Exercises Algorithms for Model Checking

1 Model Checking the Modal μ-Calculus

1. Prove, for arbitrary environment θ, arbitrary labelled transition system $\mathcal{L}$ with action set $Act \supseteq \{a\}$ that $[\![[a]\nu X.[a]X]\!]\theta = [\![\mathsf{true}]\!]\theta$.
2. Prove, for arbitrary environment θ and arbitrary labelled transition system $\mathcal{L}$ that $[\![\neg\mu X.\phi]\!]\theta = [\![\nu X.\neg\phi[X := \neg X]]\!]\theta$ for all formulae ϕ. Hint: Expand $[\![\_]\!]$ as much as possible and perform induction over the number of fixpoint-iterations.

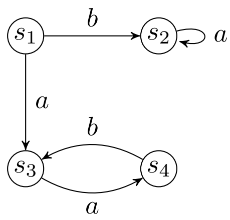


Figure 1:

3. Consider the following μ-calculus formula ϕ and the labelled transition system $\mathcal{L}$ in Fig. 1.

$$\phi := \nu X.\Big([a]X \wedge \nu Y.\mu Z.(\langle b\rangle Y \vee \langle a\rangle Z)\Big)$$

   (a) Explain in natural language the meaning of formula ϕ.
   (b) Compute the set of states of $\mathcal{L}$ where ϕ holds with the naive algorithm (give all intermediate approximations).
   (c) Compute the set of states of $\mathcal{L}$ where ϕ holds with the Emerson-Lei's algorithm (give all intermediate approximations).

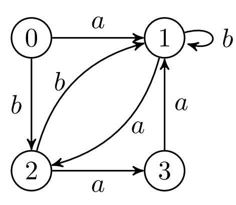


Figure 2:

4. Consider the following μ-calculus formula and the labelled transition system $\mathcal{L}$ in Fig. 2.

$$\nu X.\ \nu Y.\ \Big(\big(\langle b\rangle X\big) \wedge \big(\langle a\rangle(Y \wedge \langle a\rangle X)\big)\Big)$$

   (a) Explain in natural language the meaning of formula ϕ.

(b) Compute the set of states of $\mathcal{L}$ where ϕ holds with the naive algorithm (give all intermediate approximations).

(c) Compute the set of states of $\mathcal{L}$ where ϕ holds with the Emerson-Lei's algorithm (give all intermediate approximations).

2 Boolean Equation Systems

1. Prove, for arbitrary environment θ, arbitrary labelled transition system $\mathcal{L}$ that for arbitrary μ-calculus formula ϕ in which the formal variable A does *not* occur, the equivalence $[\![\phi]\!]\theta = [\![\nu A.\phi]\!]\theta$ holds.

2. Prove, for arbitrary μ-calculus formulae ϕ, ψ and arbitrary labelled transition system $\mathcal{L}$, that for all environments θ the equivalence $[\![\mathsf{E}\ (\phi \wedge \psi)]\!]\theta = [\![\mathsf{E}\ (\psi \wedge \phi)]\!]\theta$ holds. Hint: use structural induction and expand $[\![\_]\!]$ as much as possible.

3. Translate the model checking problem $\mathcal{L} \models \phi$ of exercise 1.3 to a Boolean equation system and solve it using Gauß Elimination.

4. Translate the model checking problem $\mathcal{L} \models \phi$ of exercise 1.4 to a Boolean equation system and solve it using Gauß Elimination.

3 Parameterised Boolean Equation Systems

1. You are given the following parameterised Boolean equation system:

$$\begin{array}{l}
\big(\nu X(i:\mathbb{N}) = X(i+1) \wedge Z(i, \mathsf{even}(i))\big) \\
\big(\mu Y(j:\mathbb{N}, b:\mathbb{B}) = (b \wedge Z(2*j, b)) \vee (\neg b \wedge Z(j, b))\big) \\
\big(\nu Z(k:\mathbb{N}, c:\mathbb{B}) = Y(k+1, \neg c)\big)
\end{array}$$

Note: the function $\mathsf{even}(i)$ is true iff i is even.

(a) Compute the set of redundant parameters of the given PBES.

