# Geometry in architecture and building

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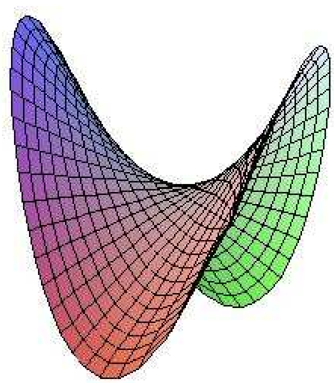

Lecture notes for '2DB60 Meetkunde voor Bouwkunde'

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# Chapter 1

# Curvature

### 1.0.1 Motivation

From calculus we recall the following. Suppose $f$ is a differentiable function of $x$ and $y$. If the partial derivatives $f_x$ and $f_y$ vanish at the point $(a, b)$, then the graph has a horizontal tangent plane at $(a, b, f(a, b))$. Sometimes the second order partial derivatives can be used to determine if $f$ has a (local) minimum, maximum or a saddle-point at $(a, b)$. For instance, if

$$f_{xx}(a,b) > 0 \quad \text{and} \quad f_{xx}(a,b) \cdot f_{yy}(a,b) - f_{xy}(a,b) > 0,$$

then $f$ has a local minimum at $(a, b)$. The larger

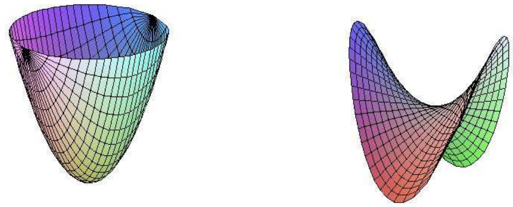

Figure 1.1: *A local minimum and a saddle-point.*

$f_{xx}(a,b) \cdot f_{yy}(a,b) - f_{xy}(a,b)$ is, the steeper the graph of $f$ is in the neighbourhood of $(a, b, f(a, b))$.

If

$$f_{xx}(a,b) < 0 \quad \text{and} \quad f_{xx}(a,b) \cdot f_{yy}(a,b) - f_{xy}(a,b) > 0,$$

then $f$ has a local maximum at $(a, b)$.

If

$$f_{xx}(a,b) \cdot f_{yy}(a,b) - f_{xy}(a,b) < 0,$$

then the graph of $f$ has a saddle-point.

From these observations, one concludes that $f_{xx}(a,b) \cdot f_{yy}(a,b) - f_{xy}(a,b)$ somehow measures the curvature of the graph of $f$; at least in stationary points. The situation in a non-stationary point can be dealt with in a similar way: first rotate the graph so that the tangent plane becomes horizontal, then analyse the situation as before, and finally rotate things back. We will not give the computational details, but just give the final result: a good measure for the *curvature* of the graph of $f$ at $(a, b, f(a,b))$ is

$$\frac{f_{xx}(a,b)f_{yy}(a,b) - f_{xy}^2(a,b)}{(1 + f_x(a,b)^2 + f_y(a,b)^2)^2}\,.$$