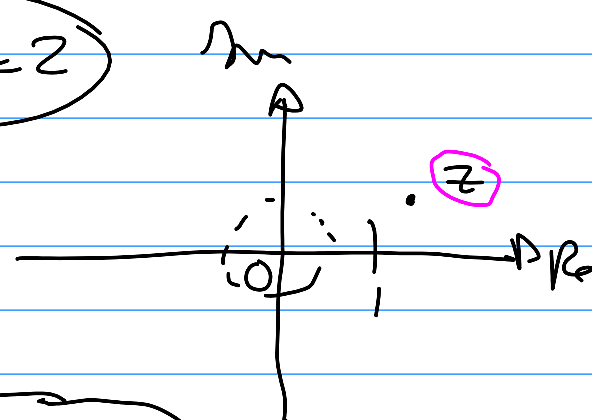
$$\frac{1}{(z-1)} = \frac{1}{z\left(1-\frac{1}{z}\right)} \rightarrow |z|>1$$

$(|z|<1)$

$z = 2$

(b) $z = 1 + w$

w small



$$f(z) = f(1+w) = \boxed{g(w)} \qquad w \simeq 0$$

$\ldots\ldots \quad w = (z-1)$

3 b) ~ Laurent series around $z = 1$

$$\sum_{n=-\infty}^{+\infty} c_n \cdot (z-1)^n$$

?

$$f(z) = \frac{1}{(z-1)^2}$$

$$e^z = e^{(1+w)} = e \cdot (e^w) =$$

$z = 1 + w$ $w = (z-1)$

$$e \cdot \sum_{n=0}^{\infty} \frac{w^n}{n!} = e \cdot \sum_{n=0}^{\infty} \frac{(z-1)^n}{n!}$$

$$\frac{1}{(z-1)^2}$$

$z = a$ — $z = a + w$

$w = (z - a)$

around $z = 2$

$z = 2 + w$ w small

$$e^{z} = e^{(2+w)} = e^{2} \cdot e^{w}$$

$$w = (z-2)$$

$$\frac{1}{(z-1)^2} = \frac{1}{((2+w)-1)^2} =$$

$$\frac{1}{(1+w)^2}$$

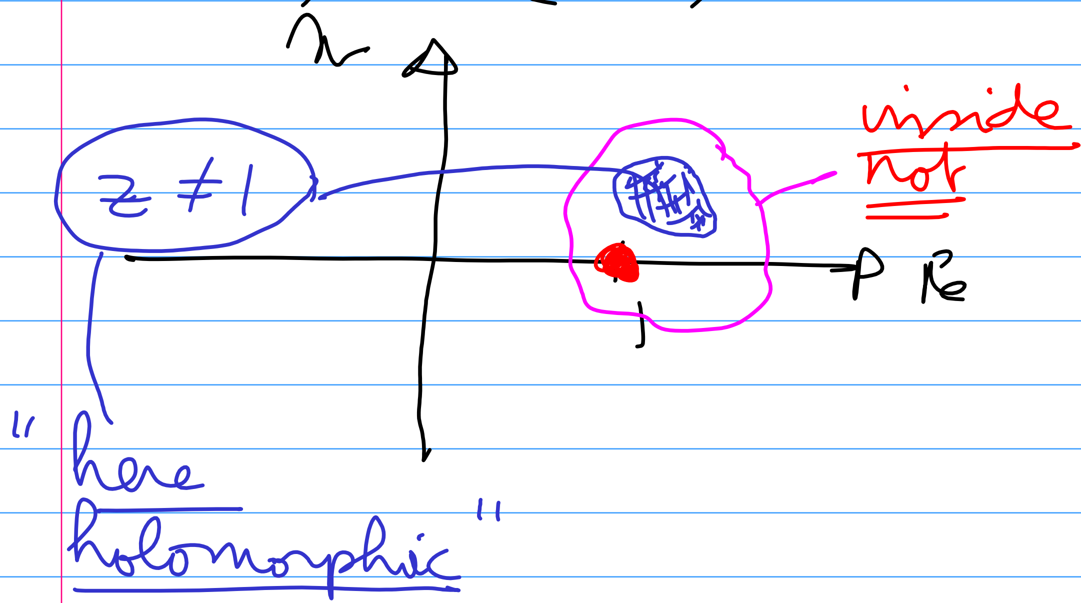
$$\frac{d}{dw}\left(\frac{-1}{1+w}\right) = \frac{d}{w}\left(\sum_{n=0}^{\infty} (-1)^{n+1} \cdot w^{n}\right) =$$

