Algorithms for Model Checking (2IW55)

Lecture 11

Parameterised Boolean Equation Systems (3)

Background material:

- *Verification of Reactive Systems via Instantiation of Parameterised Boolean Equation Systems*, B. Ploeger, J.W. Wesselink and T.A.C. Willemse (*I&C 2010/2011*)

- *Static Analysis Techniques for Parameterised Boolean Equation Systems*, S. Orzan, J.W. Wesselink and T.A.C. Willemse (*TACAS 2009*)

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Outline

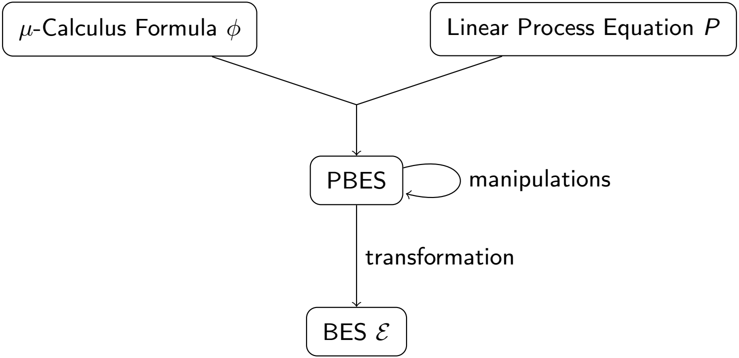
Parameterised Boolean Equation Systems

Instantiation

Manipulations

Exercise

Verification Methodology:



Solving $\mathcal{E}$ answers $P \models \phi$

Problem Description

1. Given a process $P(e)$ described by an LPE P over Act
2. Given a first-order modal μ-calculus formula $\sigma X.\phi$
3. Given environments η, ε
4. Check whether $P(e) \models \sigma X.\phi$ holds, where:

$$P(e) \models \sigma X.\phi \text{ iff } e \in [\![\sigma X.\phi]\!]\eta\varepsilon$$

5. Conversion to PBES:

$$P(e) \models \sigma X.\phi \text{ iff } e \in [\![\mathsf{E}\ (\sigma X.\phi)]\!]\eta\varepsilon(\tilde{X}) \text{ (or, more informally: } \tilde{X}(e) = \text{true)}$$

Outline

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Instantiation

How to solve PBESs

$$X_i(e) \stackrel{?}{=} \text{true in } \mathcal{E} := \;\; (\sigma_1 X_1(d_1 : D_1) = \phi_1) \cdots (\sigma_n X_n(d_n : D_n) = \phi_n)$$

Known techniques for solving/simplifying $\mathcal{E}$:

- Gauß Elimination on PBES + symbolic approximation of equations
- Instantiation to BES and subsequently solve the BES
- Using patterns
- Using under/over approximation
- Invariants

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Definition (Logical Equivalence)

Let ϕ, ψ be two predicates. Then ψ is logically equivalent to ϕ, denoted $\phi \leftrightarrow \psi$ iff

$$\forall \varepsilon, \eta : \; [\![\phi]\!]\eta\varepsilon = [\![\psi]\!]\eta\varepsilon$$

- If $\phi \leftrightarrow \psi$, then equation $\nu X(d : D) = \phi$ has the same solution as $\nu X(d : D) = \psi$ (likewise for μ)
- Useful simplifications:
  - $\mathsf{false} \wedge \phi \leftrightarrow \mathsf{false}$
  - $\mathsf{true} \vee \phi \leftrightarrow \mathsf{true}$
  - if $d \notin \mathsf{FV}(\phi)$, then $(\exists d : D.\ \phi) \leftrightarrow (\forall d : D.\ \phi) \leftrightarrow \phi$
  - One-point rule: $(\exists d : D. d = e \wedge \phi(d)) \leftrightarrow \phi(e)$
  - One-point rule: $(\forall d : D. d = e \Rightarrow \phi(d)) \leftrightarrow \phi(e)$
- Apply logical simplifications before applying PBES manipulations/solving techniques.

Instantiation to BES:

$$X_i(e) \stackrel{?}{=} \text{true in } \mathcal{E} := \quad (\sigma_1 X_1(d_1 : D_1) = \phi_1) \cdots (\sigma_n X_n(d_n : D_n) = \phi_n)$$

- Let X_i^e be a fresh propositional variable representing instance $X_i(e)$.
- The procedure below creates a BES from $\mathcal{E}$ s.t. $X_i(e) = \text{true}$ iff $X_i^e = \text{true}$
  1. For each $X_j(e_j)$ occurring in $\text{eval}(\phi_i[d_i := e])$ create a fresh variable $X_j^{e_j}$
  2. Create an equation $\sigma_i X_i^e = \tilde{\phi}_i$, where:
     - $\overline{\phi_i} = \text{eval}(\phi_i[d_i := e])$,
     - $\tilde{\phi}_i$ is $\overline{\phi_i}$ in which every $X_j(e_j)$ is replaced by $X_j^{e_j}$
  3. Repeat step 1 and 2 for every $X_j^{e_j}$ introduced in step 1, for which there is no equation
  4. Order all equations $\sigma_i X_i^e = \ldots$ according to the ordering of $\mathcal{E}$ (ordering within a block may be arbitrary)

