A Gasdynamic-Acoustic Model of a Bird Scare Gun

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1 Introduction

A bird scare gun is a relatively simple device which produces impulsive noise by means of a gas explosion. It is used for scaring birds away from areas where their presence is unwanted, like orchards, airfields, or oilfields. It is meant to operate automatically for long periods of time, with little or no human intervention. The construction is simple and robust, with as few as possible moving parts, so that a lifecycle in the order of 100,000–200,000 explosions is attainable.

The mechanism is simple. A carefully-controlled mixture of air and propane or butane gas (stoichiometric[1] mixture, or a little bit richer than that) is periodically (every 5 or 10 minutes) blown into a semi-open pot, which is the combustion chamber. This pot is connected via a small diaphragm or iris (a small hole in the wall of the combustion pot) to an exhaust pipe. After ignition, the gas burns quickly (but without detonation, *i.e.* with a subsonically moving flame front) so that pressure and temperature increase quickly. This high pressure drives the gas out of the pot via a hot jet, which issues from the diaphragm into the pipe. Acting like a piston, this jet creates a pressure wave in the cold exhaust pipe. Part of the wave reflects at the exit, and part radiates, nearly spherically, away into the open air.

An interesting detail in the design gave, without any further analysis, already insight in the gasdynamic behaviour. In order to vary the noise that is produced, the length of the exhaust pipe is made variable. The pipe consists of two shorter pipes, one of which slides inside the other, like a telescope. It appears that the gun produces a louder bang when the pipe is longer. This hints, of course, at a possible relation to nonlinear wave steepening. A pressure wave of high enough amplitude

[1] Stoichiometric mixture = just enough of each component for a complete chemical reaction.

steepens while propagating onto the pipe exit. Indeed, from fairly elementary acoustics it is known, that (at least for low frequency linear perturbations) the radiated part is proportional to the derivative of the pipe wave. Therefore, a longer pipe, leading to more advanced steepening, may be expected to produce more noise. So the effect of steepening is not only to intensify the higher frequencies -the pitch- of the radiated pulse but also to increase its amplitude.

The size of the diaphragm is also known to be very critical for the loudness of the sound. On one hand, too large a hole would allow the gas to be blown away too quickly so that not all the gas reacts, and the pressure wave remains too flat. On the other hand, with too small a hole the mass flux of the jet will remain too small, and hence the created pressure wave will be too small. Somewhere, there is obviously an optimum.

At present, the design of a bird scare gun is mainly done by trial and error using the skills of the experienced craftsmen. A mathematical model would enable the designer to understand the various physical and chemical processes better, and hence to further refine the quality of the gun.

Aspects of practical interest are, for example

- How much chemical energy is needed to produce a bang of a given loudness?
- How does the length of the exhaust pipe relate to the noise that is produced?
- How do we determine the optimum iris?
- What are the maximum pressure and temperature levels in the pot? (Although it is difficult to quantify the relation, the life of the device appears to be determined completely by wear due to fatigue of the pot.)

Most of these questions can not be fully answered by a mathematical model of manageable complexity. Three-dimensional combustion, flame front propagation, turbulent compressible hot-cold gas mixing, etc. are complex phenomena, which are very difficult to model, while any model based on first principles will be very difficult to evaluate. So we will try to get as far as possible, with the answers to the above design questions as ideal goals.

To investigate the effect of various problem parameters on the resulting noise level (Sound Pressure Level, SPL) we will develop a model that is simple enough to make progress, but still rich enough to describe the essence of the underlying mechanisms. As we will see, the problem is physically quite complicated, and requires many modelling assumptions. A great help in this respect is the fact that noise is measured logarithmically (in decibels), which turns rather crude assumptions into relatively accurate results.

2 Model

2.1 Geometry

A schematic sketch of the device is shown in figure 1. In the pot (1) a mixture of butane or propane gas and air is ignited. From the pressure difference between (2) and (1) a hot jet (j) is formed from the iris into the cold pipe. The flux of mass and momentum across a hot-cold mixing zone at $x = 0$ initiates a pressure wave in the pipe. This wave steepens and reflects at the open end ($x = L$), where an almost spherical[2] sound pulse is generated that radiates away to the far field.

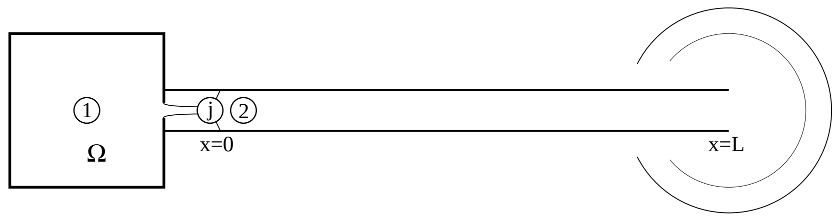

Figure 1: Sketch of geometry.