$$\sum_{n=0}^{\infty} (-1)^{n+1} n \cdot w^{n}$$

$$w = (z-2)$$

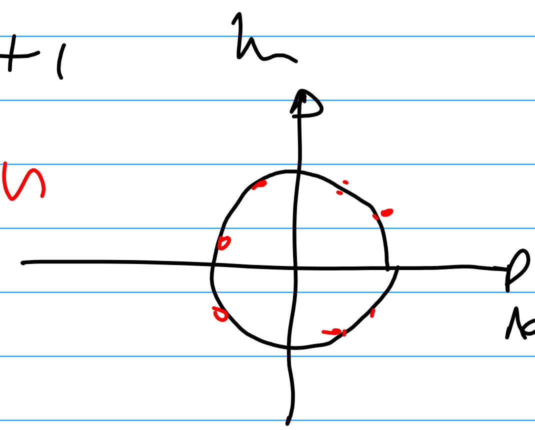
$$\sum_{n=1}^{\infty} \frac{1}{(z-1)^n} =$$

$$\frac{1}{(z-1)} + \frac{1}{(z-1)^2} \cdots + \frac{1}{(z-1)^n} + \cdots$$

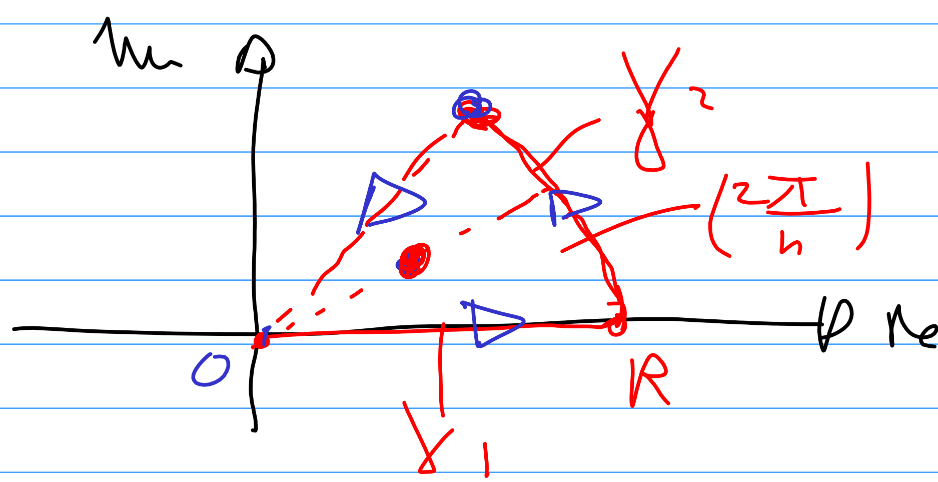


(5)

$$f(z) = \frac{1}{z^n+1}$$



$$z^n = -1$$



$$\frac{(z-z_1)\,1}{(z^n+1)} = \frac{\cancel{(z-z_1)}\,1}{\cancel{(z-z_1)}(z-z_2)\cdots(z-z_n)}$$