(b) Compute the solution to $X(i)$, for all $i \in \mathbb{N}$. Hint: you may use logic to simplify right-hand sides of the PBES equations. Again show the intermediate steps, and explain your line of reasoning.

2. You are given the following parameterised Boolean equation system:

$$\begin{array}{l}
\big(\nu X(b_0:\mathbb{B}, b_1:\mathbb{B}, b_2:\mathbb{B}, b_3:\mathbb{B}, b_4:\mathbb{B}) = \\
\qquad \forall c:\mathbb{B}.\ ((b_0 \vee c) \wedge X(c, \neg b_0, b_4, b_3, b_2)) \vee Y(b_0 \vee b_1, b_1, b_2 \vee b_3 \vee b_4)\big) \\
\big(\mu Y(c_0:\mathbb{B}, c_1:\mathbb{B}, c_2:\mathbb{B}) = \\
\qquad X(\neg c_0, c_0, c_1 \vee c_2, \neg c_1 \wedge c_2, c_1) \vee Y(\neg c_0, c_2, c_1)\big)
\end{array}$$

(a) Compute the set of redundant parameters of the given PBES.

(b) Compute the solution to $X(b_0, b_1, b_2, b_3, b_4)$ for every combination of b_0, b_1, b_2, b_3, b_4 of type $\mathbb{B}$. Again show the intermediate steps. □

3. Consider the LPE description of a lossy channel system, where actions r, s and l represent *receiving*, *sending* and *losing*, respectively.

$$\begin{array}{rcl}
C(b{:}\mathbb{B}, m{:}\mathbb{N}) & = & \sum_{k:\mathbb{N}} b \longrightarrow r(k) \cdot C(\mathsf{false}, k) \\
& + & \neg b \longrightarrow s(m) \cdot C(\mathsf{true}, m) \\
& + & \neg b \longrightarrow l \cdot C(\mathsf{true}, m)
\end{array}$$

Let ϕ be first-order modal μ-calculus formula $\nu X.\ \mu Y.(\langle l \rangle X \vee \langle \neg l \rangle Y)$.

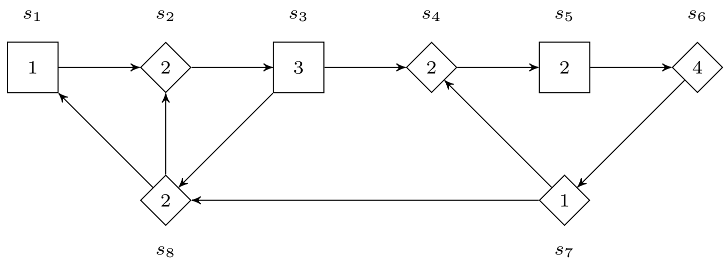
(a) Verify whether the PBES given below can be the result (up to logical equivalence) of the transformation $\mathsf{E}\ \phi$ applied to C. Clearly relate the (sub)expressions in the PBES to the (sub)expressions in ϕ, or mark the (sub)expression(s) of ϕ that demonstrate(s) an error in the transformation and correct it.

$$
\begin{array}{lcl}
\big(\nu X(b{:}\mathbb{B}, m{:}\mathbb{N}) & = & Y(b, m)\big) \\
\big(\mu Y(b{:}\mathbb{B}, m{:}\mathbb{N}) & = & (\neg b \wedge X(\mathsf{true}, m)) \\
 & \vee & (\neg b \wedge Y(\mathsf{true}, m) \\
 & \vee & (\exists k{:}\mathbb{N}.\ b \wedge Y(\mathsf{false}, k))\big)
\end{array}
$$

(b) If possible, compute and solve a Boolean Equation System from the above PBES that answers whether $X(\mathsf{true}, m_0) = \mathsf{true}$ (for given value $m_0 \in \mathbb{N}$), or clearly indicate why this cannot be done.

4 Parity Games

1. Consider the following Parity Game G, where the diamond vertices are owned by player *Even* and the square vertices are owned by player *Odd*. The priorities associated to each vertex are written inside the vertices.



   (a) Determine the set of vertices won by player *Even*.

   (b) Validate your answer by transforming the problem to a Boolean equation system.

2. Compute the Parity Games underlying the Boolean equation systems of Exercises 2.3 and 2.4.