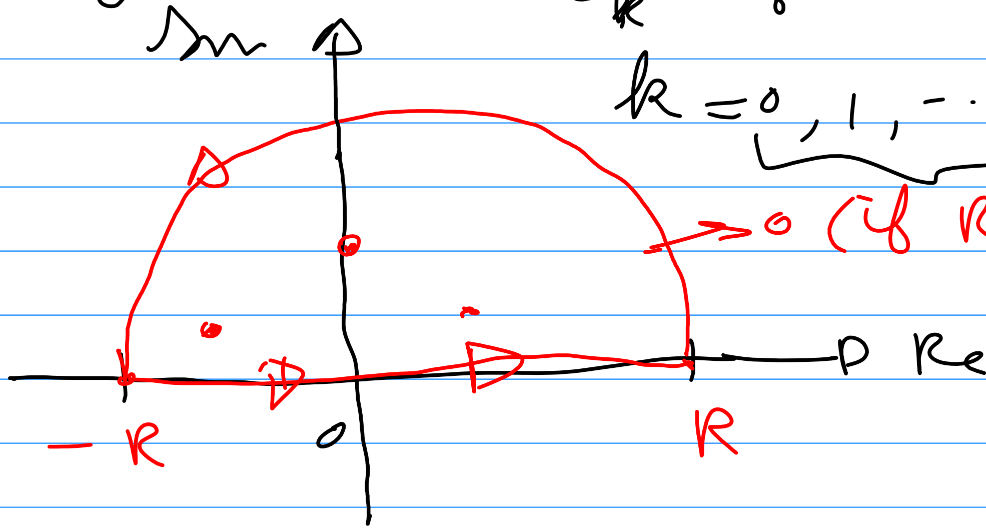
$\underline{\underline{2CAI30 - 201026}}$

(3.3) 1c)

$$\int_{-\infty}^{+\infty}\frac{1}{(x^6+1)}\,dx = \lim_{R\to\infty}\underbrace{\int_{-R}^{R}\frac{1}{x^6+1}\,dx}$$

$z^6=-1$ $\qquad z_k = \frac{\pi}{6}+\frac{2\pi}{6}\cdot k$

$\underbrace{k=0,1,\dots,5}$



$\to 0$ (if $R\to\infty$)

C_R^+: $\quad |z^6+1| \geq \big||z^6|-1\big| = \underline{R^6-1}$

$\underline{|z|=R}$ $\quad (R>1)$

$$\left|\int_{C_R^+} f(z)\,dz\right| \leq \pi R\left(\frac{1}{R^6-1}\right)\to 0$$

$\underline{\underline{\text{if } R\to\infty}}$

$$z^6+1 = (z-z_0)(z-z_1)\cdots(z-z_5)$$

$$\operatorname*{Res}_{z=z_0} f(z) = \lim_{z\to z_0} (z-z_0)\cdot f(z_0)$$

$$= \frac{1}{(z_0-z_1)(z_0-z_2)\cdots(z_0-z_5)}$$

$$\lim_{R\to\infty} \int_{-R}^{R} f(t)\,dt = 2\pi i \left(\sum_{i=0}^{2} \operatorname*{Res}_{z=z_i} f(t) \right)$$

$$\frac{A}{(z-z_0)} + \cdots + \frac{E}{(z-z_5)}$$

$$f(z) = (z^n+1) \sim \left(\frac{n}{z_0}\right)$$

$$\rightsquigarrow f(z) = f(z_0) + f'(z_0)(z-z_0) \cdots$$

$$f(z_0) = 0$$

$$(z-z_0)\frac{1}{f(z)} = \frac{1}{f'(z_0)(z-z_0) + \cdots}$$

$$= (z-z_0)\left(\frac{1}{f'(z_0) + \cdots}\right) \frac{1}{f'(z_0)}$$

$$f(z) = z^6 + 1$$

$$\frac{1}{f(z)} = \frac{1}{0 = f(z_0) + f'(z_0)(z - z_0) + \cdots}$$

$$= \frac{1}{f'(z_0)(z - z_0) + \cdots}$$

$$= \frac{1}{(z - z_0)} \cdot \frac{1}{f'(z_0)} \cdot \frac{1}{(1 + \cdots)}$$

$$\frac{(z - z_0)}{(z - z_0)} \; \frac{1}{f'(z_0)} \qquad \xrightarrow{\;\;} 1 \quad \text{if } z \to z_0$$