z_1 2nd. $(z \to z_1)$

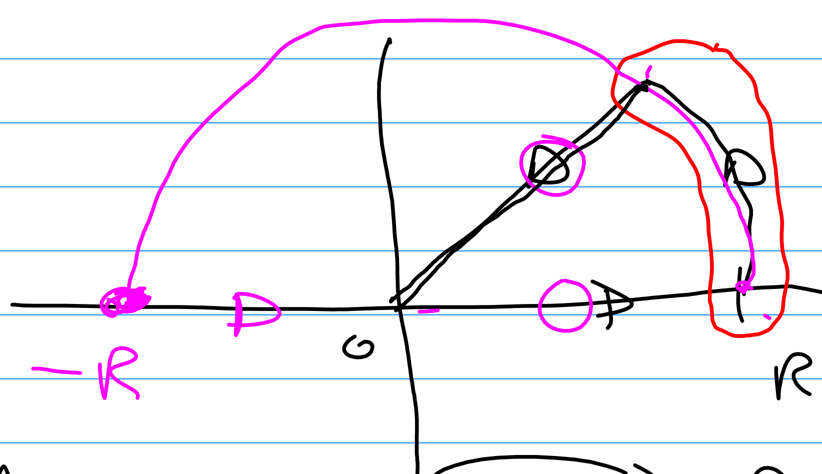
z_1 pole of order 1

$$\text{res}\, f(z) \quad \lim_{z \to z_1} (z-z_1)\cdot f(z)$$

$$2\pi i \left(\frac{1}{(z_1-z_2)(\cdots)(z_1-z_n)} \right)$$

$$\left(e^{\frac{2\pi}{n}} - e^{\frac{2\pi}{n}2}\right) \cdots$$

(d)



ML lemma

$R>1$ $R \to \infty$

$$|z^n+1| \geq |R^n-1| = (R^n-1)$$

$$\left| \int_{C_R} \cdots dz \right| \le \frac{2\pi \cdots R^{1}}{(R^{n}-1)} \to 0$$

