(1.1) – 7c):

$$z^5 - i\cdot z^3 + i z^2 + 1 = 0$$

at most 5 diff. solutions.
(5th order polynomial)

$\cancel{z = 1}$; $1 - i + i + 1 \neq 0$

$z = i$; $z^2 = -1$, $z^3 = -i$

$z^4 = +1$, $z^5 = i$

$$i - i(-i) + i(-1) + 1 =$$

$$i - 1 - i + 1 = 0$$

$$(z - i)(\cdots\cdots\quad) = 0$$

4th order polyn.

$$z^5 - i z^3 + i z^2 + 1 = 0$$

$$z^2(z^3 + i) - i\cdot z^3 + 1 = 0$$

$$z^2(z^3 + i) - i(z^3 + i) = 0$$

$$(z^2 - i)\cdot(z^3 + i) = 0$$

1) $z^2 - i = 0$

2) $z^3 + i = 0$

$$z^3 + i = 0 \qquad ((z = i)!)$$

$$(z^3 + i) : (z - i) = z^2 + iz - 1$$

$$\begin{array}{l} \underline{z^3 - iz^2} \; - \\ \quad iz^2 + i \\ \quad \underline{iz^2 + z} \; - \\ \qquad -z + i \\ \qquad \underline{-z + i} \; - \\ \qquad\quad 0 \end{array}$$

$$(z - i)(z^2 + iz - 1) = 0$$

$$z^2 + iz - 1 = 0$$

$$\left(z + \frac{i}{2}\right)^2 + \frac{1}{4} - 1 = 0$$

$$\left(z + \frac{i}{2}\right) = \left(\frac{3}{4}\right)$$

$$z = -\left(\frac{i}{2}\right) \pm \sqrt{\frac{3}{4}}$$

$$z_1 = i, \quad z_2 = -\frac{i}{2} + \frac{1}{2}\sqrt{3}$$

$$z_3 = -\frac{i}{2} - \frac{1}{2}\sqrt{3}$$

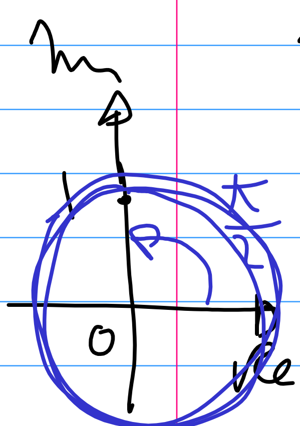
$$z^2 - i = 0 \qquad z = R \cdot e^{i\varphi}$$

$$z^2 = i \qquad z^2 = R^2 \cdot e^{(i2\varphi)}$$

$$\left|R^2 \cdot e^{(i2\varphi)}\right| = |i| \qquad \left(R \in \mathbb{R},\ R > 0\right)$$

$$\left|e^{2i\varphi}\right| = 1 \qquad |i| = 1$$

$$R^2 = 1 \Rightarrow R = 1$$




$$e^{2i\varphi} = i = \exp\left(i\left(\frac{\pi}{2} + k \cdot 2\pi\right)\right)$$

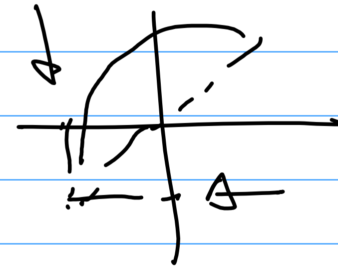
$$k \in \mathbb{Z}$$

$$2i\varphi = i\left(\frac{\pi}{2} + k \cdot 2\pi\right)$$

$$\varphi_k = \left(\frac{\pi}{4} + k\cdot\pi\right), \quad k \in \mathbb{Z}$$

$$z_k = 1\cdot \exp(i\cdot\varphi_k), \quad k \in \mathbb{Z}$$

$$= \exp\left(i\left(\frac{\pi}{4} + k\cdot\pi\right)\right)$$



$\underline{k=0}$: $$z_0 = \exp\left(i\frac{\pi}{4}\right) = \frac{1}{2}\sqrt{2} + i\,\frac{1}{2}\sqrt{2}$$

$\underline{k=1}$: $$z_1 = \exp\left(i\left(\frac{5\pi}{4}\right)\right) = -\frac{1}{2}\sqrt{2} - i\cdot\frac{1}{2}\sqrt{2}$$

5!

**all different solutions:**

$z_1 = i$

$z_2 = \frac{-i}{2} + \frac{1}{2}\sqrt{3}$

$z_3 = \frac{-i}{2} - \frac{1}{2}\sqrt{3}$

$z_4 = \frac{1}{2}\sqrt{2}(1+i)$

$z_5 = \frac{-\sqrt{2}}{2}(1+i)$