$$\boxed{z_0^6 = -1}$$

$$f'(z) = n \cdot z^{(n-1)}$$

$n = 6$

$$f'(z_0) = n (z_0)^{n-1} = \left(-\frac{n}{z_0}\right)$$

$$\alpha = \left(-\frac{6}{z_0} - \frac{6}{z_1} - \frac{6}{z_2}\right)$$

$$\boxed{\left(\frac{1}{\alpha}\right)}$$

Exam Wed 22 Jan 2020

(1 b)

$$\sum_{n=0}^{n} \alpha \cdot (n) \quad \alpha \cdot (1 + 2 \cdots)$$

$$(\text{ape} \cdot \text{cow})^n = \text{ape}^n \cdot \text{cow}^n$$

$$\left|\frac{z(z-3)}{4}\right| < 1$$

$$(z(z-3))^n$$

$$\left(\frac{5}{4}\right)(z-3) \cdot \sum_{n=0}^{\infty} \left(\frac{z(z-3)}{4}\right)^n$$

$$\frac{5}{4}(z-3) \cdot \frac{1}{1 - \left(\frac{z(z-3)}{4}\right)}$$

$$|z(z-3)| < 4 \, ?$$

$$\to |z| < 1 \qquad \underbrace{|z^2 - 3z|} < 4 \, ?$$

$$\Big|\underbrace{|z^2| - |3z|}\Big| \le |z^2 - 3z|$$

$\cancel{(\;)}$

$$\big|\, |z|^2 - 3\cdot|z| \,\big| =$$

$$|z|\cdot\big|\underbrace{|z|-3}\big| = |z|\cdot(3-|z|)$$

$$|z|<1$$

$$|3-|z|| \le 4.$$

(jan. 2019) (4)

$$f(z) = \frac{\sin(z)}{(\cosh(z)-1)^2}$$

$$\rightarrow \quad \boxed{\cosh(z) - 1 = 0}$$

$$\frac{e^z + e^{-z}}{2} = 1$$

$$\left(\frac{e^z}{2} + \frac{1}{2\cdot e^z} = 1\right) e^z$$

$$2\cdot\left(\frac{(e^z)^2}{2} - e^z + \frac{1}{2} = 0\right)$$

$$e^z = w$$

$$w^2 - 2w + 1 = 0$$

$$(w-1)^2 = 0$$

$$w = 1 \qquad \boxed{e^z = 1}$$

$$\cancel{z = 0}$$

$$\cosh z - 1 = 0$$

$$(e^z - 1)^2 = 0$$

$$\underbrace{\left(\frac{e^z + e^{-z}}{2} - 1\right) \cdot 2e^z}_{} =$$

$$(e^z - 1)^2$$

$$(\cosh(z) - 1)^2 = \left(\frac{(e^z - 1)^2}{2 \cdot e^z}\right)^2$$

$$\frac{1}{(\ldots)^2} = \left(\frac{4(e^z)^2}{(e^z - 1)^4}\right)$$

$$\frac{\sin(z)}{(\cosh(z)-1)^2} = \frac{\sin(z)\cdot 4(e^z)^2}{(\underbrace{e^z-1}_{(z+\dots)})^4} =$$

$$\boxed{\left(\frac{\sin(z)}{z} \to 1\right)} \text{ if } z \to 0$$

$$\frac{\sin(z)}{z} \frac{4(e^z)^2}{(z^3+O(z^4))}$$

z^2

$$\frac{(e^z)^2}{z^3\dots} = \frac{e^{2z}}{z^3\dots} = \frac{1+2z+\frac{(2z)^2}{2!}+\dots}{z^3}$$

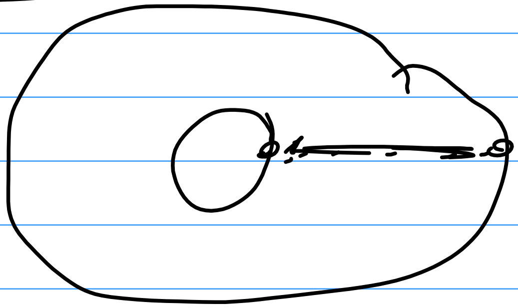
z^3

$\left(\frac{1}{z}\right) \to$ Res

b) Region of conv

(1/0) $z=0$ singularity.

