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CH(1.1) – Ex. 9:

- $z, w \in \mathbb{C}$, $z \neq 0$, $w \neq 0$
- given: $z \cdot \overline{w} - \overline{z} \cdot w = 0$

$(z = c \cdot w \,,\; c \in \mathbb{R})$

i) $z = a + ib$

$w = \alpha + i \cdot \beta$

$(a + ib) \cdot (\alpha - i\beta)$

$- (a - ib) \cdot (\alpha + i\beta) = 0$

$\cancel{a \cdot \alpha} - i a \cdot \beta + i b \cdot \alpha + \cancel{b \cdot \beta}$

$- (\cancel{a \alpha} + i a \cdot \beta - i b \cdot \alpha + \cancel{b \cdot \beta}) = 0$

$-2i a \cdot \beta + 2 i \cdot b \cdot \alpha = 0$

(s.n.p)

$-a \cdot \beta + b \cdot \alpha = 0$

$a \cdot \beta = b \cdot \alpha \Rightarrow a = \left(\frac{b \cdot \alpha}{\beta}\right)$

$\beta \neq 0$ (Special; $\beta = 0$??)

$$z = a + ib = \left(\frac{b \cdot \alpha}{\beta} + ib\right) =$$

$$\frac{(b \cdot \alpha + i b \beta)}{\beta} =$$

$$\left(\frac{b}{\beta}\right)\underbrace{(\alpha + i\beta)}_{w} = \left(\frac{b}{\beta}\right) \cdot w$$

$$c = \frac{b}{\beta} \quad (\beta \neq 0 \,!)$$

$c \in \mathbb{R}$ $\quad z = c \cdot w$ $\quad$ &

$\beta = 0$?? $\quad a \cdot \beta = b \cdot \alpha$

$\beta = 0$ $(\alpha \neq 0)$ $\quad w \neq 0$

$$a \cdot 0 = b \cdot \alpha \Rightarrow b = 0$$

$\beta = 0 \rightsquigarrow w = \alpha \Rightarrow z = a$

$$z = \left(\frac{a}{\alpha}\right)\alpha = \left(\frac{a}{\alpha}\right) w$$ (s.n.p.)

$\beta = 0 \Rightarrow w = \alpha \in \mathbb{R}$

$b = 0 \Rightarrow z = a \in \mathbb{R}$

$z = \left(\frac{a}{\alpha}\right) \cdot w$ &

So if $z \neq 0, w \neq 0 \Rightarrow$

$z = c \cdot w$ with $\underline{\underline{c \in \mathbb{R}}}$

$\boxtimes$

(ii) <u>polar coordinates:</u>

$z = R \cdot e^{i\varphi}$ $R \neq 0$

$w = S \cdot e^{i\psi}$ $S \neq 0$

$0 = z \cdot \overline{w} - \overline{z} \cdot w = R \cdot e^{i\varphi} \cdot S \cdot e^{-i\psi}$

$- (R \cdot e^{-i\varphi}) \cdot S \cdot e^{i\psi}$

$R \cdot S \left(\exp(i(\varphi - \psi)) \right.$

$\left. - \exp(i(-\varphi + \psi)) \right) = 0$

(1.n.p.)

$$\exp(i(\varphi - \psi)) = \exp(i(\underline{-\varphi + \psi}))$$

multiply: $\exp(i(\underline{\varphi - \psi}))$

$$\exp(2i(\varphi - \psi)) = \exp(0) = 1$$

$$= \exp(i(2\cdot k\cdot \pi)), \quad k \in \mathbb{Z}$$

$$2i(\varphi - \psi) = 2ik\cdot\pi$$

$$\varphi - \psi = k\cdot\pi, \quad k \in \mathbb{Z}$$

$$\varphi = (\psi + k\cdot\pi)$$

$$z = R\cdot\exp(i(\psi + k\cdot\pi))$$

$$= R\cdot\underline{\exp(i(\psi))}\cdot\exp(ik\pi)$$

$$= \left(\frac{R}{S}\right)\overbrace{\boxed{S\cdot\exp(i\cdot\psi)}}^{w}\cdot(-1)^k$$

$$= \boxed{\left(\frac{R}{S}\right)\cdot(-1)^k\cdot}\; w$$

(s.n.p)

$$z = c \cdot w \quad , \quad c = \left(\frac{R}{S}\right) \cdot (-1)^{k} \in \mathbb{R}$$

