LNMB-course Algorithmic Methods in Queueing Theory (AlQT)

Homework assignment (part 2)

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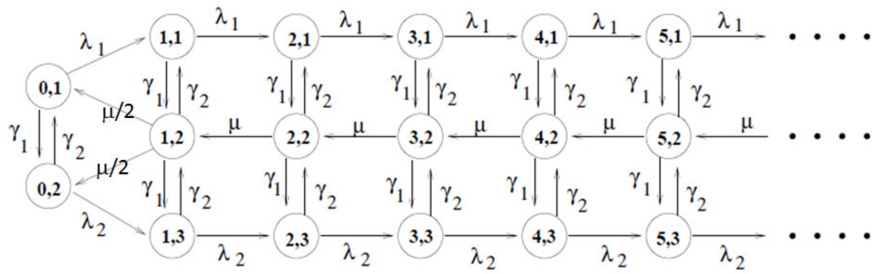
Exercise 2: Matrix geometric method for QBD processes

Consider the following example transition diagram for the QBD process that was discussed during the third lecture by Rob.



2.1 The Q-matrix of this QBD process has a block structure. Write down the Q-matrix.

2.2 What are the stability conditions for the QBD process (Neuts' drift condition)? Explain why this condition does not depend on γ_1 and γ_2.

2.3 What are the balance equations for this process?

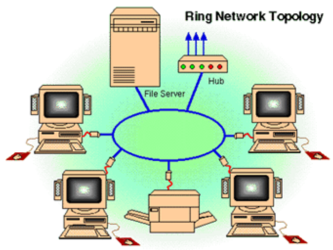
2.4 Formulate the equation that characterizes the R-matrix.

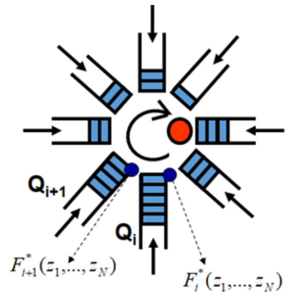
2.5 Write a program that calculates the equilibrium distribution of the QBD process via the matrix-geometric method. Add the source code to the assignment.

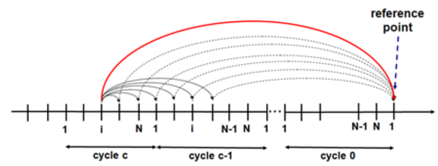
2.6 Assume that the model parameters take the following values:
$\lambda_1 = \lambda_2 = 2, \quad \mu = 6, \quad \gamma_1 = \gamma_2 = 3.$
Use the program written in 2.5 to calculate the equilibrium probabilities.

Hint: to validate the correctness of the program, note this model instance is symmetric with respect to sub-index 1 and 2.

2.7 Assume that the model parameters take the following values:
$\lambda_1 = 1, \quad \lambda_2 = 4, \quad \mu = 6, \quad \gamma_1 = 3, \quad \gamma_2 = 5.$
Use the program written in 2.5 to calculate the equilibrium probabilities.







<u>Exercise 3</u>: Waiting-time analysis in polling models with the Buffer Occupancy Method and the Descendant Set Approach

Consider the following three-queue polling model. Arrival occur according to independent Poisson processes with *relative* arrival rates 1:2:3 for queues 1, 2 and 3, respectively. The service times at the each of three queues are exponentially distributed with mean 1. Queue 1 receives gated service and queues 2 and 3 are served exhaustively. The switchover times from queue 1 to queue 2, from queue 2 to queue 3, and from queue 3 to queue 1 are all independent and exponentially distributed with means 0.10, 0.25 and 1.00, respectively.

3.1. What are the (absolute) arrival rates when the load of the system is set to $\rho = 0.8$? What is the mean switch-over time per cycle? What is the second moment of the total switch-over time per cycle? What are the first two moments of the service times of an *arbitrarily* chosen customer in the system (i.e., weighted proportionally to the arrival rates)?

3.2. A necessary and sufficient condition for the stability of the system is $\rho < 1$. Remarkably, this condition does not depend on the switch-over time distributions. Give an intuitive explanation for this (no proofs needed).

3.3. Formulate the pseudo-conservation law (PCL) for this model. Make sure that all the parameters that you mention are well-defined.

3.4. Assume $\rho = 0.8$. Write a program that calculates the expected waiting time at *each* of the three the queues by using the Buffer Occupancy Method (BOM). Write down the formula's and equations you use.

<u>Hint</u>: Validate the correctness of your results by comparing the left-hand side and right-hand side of the PCL formulated in exercise 3.3.

3.5. For the same model, work out the details of Descendant Set Approach (DSA) and write a program that calculates the expected waiting time queue 1. Check the correctness of the results by comparing the outcome to the results from exercise 3.4.

3.6. Suppose now that queue 1 is also served exhaustively instead of gated (while queues 2 and 3 are still served exhaustively). Intuitively, what would you expect to happen to the mean waiting time at each of the three queues?

3.7. Calculate the mean waiting times at each of the queues by using the BOM, and check the correctness of the results by substituting the results into the PCL formulated in Exercise 3.3. Do the results meet your expectations? Explain what you see.