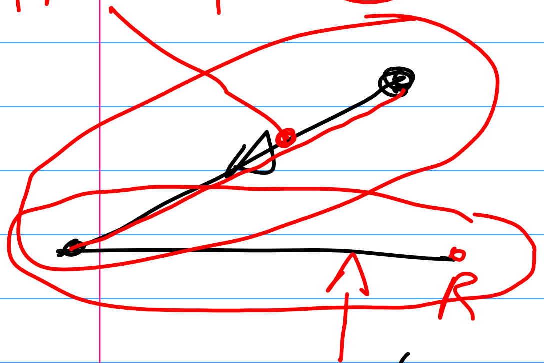
R^{-n}

$n \ge 2$

$$\int_{-R}^{0} dx \quad \int_{0}^{R} dx \quad ??$$

n even $\quad f(x) = \dfrac{1}{(x^n+1)}$

$|R-s| \le R$



$$f(-x) = \frac{1}{((-x)^n+1)} = \frac{1}{x^n+1} = f(x)$$

$$z = (R-s)e^{\left(\frac{2\pi}{n} i\right)}, \quad 0 \le s \le R$$

$$f(z) = \frac{1}{z^n+1} \quad ??$$

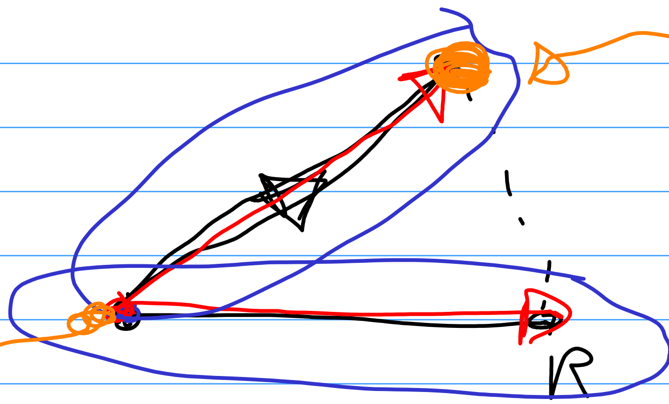
$$\frac{1}{(R-s)^n e^{2\pi i} + 1}$$

$s = R$

$f|_{\gamma(s,R)}$ $\quad dz =$

$|f| \le 1, \; R$

$\gamma_1 \longrightarrow \; |f| \le \frac{1}{(R^n - 1)}$



$\gamma_3 : \quad (R-s)\, e^{\frac{2\pi i}{n}}, \quad 0 < s \le R$

$s = 0$

$s = R$

$\longrightarrow \quad s \cdot e^{\frac{2\pi i}{n}}$

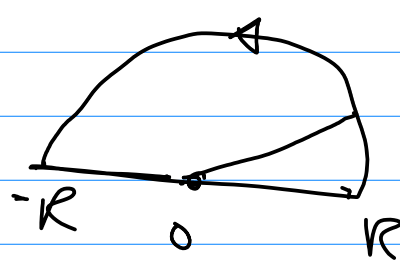
$0 < s \le R$

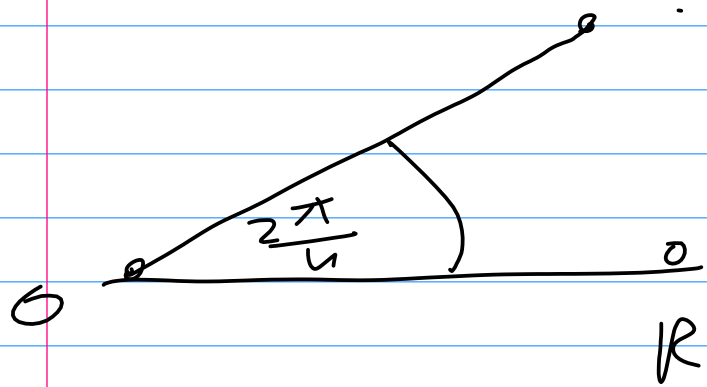
$n = \text{even}$

$(-x)^n = x^n$

$(-x)^n = -(x^n)$

$n = \text{odd}$





$$F_3 = \int_{\gamma_{3,R}} \frac{1}{z^n+1}\,dx \quad ??$$

$$\to z = (R-s)e^{\frac{2\pi i}{n}} \qquad 0 \le s \le R$$

$$dz = -e^{\frac{2\pi i}{n}}\,ds$$

$$F_3 = \int_0^R \frac{-e^{\frac{2\pi i}{n}}}{(R-s)^n e^{2\pi i} + 1}\,ds =$$

$$(e^{2\pi i} = 1)$$

$$= \int^R \frac{-e^{\frac{2\pi i}{n}}}{(R-s)^n + 1}\,ds =$$

$$-e^{\frac{2\pi i}{n}} \cdot \int_0^R \frac{1}{(R-s)^n + 1}\,ds$$

0

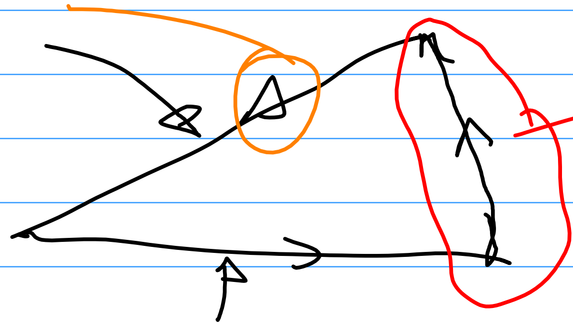
$$t = (R-s) \qquad s = 0 \to t = R$$

$s = R \to t = 0$

$$+ e^{\frac{2\pi i}{n}} \cdot \int_R^0 \left(\frac{1}{t^n+1}\right) \cdot dt$$

$$- e^{\frac{2\pi i}{n}} \cdot \int_0^R \frac{1}{t^n+1}\, dt$$

~~write out~~



$\to 0$ if $R \to \infty$

$$\int_{\gamma_1} f(z)\,dz = 2\pi i \operatorname{Res}_{z=?}$$

$$\int_0^R \frac{1}{x^n+1}\,dx - e^{\frac{2\pi i}{n}} \int_0^R \frac{1}{t^n+1}\,dt$$

$$\left(1 - e^{\frac{2\pi i}{n}}\right) \int_0^R \cdots dt = 2\pi i \operatorname{Res}_{z}$$