2.2 Pot

For a small enough iris any flow in the pot is negligible. Also, the size of the pot seems to be just small enough to allow us to model the gas in the pot using mean quantities. We have a volume Ω of gas of density ρ_1, pressure p_1 and temperature T_1. The jet has cross section S_j, (average)

[2]Well, not exactly. In general the situation may be very complex. The highest frequency components tend to propagate straight out the exit, like rays, whereas the lowest frequencies spread out spherically. Furthermore, high amplitude waves lose acoustic energy by vortex shedding at the sharp exit edges. Measurements indicate a variation of a few dB between the forward and backward sound field. Therefore we start with the simplest asumption of spherical radiation.

velocity v_j and density ρ_j, so conservation of mass requires

$$\Omega \frac{\mathrm{d}\rho_1}{\mathrm{d}t} = -\rho_j v_j S_j. \tag{1}$$

The jet cross section S_j is smaller than the diaphragm S_d because of the vena-contracta effect (radial inertia narrowing the jet). Typically, S_j/S_d is of the order 0.6, but the actual value depends on various parameters like the Reynolds number, the Mach number, and the sharpness of the iris edges [2].

The typical duration of pressure build up and release is a few milliseconds, which corresponds to a pressure wave length of several decimeters. (The sound speed is of the order of 34 cm/ms.) This is large compared to the iris and jet size. On the other hand, the jet is small compared to the pipe length. So the jet may be described quasi-stationary and locally. Furthermore, a typical jet velocity is high (nearly sonic) so viscous effects are small, and the flow from pot to jet is compressible. Assuming a nearly optimal situation, such that there is no loss of mechanical energy by shock formation, the flow is isentropic, for which Bernoulli's law and the condition of isentropy are valid:

$$\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} = \tfrac{1}{2}v_j^2 + \frac{\gamma}{\gamma-1}\frac{p_j}{\rho_j}, \tag{2}$$

$$\frac{p_1}{\rho_1^\gamma} = \frac{p_j}{\rho_j^\gamma}. \tag{3}$$

The ideal gas law and the definition of sound speed are valid everywhere (pot, pipe, outside):

$$p = \rho R T, \tag{4}$$

$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_S = \frac{\gamma p}{\rho} = \gamma R T. \tag{5}$$

The heat capacity at constant pressure $C_P = 1000.0\,\mathrm{J/kg\,K}$ and constant volume $C_V = 713.26\,\mathrm{J/kg\,K}$, their ratio $\gamma = C_P/C_V = 1.402$, and their difference, the gas constant, $R = C_P - C_V = 286.73\ \mathrm{J/kg\,K}$ are taken constant, although in reality they vary slightly with temperature.

The combustion appears to be very difficult to model from first principles. The finite duration is acoustically important, so a description of the propagating flame front seems required. This is very complicated if the effects on temperature and pressure of a decreasing density due to the outflow from the iris are to be included.

To make progress, some experimental input is used at this point. The effect of the increase of thermal energy by the combustion is described by an increase of the entropy, in the form of a suitably chosen function $\beta(t)$

$$\beta(t) = \beta_0 + (\beta_1 - \beta_0)b(t/\tau) = \frac{p_1}{\rho_1^\gamma}. \qquad (6)$$

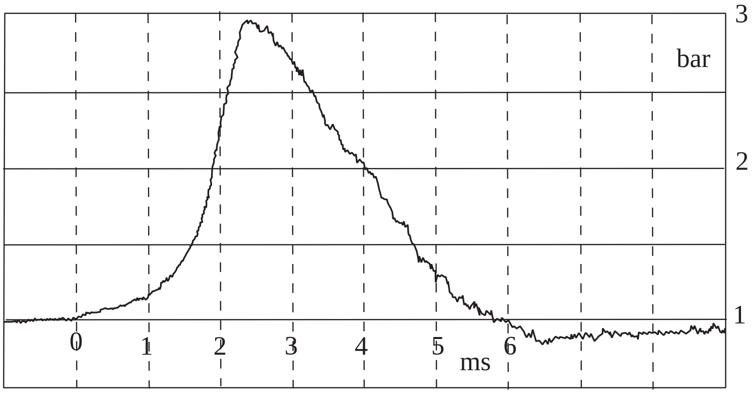

Figure 2: Measured pressure inside the pot.

Guided by experiments (figure 2), the following shape appeared to be realistic (see figure 3)

$$b(\zeta) = \begin{cases} 0 & \zeta \in (-\infty, 0.5] \\ \frac{37\exp(4-(\zeta-2.5)^2)/(1+9(\zeta-2.5)^2)-1}{37\exp(4)-1} & \zeta \in [0.5, 2.5] \\ 1 & \zeta \in [2.5, \infty). \end{cases}$$

The initial and final levels β_0 and β_1 and the combustion time τ are to be found from experiments. β_0 may be found directly from p_∞ and ρ_∞. If only the pressure in the pot can be measured, and not the temperature or the density, β_1 can be found by trial and error, until the maximum pot pressure fits the measured value. The detailed description of $b(\zeta)$ is of minor importance.