$\boxtimes$

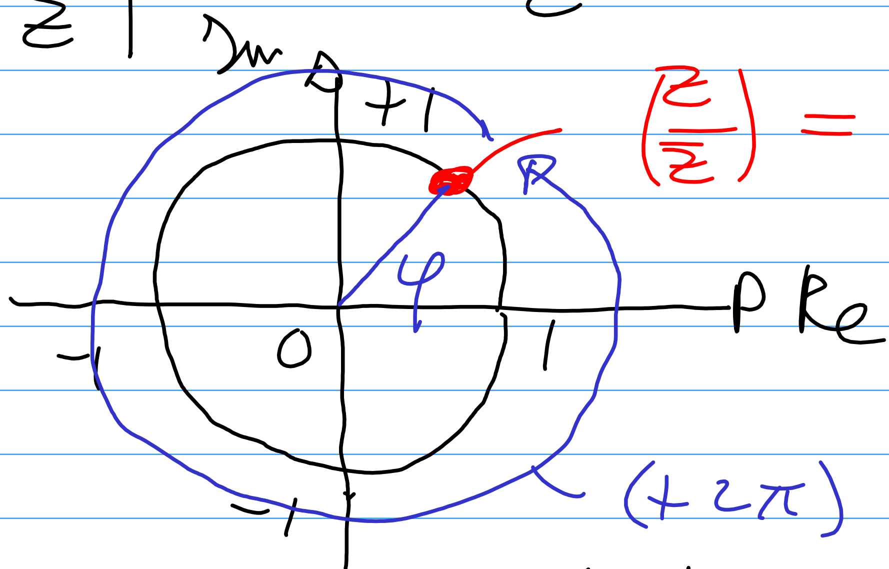
Ideas??

$$z \cdot \overline{w} - \overline{z} \cdot w = 0$$

$$z \cdot \overline{w} = \overline{z} \cdot w$$

$$\frac{z}{(\overline{z})} = \frac{w}{(\overline{w})}$$

$$\left|\frac{z}{\overline{z}}\right| = 1 \qquad \frac{z}{\overline{z}} \in \text{unit circle}$$



$$\left(\frac{z}{\overline{z}}\right) = \left(\frac{w}{\overline{w}}\right)$$

What to do with it, I don't know

See next page!! (s.n.p)

I stopped and a few minutes later I got some idea, what to with that last idea!!

$$\frac{z}{\bar{z}} = \frac{w}{\bar{w}} \Longrightarrow z = \left(\frac{\bar{z}}{\bar{w}}\right) w$$

my question became

is $c = \left(\frac{\bar{z}}{\bar{w}}\right)$?? Better question

is $\left(\frac{\bar{z}}{\bar{w}}\right) \in \mathbb{R}$?? (~~if~~ $z\cdot\bar{w} - \bar{z}\cdot w = 0$)

So $z = \left(\frac{\bar{z}}{\bar{w}}\right) w$, fill in in the given equation

$$\underbrace{\left(\frac{\bar{z}}{\bar{w}}\right)\cdot w}_{z}\cdot \bar{w} - \underbrace{\overline{\left(\frac{\bar{z}}{\bar{w}}\cdot w\right)}}_{\bar{z}}\cdot w = 0 \quad {*1} \Longrightarrow$$

$$\left(\frac{\bar{z}}{\bar{w}}\right)|w|^2 - \overline{\left(\frac{\bar{z}}{\bar{w}}\right)}\cdot|w|^2 = 0$$

(s.n.p.)

$$\left(\frac{\bar{z}}{\bar{w}}\right) = \overline{\left(\frac{z}{w}\right)} \quad \overset{*_2}{\Longrightarrow}$$

$$\left(\frac{z}{w}\right) \in \mathbb{R}$$

$*_2$ $z = x + iy$, $\bar{z} = x - iy$ } if $z = \bar{z}$ $\Rightarrow$ $x+iy = x-iy$ $\Rightarrow y = 0$ So $z = x \in \mathbb{R}$

$*_1$ $(z \cdot \bar{z} = x^2 + y^2)$

Or shall we do it into 2 lines!! $z \neq 0$, $w \neq 0$ and

$z \cdot \bar{w} - \bar{z} \cdot w = 0 \Rightarrow z \cdot \bar{w} = \bar{z} \cdot w$

$\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}} = \frac{z}{w} \Rightarrow \left(\frac{z}{w}\right) \in \mathbb{R}$ and

$z = \left(\frac{\bar{z}}{\bar{w}}\right) \cdot w$ so $c = \frac{\bar{z}}{\bar{w}} \in \mathbb{R}$. $\boxtimes$