$\boxed{R \to \infty}$



c) z^{-m}

$$\frac{1}{(\cos(z)-1)}$$

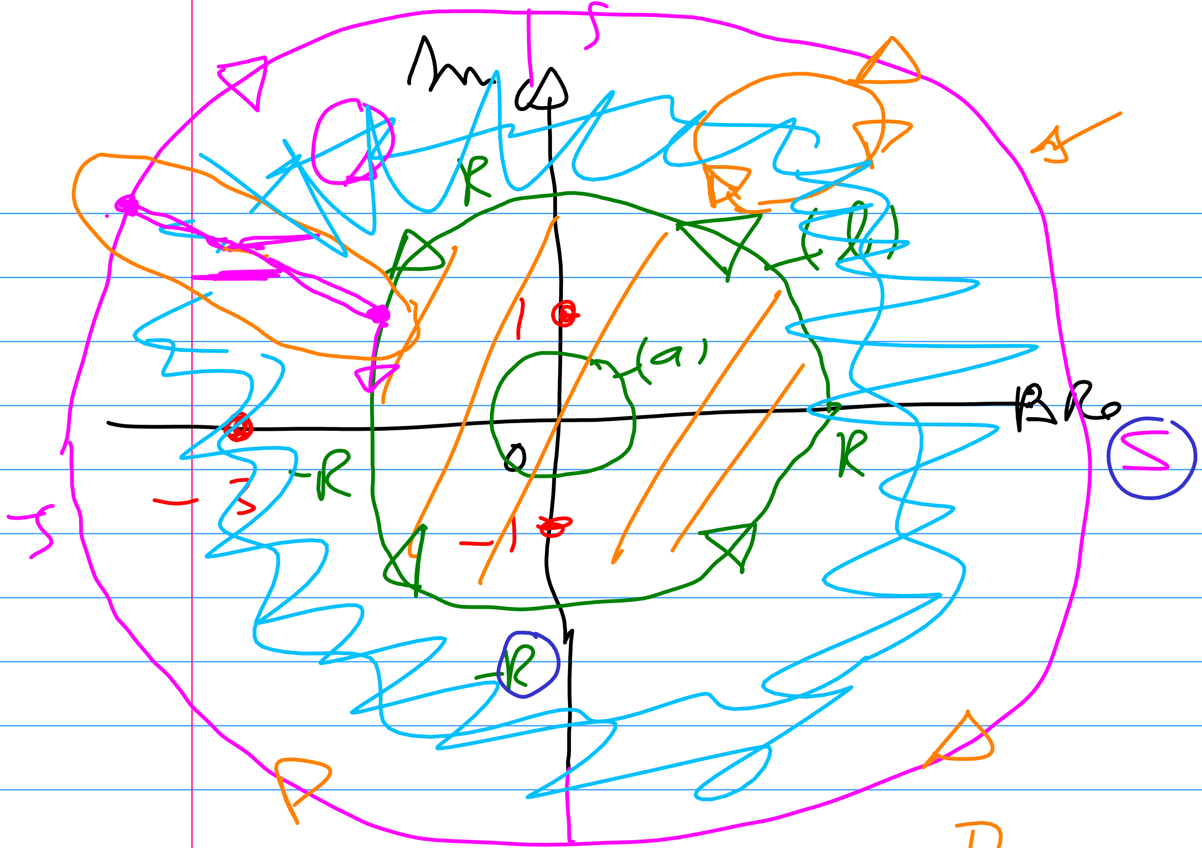
$z = 0,$ $\quad z = 2\pi$

$0 < |z| < 2\pi$

$(3-1)(3)$

$z = -3$

$$\frac{1}{(z^2+1)^3 \cdot (z+3)} =$$



D

$$\int_{|z|=R} + \int_{|z|=S} \cdots = 2\pi i \operatorname{Res}_{z=3} f(z)$$