An estimate for the increase in entropy may be found as follows. In the *theoretical* case of a completely closed pot, so $\rho_1 = \rho_\infty =$ constant, the variation in entropy ds and internal energy de are given by the fundamental law of thermodynamics for reversible processes, and the relation for a perfect gas

$$T\mathrm{d}s = \mathrm{d}e + p\mathrm{d}\rho^{-1} = \mathrm{d}e = C_V\mathrm{d}T = \frac{C_V}{\rho_\infty R}\mathrm{d}p.$$

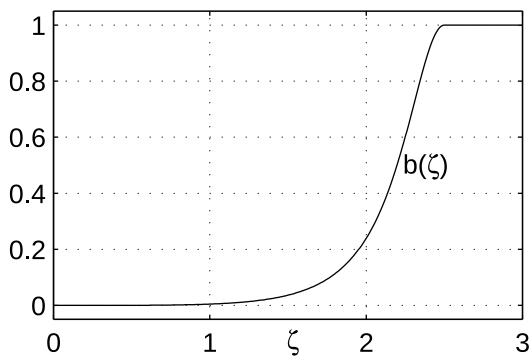

Figure 3: Shape function entropy increase.

So the energy increase ΔQ_1, and the entropy increase Δs_1, are functions only of the initial and theoretical final temperature or pressure:

$$\Delta Q_1 = \int T \, \mathrm{d}s = C_V(T_{th} - T_\infty) = \frac{p_{th} - p_\infty}{\rho_\infty(\gamma - 1)},$$

$$\Delta s_1 = \beta_1 - \beta_0 = \frac{p_{th} - p_\infty}{\rho_\infty^\gamma}.$$

It follows that if we knew the total amount of chemical energy transferred into heat, and the pot were closed, then we would know the increase of entropy. In practice this is an upper limit, because the pot is open: we do not know exactly the amount of gas that reacts, since a part of it is blown away into the pipe, and an a priori unknown part of the energy increase ($p\mathrm{d}\rho^{-1}$) is used for a volume change.

2.3 Jet

When the jet becomes turbulent, the flow is decelerated, and the jet widens until it fills the whole cross section S_2 of the pipe. The transition from the jet (j) to the pipe (2) is described by cross wise averaged conservation laws [1, 2]:

$$S_j\rho_j v_j = S_2\rho_2 v_2 \quad \text{(mass)}, \tag{7}$$

$$S_2 p_j + S_j\rho_j v_j^2 = S_2 p_2 + S_2\rho_2 v_2^2 \quad \text{(momentum)}, \tag{8}$$

$$S_j\rho_j v_j(\tfrac{1}{2}v_j^2 + w_j) = S_2\rho_2 v_2(\tfrac{1}{2}v_2^2 + w_2) \quad \text{(energy)}, \tag{9}$$

where $w = e + p/\rho$ denotes the heat function [4]. It should be noted that the pressure p_j inside the jet is equal to the value outside. For a perfect gas the heat function is given by [4, p.315] $w = \frac{\gamma}{\gamma-1}\frac{p}{\rho}$, and the energy integral may be further simplified to

$$\tfrac{1}{2}v_j^2 + \frac{\gamma}{\gamma-1}\frac{p_j}{\rho_j} = \tfrac{1}{2}v_2^2 + \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2} \tag{10}$$

Note that this equation is similar to Bernoulli's equation (the integral of the momentum equation along a streamline). However, Bernoulli is not valid here because the flow is not isentropic. Energy is lost in the form of conversion into turbulence (which has not been modelled).

At a later stage in the process, the pressure in the pipe may be increased by the reflected wave, and the pot pressure may become lower than the pipe pressure, leading to a jet flow *in*to the pot. Assuming the iris to behave symmetrically, the equations are the same as above, apart from a sign in the velocity. Note that this inward jet forms a sink of acoustic energy.

2.4 Pipe

When the hot jet enters the cold pipe, a hot-cold mixing zone is created. This is gradually convected into the pipe, and describing this process calls again for serious modelling assumptions.

Since the jet velocity v_j is at most sonic, and since the iris S_d is much smaller than the pipe cross section S_2, the pipe velocity v_2 is much smaller than the sound speed, and the propagation of the mixing zone can be ignored compared to the generated pressure wave. Thus only the flux of momentum across the mixing zone needs to be taken into account.