Example

PBES: $(\nu X(n : Nat) = n \leq 2 \wedge Y(n))$ $(\mu Y(n : Nat) = \text{odd}(n) \vee X(n + 1))$
Instantiation starting at e.g. $X(0)$ .... introduce X^0

1. $Y(0)$ occurs in $\text{eval}((n \leq 2 \wedge Y(n))[n := 0])$ .... introduce Y^0
2. Introduce $\nu X^0 = \text{eval}(0 \leq 2 \wedge Y^0)$ .... $\nu X^0 = Y^0$
3. $X(1)$ occurs in $\text{eval}((\text{odd}(n) \vee X(n + 1))[n := 0])$ .... introduce X^1
4. Introduce $\mu Y^0 = \text{eval}(\text{odd}(0) \vee X^1)$ .... $\mu Y^0 = X^1$
5. $Y(1)$ occurs in $\text{eval}((n \leq 2 \wedge Y(n))[n := 1])$ .... introduce Y^1
6. Introduce $\nu X^1 = \text{eval}(1 \leq 2 \wedge Y^1)$ .... $\nu X^1 = Y^1$
7. no variable occurs in $\text{eval}((\text{odd}(n) \vee X(n + 1))[n := 1])$ .... end
8. Introduce $\mu Y^1 = \text{eval}(\text{odd}(1) \vee X^2)$ .... $\mu Y^1 = \text{true}$
9. Order equations: first X^i, then Y^j
10. Resulting BES: $(\nu X^0 = Y^0)$ $(\nu X^1 = Y^1)$ $(\mu Y_0 = X^1)$ $(\mu Y_1 = \text{true})$

Outline

Definition (Simple Formula)

A simple formula is a formula not containing predicate variables

Observations:

1. Consider the equation $\nu X(n : Nat) = \text{true} \wedge X(n+1)$
   - X has solution Nat (check!)
   - Consider formal parameter n:
   - It does not affect the value of the simple subformula true
   - It appears to be redundant for the solution to X
2. Consider the equation $\nu X(n : Nat, m : Nat) = n \leq 5 \wedge X(n+m, m)$
   - X has solution $\{(n, 0) \in Nat \times Nat \mid n \leq 5\}$ (check!)
   - Consider formal parameter m:
   - It does not affect the value of the simple formula $n \leq 5$
   - Via a single recursion through X, it does affect the value of $n \leq 5$
   - It appears to become significant for the solution to X

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PBES Manipulation

- Identify all obvious significant formal parameters . . . . sig
- Identify the dependencies . . . . dep

$$
\begin{aligned}
\mathsf{sig}(b) &= \mathsf{FV}(b) \\
\mathsf{sig}(X(e)) &= \emptyset \\
\mathsf{sig}(\phi \wedge \psi) &= \mathsf{sig}(\phi) \cup \mathsf{sig}(\psi) \\
\mathsf{sig}(\phi \vee \psi) &= \mathsf{sig}(\phi) \cup \mathsf{sig}(\psi) \\
\mathsf{sig}(\forall d{:}D.\ \phi) &= \mathsf{sig}(\phi) \setminus \{d\} \\
\mathsf{sig}(\exists d{:}D.\ \phi) &= \mathsf{sig}(\phi) \setminus \{d\}
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{dep}(b) &= \emptyset \\
\mathsf{dep}(X(e)) &= \{X(e)\} \\
\mathsf{dep}(\phi \wedge \psi) &= \mathsf{dep}(\phi) \cup \mathsf{dep}(\psi) \\
\mathsf{dep}(\phi \vee \psi) &= \mathsf{dep}(\phi) \cup \mathsf{dep}(\psi) \\
\mathsf{dep}(\forall d{:}D.\ \phi) &= \mathsf{dep}(\phi) \\
\mathsf{dep}(\exists d{:}D.\ \phi) &= \mathsf{dep}(\phi)
\end{aligned}
$$

Examples:

- $\mathsf{sig}(\mathsf{true} \wedge X(n+1)) = \emptyset$, $\mathsf{sig}(n \leq 5 \wedge X(n+m, m)) = \{n\}$
- $\mathsf{dep}(\mathsf{true} \wedge X(n+1)) = \{X(n+1)\}$, $\mathsf{dep}(n \leq 5 \wedge X(n+m, m)) = \{X(n+m, m)\}$

Assume the following PBES:

$$\mathcal{E} := \ (\sigma_1 X_1(d_1 : D_1) = \phi_1) \cdots (\sigma_n X_n(d_n : D_n) = \phi_n)$$

- arity(X_i): the length of vector d_i
- $d_i[j]$ denotes the j-th element of vector d_i