$S \to \infty$

$$\left| \int_{|z|=S} \cdots dz \right| \le \frac{2\pi S}{(S^2-1)^8 (S-3)}$$

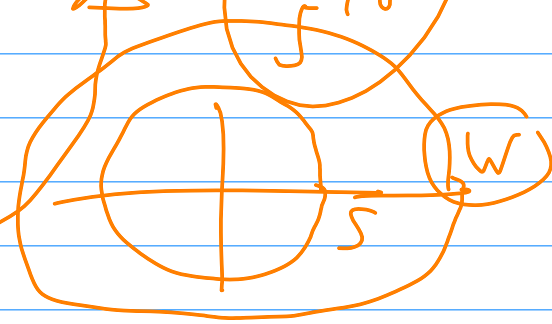
$S \to \infty$

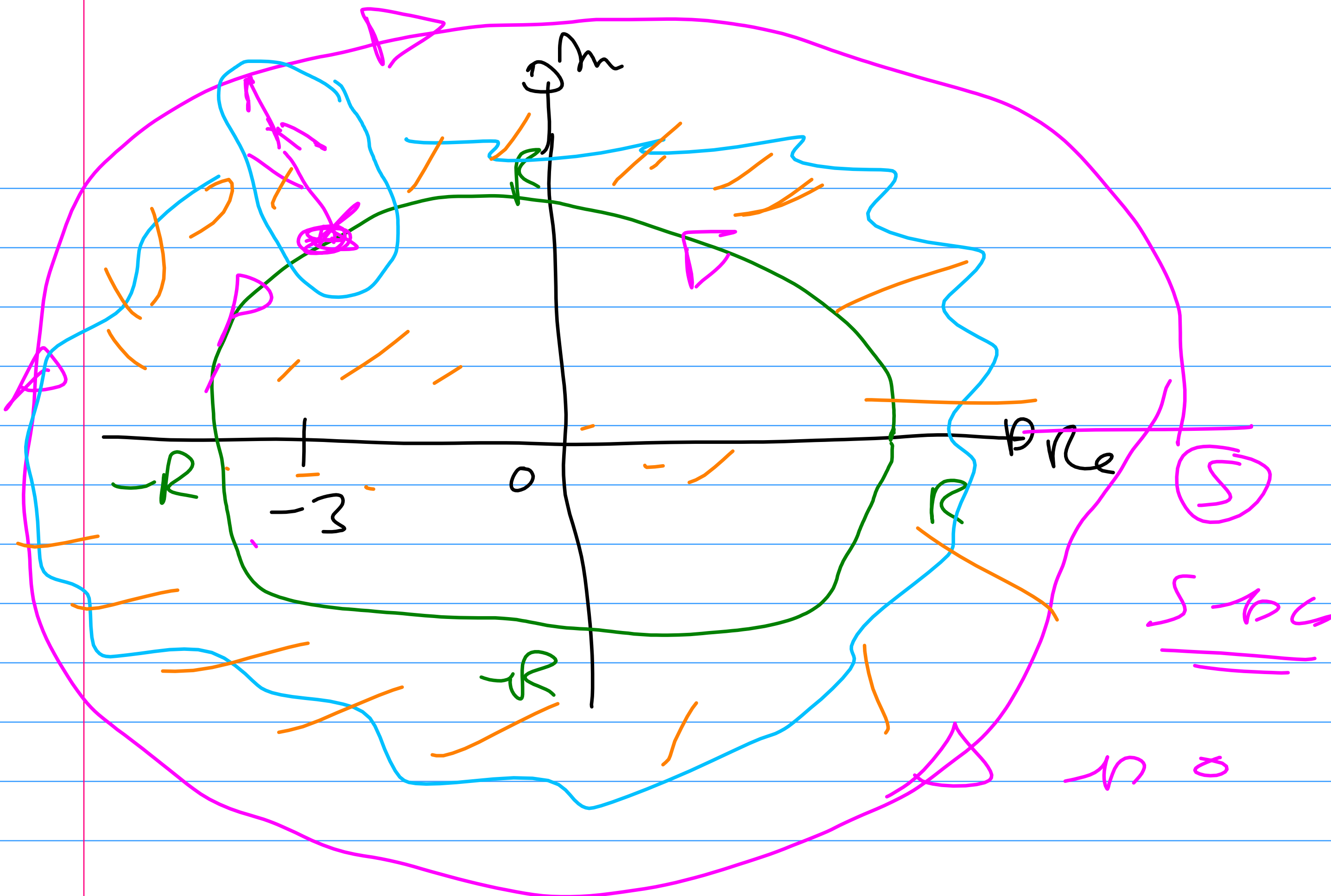
$$\left(\frac{1}{10}\right)^8 \cdot 2\pi i$$