Since the mixing zone is small, the flux of momentum can be described in a quasi-stationary manner by continuity of pressure. Together with continuity of mass flux we have the following relations between the values at the hot side (subscript 2) and the cold side (subscript 0, referring to $x = 0$)

$$p_2 = p_0, \tag{11}$$

$$\rho_2 v_2 = \rho_0 v_0. \tag{12}$$

Note that we defined $x = 0$ as the position of the mixing zone between the hot jet and the cold pipe gas. This position is difficult to identify, but on the other hand, the jet is relatively small. Therefore we take $x = 0$ simply at the position of the iris.

Inside the pipe we assume that the generated pressure wave (and its reflections) of linear acoustic or weakly non-linear type, such that the temperature and sound speed are approximately constant, but not exactly (no isothermal changes). So we cannot put p_0 equal to $\rho_0 R T_\infty$.

Let us start by assuming that the amplitude of the wave in the pipe is small enough for an acoustic (linear, isentropic) one-dimensional approximation to be valid. The gas dynamic equations are then

$$\rho_t + \rho_\infty v_x = 0, \quad \rho_\infty v_t + p_x = 0, \quad \mathrm{d}p = c_\infty^2 \mathrm{d}\rho,$$

where the indices x and t indicate partial derivatives with respect to x and t. The values at $x = 0$ are as yet unknown, as they are coupled to the jet flow, but at $x = L$ the flow leaves the exit nearly spherically (at least for small pipe diameter). The outside velocity drops very quickly so that the pressure near the exit plane is practically equal to the ambient pressure. So we take the boundary condition of a vanishing acoustic pressure

$$p(L, t) - p_\infty = 0.$$

The effective position of the open end (as seen by the wave incident from the pipe) is slightly outside the exit plane. Typically, for low frequencies and a thin pipe wall, it is located at $x = L + 0.6a$ where a is the pipe radius, and $0.6a$ is called the "end correction". The error made by ignoring the end correction compensates to some extent the above error made in the position $x = 0$.

When we eliminate p, ρ or v from the above linearised equations, we obtain the 1-D wave equation

$$p_{tt} - c_\infty^2 p_{xx} = 0,$$

which has the well known solution of d' Alembert [4, p.246]. This leads, combined with the boundaty condition at $x = L$, to

$$p(x,t) = p_\infty + \rho_\infty c_\infty^2 \Big[f\Big(t - \frac{x}{c_\infty}\Big) - f\Big(t + \frac{x - 2L}{c_\infty}\Big)\Big] \tag{13}$$

$$\rho(x,t) = \rho_\infty + \rho_\infty \Big[f\Big(t - \frac{x}{c_\infty}) - f\Big(t + \frac{x - 2L}{c_\infty}\Big)\Big] \tag{14}$$

$$v(x,t) = c_\infty \Big[f\Big(t - \frac{x}{c_\infty}\Big) + f\Big(t + \frac{x - 2L}{c_\infty}\Big)\Big] \tag{15}$$

with a shape function f that is to be determined. Since we have silence for all $x \in [0, L]$ when $t < 0$, f satisfies the condition of causality

$$f(z) = 0 \quad \text{for } z \le 0. \tag{16}$$

One should be aware of the fact that the exit boundary condition of vanishing acoustic pressure is quite sufficient to determine the rough amplitude of the reflected wave, but in other respects is very crude. For example, it leads to equal amplitudes of the incident and reflected waves, which means that as much energy is incident as is reflected. In other words, there is no energy loss at the exit, and the wave would bounce back and forth for ever, if there was no vortex shedding from the iris,

In practice there is energy loss by radiation and vortex shedding, all of which are rather difficult phenomena to model. Therefore, it may be better to reduce the reflected amplitude a little, by writing $Rf(t+\frac{x-2L}{c_\infty})$ where R is a number slightly smaller than 1. The problem here, of course, is that we have little information on the appropriate value to be taken (especially because it is an ad-hoc "fix", and is not based on any modelling of the physics). For this reason, we will not pursue the matter further here.

2.5 Radiated Field

Outside the pipe we have relatively low amplitudes of the perturbation $p-p_\infty$, and the gas dynamic problem certainly simplifies to an acoustic one, with governing equation

$$p_{tt} - c_\infty^2 \nabla^2 p = c_\infty^2 Q_L'(t)\delta(\mathbf{x})$$

where $Q_L(t)$ is the mass flux from the pipe exit, while the co-ordinate system has been changed a little, so that the origin is now taken to be at the exit plane. From symmetry, the field depends only on t and r, where $|\mathbf{x}| = r$. The solution (for outgoing waves) is [4, p.265]

$$p(r,t) = p_\infty + \frac{1}{4\pi r} Q_L'\Big(t - \frac{r}{c_\infty}\Big).$$

Since the mass flux is equal to

$$Q_L(t) = \rho_\infty S_2 v(L,t) = 2S_2\rho_\infty c_\infty f\Big(t - \frac{L}{c_\infty}\Big), \tag{17}$$

we have for the outer field

$$p(r,t) = p_\infty + \frac{\rho_\infty c_\infty S_2}{2\pi r} f'\Big(t - \frac{L}{c_\infty}\Big), \tag{18}$$

which shows that the radiated field is proportional to the derivative of the pipe field.

