Exercise 5.(1.3) 3

a) $\lim_{R \to 0} f(1 + R \cdot e^{i\theta}) =$

$$\lim_{R \to 0} \frac{(1 + 2Re^{i\theta} + R^2 e^{2i\theta} - 1)}{|Re^{i\theta}|}$$

$= 2 \cdot e^{i\theta}$ depends on $\arg(z)$ (path-dependent)

b) not exists, because it is not path-independent.

$R \cdot e^{i\theta} \rightsquigarrow$ straight lines to $z = 0$ if $R \to 0$. (θ fixed.)

$$f\left(1 + R \cdot e^{i\left(\frac{\theta}{R}\right)}\right) \qquad (\theta \neq 0)$$

$R \cdot e^{i\left(\frac{\theta}{R}\right)} \rightarrow$ spiral going to $z = 0$ if $R \to 0$

$$\lim_{R \to 0} f\left(1 + R \cdot e^{i\frac{\theta}{R}}\right) =$$

$$\lim_{R \to 0} 2 \cdot e^{i\left(\frac{\theta}{R}\right)}$$

not exists !!

if $R \to 0$ then $e^{i\left(\frac{\theta}{R}\right)}$ is walking on unitcircle in $\mathbb{C}$.

so not just one value