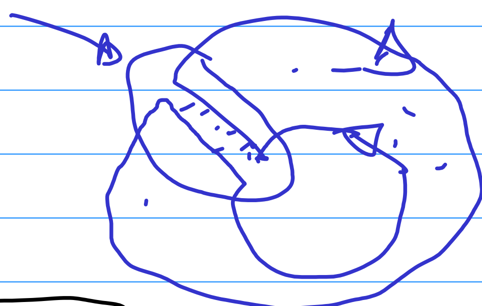
C) 0

$S > 3$

$\frac{1}{S^{16}}$







$$\boxed{f(z) = \frac{\sin(z)}{z}}$$ $\boxed{z = 0}$

$z = 0$

$$\frac{\cos(\pi z)}{(2z - 1)}$$

$z = \frac{1}{2}$

$\boxed{\frac{0}{0}}$ $\left| \frac{z^2}{z}, \frac{z}{z}, \frac{z}{z^2} \right.$

$$\boxed{\frac{f(z)}{(z-a)} \qquad \lim_{z\to a} \frac{f(z)}{(z-a)} = c}$$

$\underline{z = a \text{ remo??}}$

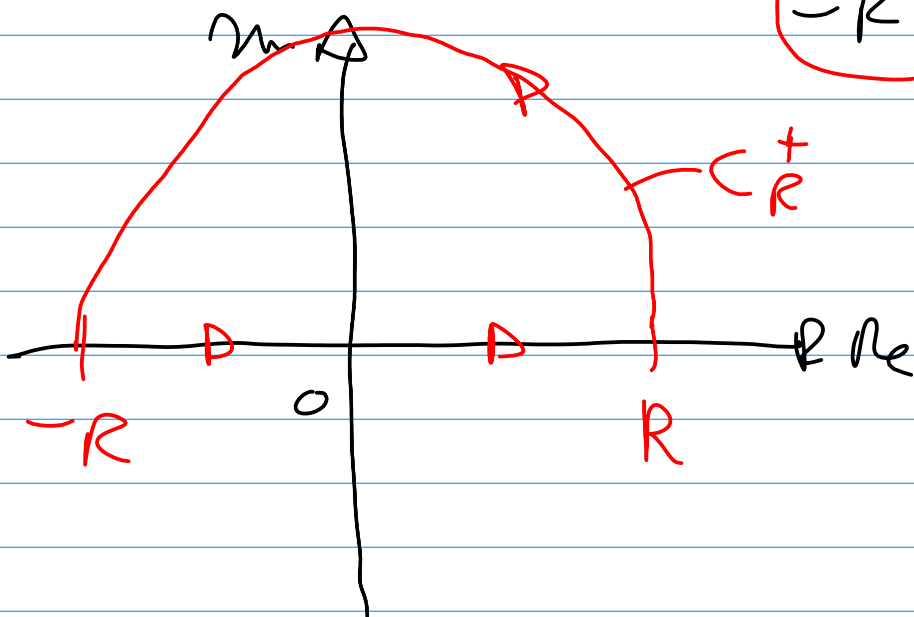
<u>if limit exists</u>

<u>singularity remov.</u>

<u>jan(2019)</u> 5a

(5b)