Finally, we wish to assign a single number to the sound field, in order to measure its loudness. This is not straightforward, because the time history of the pressure is a continuous function, which may be interpreted as an infinite dimensional vector. (Or, if sampled, a finite but high dimensional vector.) The problem is to find a norm, suitably fitting with the physiology of our ear and brains. Spectral distribution, duration, presence of pure tones, and many more factors may play a rôle, and as a result there exist many definitions of noise level.

Since the range of our audible sensitivity is incredibly large (10^{14} in energy), the loudest and quietest levels are almost practically infinitely far away, and we have no reference, or scaling level, to compare with, other than the sound itself we are hearing. As a result, sound intensity is (necessarily) perceived logarithmically. Therefore, the various definitions of sound level always include a (base-10) logarithm.

One of the most elementary and widely used definitions is the SPL (short for Sound Pressure Level). It is based on the acoustic energy density, averaged over a short, but not too short period of time. Since acoustic energy is proportional to $(p - p_\infty)^2$, this averaged energy is (apart from a square) equivalent to the root-mean-square (rms) of $p - p_\infty$. This averaged acoustic pressure is then compared to a reference level of $2 \cdot 10^{-5}$ Pascal, which corresponds (for good ears and optimal conditions) to the threshold of hearing.

By taking the logarithm of the ratio $(\text{rms}/2 \cdot 10^{-5})^2$ we obtain the level in Bels, and hence in decibels if we multiply this by 10.

The noise level (SoundPressure Level, SPL) at a distance r is thus defined by

$$SPL = 20 \log_{10}\left[\left\{\frac{1}{T_{ref}} \int_0^{T_{ref}} (p(r,t) - p_\infty)^2 \,\mathrm{d}t\right\}^{1/2} / 2 \cdot 10^{-5}\right] \text{dB}, \quad (19)$$

where by definition $T_{ref} = 1$ sec. for "slow" measurements (noise of long duration), $T_{ref} = \frac{1}{8}$ sec. for "fast" measurements (short duration), and $T_{ref} = 35$ msec. for "impulsive" measurements (impulsive sound). The present noise character is in general of impulsive type, since the main pressure variations (the part that contributes the most to the SPL value) occur within 35 msec. If the noise duration is less than 35 msec, the difference between these three measuring definitions is of course just 9 dB and 5.5 dB. On the other hand, if the problem parameters were changed such that the pot would be emptied much slower (for example by reducing the size of the iris), then the time history would extend much beyond the 35 msecs, and it would be necessary to use another definition.

These formal definitions of SPL have absolutely no bearing on the physical phenomena, of course, but they indicate that the typical accuracy of any suitable model is in the order of decibels, which is rather crude from a gas dynamics point of view.

2.6 Nonlinear correction in the pipe

A very important observation we can make from equation (18) concerns the fact that the outside pressure depends on the derivative f', rather than on f itself. So any steepening of the wave inside the pipe is immediately coupled to the outside noise level. Therefore, it seems relevant, and maybe necessary, to investigate here the possibility of incorporating an extension of the foregoing model in which we include the acoustic effects of the wave steeping.

The general problem of how the left and right running compression waves behave is too difficult for any analytical approach. The main effect here, however, is the creation of a wave generated by the jet, and incident to the pipe exit. Therefore, we consider a 1-D isentropic gas flow, with a perturbation propagating in positive direction, given by

$$\rho_t + (\rho v)_x = 0, \quad \rho(v_t + v v_x) + p_x = 0, \quad \mathrm{d}p = c^2 \mathrm{d}\rho. \tag{20}$$

If we assume velocity and pressure to be functions of ρ only, and follow the disturbance along $x = X(t)$, we have [4, p.366]

$$\rho_t + \rho v' \rho_x + \rho_x v = 0, \quad \rho v' \rho_t + \rho v v' \rho_x + c^2 \rho_x = 0,$$

so that

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\rho(X(t),t) = \rho_t + \dot{X}\,\rho_x = \rho_t + (\rho v' + v)\rho_x = \rho_t + \left(v + \frac{c^2}{\rho v'}\right)\rho_x = 0,$$

which is possible only if

$$\rho v' = \pm c$$

leading to

$$X = X_0 + (v \pm c)t$$

because ρ and hence v and p are constant along $x = X(t)$. Furthermore, isentropy implies that $2c^{-1}\mathrm{d}c = (\gamma - 1)\rho^{-1}\mathrm{d}\rho$, so that

$$\frac{\mathrm{d}v}{\mathrm{d}c} = \frac{\mathrm{d}v}{\mathrm{d}\rho}\frac{\mathrm{d}\rho}{\mathrm{d}c} = \pm\frac{2}{\gamma - 1},$$

which leads to

$$c = c_0 \pm \tfrac{1}{2}(\gamma - 1)(v - v_0).$$

Since v_0 is usually much smaller than c_0, the development of a nonlinear acoustic perturbation, propagating from $x = x_0$ at $t = t_0$ to a point x at time t, is therefore described by [3, p.569]

$$x - x_0 = (c_0 + \tfrac{1}{2}(\gamma + 1)v)(t - t_0). \tag{21}$$

In other words, the perturbations to p_0 and v_0 that start at $x_0 = 0$ at time t_0 arrive at $x = L$ at time

$$t_L = t_0 + \frac{L}{c_0(t_0) + \frac{1}{2}(\gamma + 1)v_0(t_0)} \tag{22}$$

We see that perturbations with a larger velocity $v_0(t_0)$ propagate faster (t_L is smaller). Figure 4 illustrates this phenomenon: in the first picture the excess density is plotted against x. The top of the wave travels faster than the bottom and so it steepens. In the second and third pictures the excess density is plotted against t. We observe that the wave passes through quicker at $x = x_2$ than at $x = x_1$.

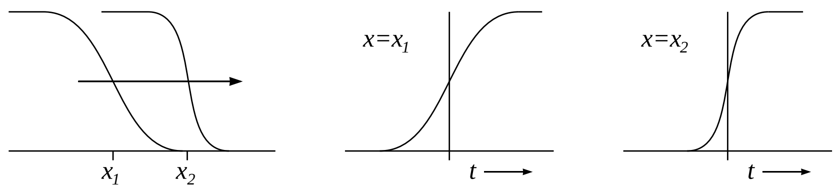

Figure 4: Sketch of nonlinear wave steepening.

When t is small enough, the wave propagates in the pipe only in the forward direction, so the present approach is certainly valid. When the backward running, reflected part is added, the amplitudes change and so do the propagation speeds. We will, however, ignore this aspect of the problem and assume the forward and backward running waves to be uncoupled, because this stage of the process is already of less importance acoustically.

As a result, the acoustic field (13,14,15) is modified such that f follows from

$$v(L, t) = 2c_\infty f\Big(t - \frac{L}{C^+(\tilde{v}_0)}\Big) \tag{23}$$

where

$$C^{\pm}(v) = c_\infty \pm \tfrac{1}{2}(\gamma + 1)v.$$

and $\tilde{v}_0$ is the value of v at $x = 0$ a time $L/C^{+}(\tilde{v}_0)$ earlier. Furthermore,

$$v_0 = v(0,t) = c_\infty\Big[\, f(t) + f\Big(t - \frac{L}{C^{+}(\hat{v}_0)} - \frac{L}{C^{-}(\hat{v}_L)}\Big)\Big] \qquad (24)$$

and $\hat{v}_0$ is the value of v at $x = 0$ a time $L/C^{+}(\hat{v}_0) + L/C^{-}(\hat{v}_L)$ earlier, and $\hat{v}_L$ is the value of v at $x = L$ a time $L/C^{-}(\hat{v}_L)$ earlier. Once we have v, similar expressions for p and ρ follow immediately.

This process is easily evaluated sequentially in time, because of the causality condition (16).

3 Analysis

To finally obtain results, we have to couple the pot (section 2.2) and the jet (section 2.3) with the pipe (sections 2.4 and 2.6) and the radiated field (section 2.5). By combining equations (2), (3), and (6) we can eliminate v_j to obtain

$$v_j = \left(\frac{2\gamma\beta}{\gamma - 1}\right)^{1/2} \left(\rho_1^{\gamma-1} - \rho_j^{\gamma-1}\right)^{1/2}.$$

Note that this cannot be valid anymore if $\rho_1 < \rho_j$. Of course, this is because in that case the jet has changed direction and the flow is from pipe into the pot. This is a situation that only occurs in a rather late stage of the process, when the pot has emptied itself almost completely, and the reflected wave in the pipe creates temporarily a slightly higher pressure. Since this is acoustically rather unimportant, we will cure this problem by a simple measure and just change the flow direction, ignoring any further details of the flow. So we have

$$v_j = \mathrm{sign}(\rho_1 - \rho_j) \left(\frac{2\gamma\beta}{\gamma - 1}\right)^{1/2} \left|\rho_1^{\gamma-1} - \rho_j^{\gamma-1}\right|^{1/2}. \qquad (25)$$

Thus v_j depends only on ρ_1 and ρ_j, and we have for equation (7) the following form

$$\frac{\mathrm{d}\rho_1}{\mathrm{d}t} = F(\rho_1, \rho_j) = -\frac{S_j}{\Omega}\rho_j v_j, \qquad (26)$$

which can be solved numerically for t as soon as we know the relationship between ρ_1 and ρ_j. From equations (7), (8), and (10) we can find expressions for $\rho_2 v_2$ and $\rho_2 v_2^2$

$$\rho_2 v_2 = \sigma \rho_j v_j,$$

$$\rho_2 v_2^2 = \sigma \frac{2\gamma\beta}{\gamma - 1}\Big(\rho_1^{\gamma-1} - \rho_j^{\gamma-1}\Big)\rho_j + \beta\rho_j^\gamma - p_2 = \frac{2\gamma}{\gamma - 1}\Big(\beta\rho_2\rho_1^{\gamma-1} - p_2\Big),$$

where $\sigma = S_j/S_2$.