$$\int_{-\infty}^{+\infty} \cdots dx = \lim_{R\to\infty} \boxed{\int_{-R}^{R} f(x)\,dx}$$



$$f(z) = \frac{\sin(z) - z\cdot\cos(z)}{z^3}$$

$$\int_{-R}^{R} \frac{\sin(x) - x\cdot\cos(x)}{x^3}\,dx$$

$C_R^+ \rightarrow \frac{\sin(x)}{\quad} \quad \frac{\cos(x)}{\quad}$

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

e^{ix}, e^{-ix} $\qquad z = x + iy$

e^{iz} $\quad e^{-iz}$ $\qquad y > 0$

$$\left|e^{i(x+iy)}\right| = \left|e^{ix}\cdot{}^{-y}\right| = e^{-y}$$

$y > 0$

$$\left|e^{-iz}\right| = \left|e^{-i(x+iy)}\right| = \left|e^{-ix}\cdot e^{y}\right| = e^{y}$$

$$\int_{-R}^{R} \frac{\sin(x)}{x^3}\,dx = \int \frac{e^{ix} - e^{-ix}}{x^3}\,dx$$

$$\mathrm{Im}\ \boxed{e^{ix}} = \sin(x)$$

$$\int_{-R}^{R} \frac{\sin x}{x^3}\,dx = \operatorname{Im}\left(\int_{-R}^{R} \frac{e^{ix}}{x^3}\,dx\right)$$

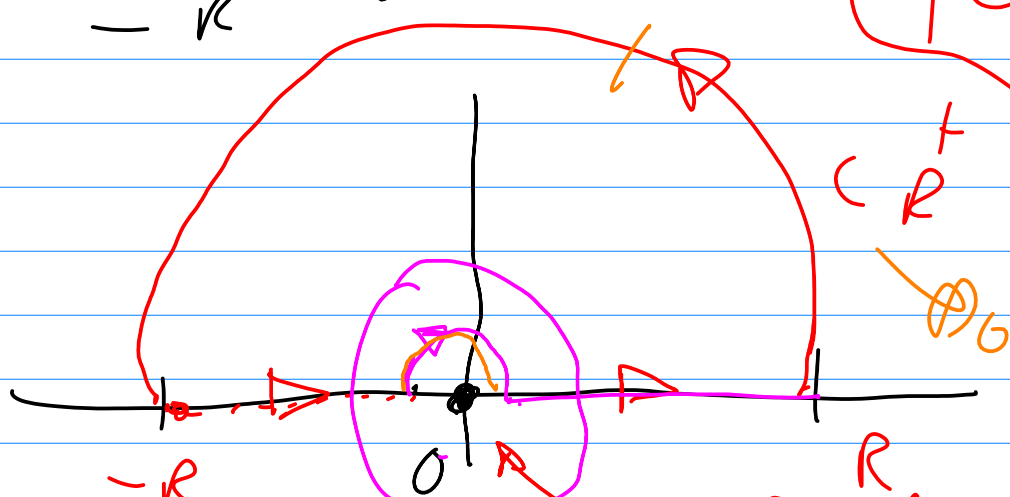
$$\boxed{\operatorname{Im}(e^{ix}) = \sin(x)}$$

$$\cos(x) = \operatorname{Re}(e^{ix})$$

$$\int_{-R}^{R} \frac{-x\cos x}{x^3}\,dx = \operatorname{Re}\left(\int_{-R}^{R} \frac{-x\cdot e^{ix}}{x^3}\,dx\right)$$

$$\int_{-R}^{R} \frac{e^{ix}}{x^3}\,dx$$

$$|e^{ix}| = e^{-y} \to 0 \quad (y \to \infty)$$



$\frac{1}{2}$ half circle

$$\boxed{-\frac{1}{2}\operatorname{Res}_{z=0}(f(z))}$$

Be careful at exam!

I hope everything goes weel