Now we can eliminate ρ_2

$$\rho_2 = \sigma\rho_j + \rho_1^{1-\gamma}\left(\frac{\gamma+1}{2\gamma\beta}p_2 + \Big(\frac{\gamma-1}{2\gamma} - \sigma\Big)\rho_j^\gamma\right). \tag{27}$$

For given p_2, the relation $\rho_1 = \rho_1(\rho_j)$ is given by the algebraic equation

$$G(\rho_1, \rho_j) = \sigma\rho_1^{\gamma-1}\rho_j\left[\frac{p_2}{\gamma\beta} + \Big(\frac{\gamma-1}{\gamma} - \sigma\Big)\rho_j^\gamma\right] + \Big(\frac{\gamma-1}{2\gamma} - \sigma\Big)^2\rho_j^{2\gamma}$$
$$+\Big(\frac{\gamma-1}{2\gamma} - \sigma\Big)\rho_j^\gamma\frac{p_2}{\gamma\beta} - \frac{1}{4}(\gamma^2 - 1)\Big(\frac{p_2}{\gamma\beta}\Big)^2 = 0. \tag{28}$$

To obtain for given ρ_1 the zero ρ_j, we should note that $G(\rho_1, 0) = -\frac{1}{4}(\gamma^2 - 1)(p_2/\gamma\beta)^2 < 0$, and $G = O(\rho_j^{2\gamma}) > 0$ for $\rho_j \to \infty$, so there is at least one zero. If $\sigma > 1 - \gamma^{-1}$ there may be more, in which case we need the smallest one. This one is always less than $(p_2/\gamma\beta)^{1/\gamma}(\sigma - 1 + \gamma^{-1})^{-1/\gamma}$, and there is only one.

Taking everything together, we have now a differential equation (26) coupled with a system of algebraic equations (25,28,6,11,12,27) and delay equations (24,23) and analogous equations for p and ρ, which can be solved numerically, to yield the outside pressure (SPL) field (18) and (19).

We have combined a standard root-finding routine for $G(\rho_1, \rho_j) = 0$ (equation 28), producing $\rho_j(\rho_1)$, with a standard 4th order Runge-Kutta integration routine for differential equation (26). Since previous values of f are necessary to include reflections, the obtained values $f(z_j)$ at z_j are stored in an array. By interpolation, these values are used to define $f(z)$ for positive z. We know already that $f(z) = 0$ for negative z.

4 Results

We note that the radiated sound field depends on f' and *not* on f itself. So the absolute value of the pressure in the pipe plays a minor rôle. That is why the wave steepening itself is important, because with increasing (negative) gradients more sound is radiated. Therefore the pipe length is also important to the noise level. In the same way the explosion time τ (inherent in $\beta(t)$) is important.

For typical values of the parameters (β_1 = 4.5, explosion time τ = 2.44 ms, Ω = 2092 cm^3, L = 60 cm, S_j = 9.5 cm^2, S_2 = 63.6 cm^2, p_∞ = 1 bar, ρ_∞ = 1.2 kg/m^3, T_∞ = 20°C, γ = 1.4, r = 1 m) a very representative noise level $SPL = 114.2$ dB is found. The corresponding time histories (time in milliseconds) are given in figure 5.

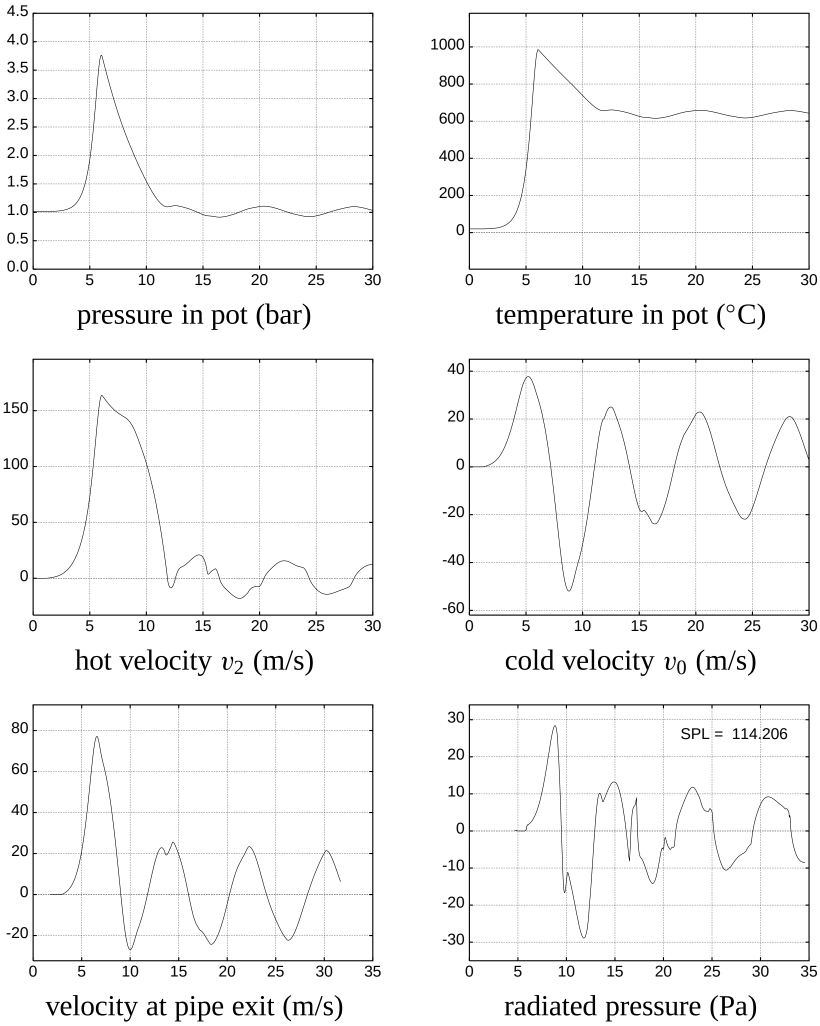

Figure 5: Time histories of a typical case.

We see in the pot pressure after the initial explosion some residual wiggles of the reflected waves. However small these are, they give rise to some in- and out-flow through the iris (positive and negative v_2 and

v_0). After the initial pulse the decay in the pipe is rather fast, justifying the assumption of weak nonlinearity.

The found SPL is exactly in the range of 112–115 dB which is found experimentally. It should be noted that the experiments are, at least under normal operational conditions, not very well repeatable. This is propably because of wind and poorly controllable combustion. So there is evidence that we have indeed modelled the dominating effects. Unfortunately, experimental input of the entropy change β_1 and the explosion time τ is necessary. Therefore, the possibility of studying the effects of pot size and iris diameter are rather limited.

5 Conclusions and suggestions for further work

An attempt has been made to model the sound production mechanism of a bird scare gun. The model is not based on a systematic reduction of a more complete model. We have built our problem description step by step by adding effects and elements like building blocks, until the required accuracy or adequacy is obtained (at least, this was our aim!)

The model consists of four parts. First (1) an increase of energy in the pot is created by combustion of gas. Since combustion is difficult to model on first principles, the resulting increase of entropy is taken from exprimental data. Then (2) the high pressure inside the pot drives a hot jet flow from the pot into the pipe. This jet acts like a piston and creates (3) a weakly nonlinear wave in the pipe, which (4) radiates away from the exit to the observer outside.

The results are well in agreement with what could be expected. It is in particular very satisfying to find the resulting SPL to be right among the measured levels. Nevertheless, it is necessarily a little on the high side, because there is no damping at the open-end reflection in our model. In reality the sound wave loses energy by radiation and vortex shedding at each reflection. If we consider only the first few reflections, this effect is not important, but for longer duration, the sound level remains incorrectly at a too high level. It may be noted that the jet from the iris, primarily included to model the generation of the pressure wave, at the same time models a certain amount of vortex shedding at each reflection against the iris. So at the iris there is some loss of acoustic energy.

It is not very difficult to include some exit radiation damping on an ad-hoc basis. We simply multiply the terms in formulas (13,14,15) that correspond to the reflected wave, by a reflection coefficient R, slightly smaller than 1. In general, however, this reflection involves 3-D effects, which calls for further modelling.

A very important omission in the present model is obviously the combustion. This should be reconsidered seriously, if possible from first principles. Only then is a real study of the effects of iris and pot size possible.

When the pipe length is increased, the predicted noise level increases too, as expected. However, when the pipe is too long, the results become unpredictable. This is probably caused by the wave steepening making the wave more than weakly nonlinear, and probably a fully nonlinear method is necessary. This goes together with a full numerical solution, though.

An interesting variant, well within the scope of the present model, would be a slowly varying pipe diameter (a horn). Until now, the design has been deliberately simple and robust. If the acoustic properties of a variable duct are very favourable, it may become an interesting alternative design.

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