- Construct a marked influence graph $G(\mathcal{E}) = \langle V, \longrightarrow, M \rangle$:
- $V = \{(X_i, j) \mid 1 \leq j \leq \text{arity}(X_i)\}$ is the set of vertices
- $(X_i, k) \longrightarrow (X_j, l)$ iff for some expression e: $X_j(e) \in \text{dep}(\phi_i)$ and $d_i[k] \in \text{FV}(e[l])$
- $M = \{(X_i, j) \mid 1 \leq i \leq n \text{ and } d_i[j] \in \text{sig}(\phi_i)\}$ is the marking

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PBES Manipulation

Definition (Positively redundant parameters)

Given a Marked Influence Graph $G(\mathcal{E}) = \langle V, \longrightarrow, M \rangle$.
The set of positively redundant parameters of $\mathcal{E}$ is:

$$\mathcal{R} = \{d_i[j] \mid \neg(\exists (X_k, l) \in M : \ (X_i, j) \longrightarrow^* (X_k, l))\}$$

- Computing the set $\mathcal{R}$ requires $\mathcal{O}(| \longrightarrow |)$ steps at most
- $\mathcal{R}$ can be computed using a standard least fixed point computation, a depth-first search or a breadth-first search.

Given closed equation system $\mathcal{E}$ with no unbound data variables

Procedure for eliminating redundant parameters in $\mathcal{E}$

1. Step 1 (compute redundant parameters)
   1.1 Construct Marked Influence Graph of $\mathcal{E}$
   1.2 Compute the set $\mathcal{R}$ of positive redundant parameters of $\mathcal{E}$
2. Step 2 (remove redundant parameters): for every equation $\sigma_i X_i(d_i{:}D_i) = \phi_i$ in $\mathcal{E}$:
   2.1 remove parameter $d_i[j]$ from $X_i(d_i{:}D_i)$ iff $d_i[j] \in \mathcal{R}$
   2.2 remove expression $e[j]$ from an occurrence $X_k(e)$ in ϕ_i iff $d_k[j] \in \mathcal{R}$

Theorem (Redundancy)

The modified equation system $\mathcal{E}$ has the "same" solution as $\mathcal{E}$, i.e., the solution of a variable X does not depend on the parameters that have been identified as positively redundant.

PBES Manipulation

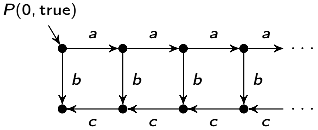
Example

- $\nu X(b{:}Bool, n{:}Nat) = b \wedge X(b, n+1)$ has solution $f = \{(c, v) \in Bool \times Nat \mid c = \text{true}\}$
- $\nu X(b{:}Bool) = b \wedge X(b)$ has solution $g = \{c \in Bool \mid c = \text{true}\}$
- For all $c{\in}Bool, v{\in}Nat$, $(c, v) \in f$ iff $c \in g$.

Example

Consider the following process:

$$\begin{array}{rl} & P(n\!:\!Nat, d\!:\!Bool) \\ = & d \longrightarrow a \cdot P(n+1, d) \\ + & d \longrightarrow b \cdot P(n, \neg d) \\ + & \neg d \wedge n > 0 \longrightarrow c \cdot P(n-1, d) \end{array}$$



Along every a path, always a b action is attainable:

$$\nu V.\ ([a]V \wedge \mu W.\ (\langle a \rangle W \vee \langle b \rangle \text{true}))$$

$$\text{PBES:} \quad \begin{array}{ll} (\nu V(n\!:\!Nat, d\!:\!Bool) = & (d \implies V(n+1, d)) \wedge W(n, d)) \\ (\mu W(n\!:\!Nat, d\!:\!Bool) = & d \vee (d \wedge W(n+1, d))) \end{array}$$

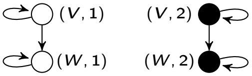
Instantiation of the PBES does not terminate.

PBES Manipulation

Example (Cont'd)

PBES: $(\nu V(n{:}Nat, d{:}Bool) = (d \implies V(n+1,d)) \wedge W(n,d))$
$(\mu W(n{:}Nat, d{:}Bool) = d \vee (d \wedge W(n+1,d)))$

- $\mathsf{dep}((d \implies V(n+1,d)) \wedge W(n,d)) = \{V(n+1,d), W(n,d)\}$
- $\mathsf{dep}(d \vee (d \wedge W(n+1,d))) = \{W(n+1,d)\}$
- Marked Influence Graph ($\mathsf{arity}(V) = \mathsf{arity}(W) = 2$):



Marked states are black; non-marked white

- $\mathcal{R} = \{(V,1), W(1)\}$, i.e., parameter n is positively redundant for V and W.
- Reduced PBES: $(\nu V(d{:}Bool) = (d \implies V(d)) \wedge W(d))$
  $(\mu W(d{:}Bool) = d \vee (d \wedge W(d)))$
- Instantiation of the above PBES terminates

Outline

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Consider the lossy channel system described by the following LPE:

$$
\begin{array}{rcl}
C(b:Bool, m:M) & = & \sum_{k:M} b \longrightarrow r(k)\cdot C(\text{false}, k) \\
& + & \neg b \longrightarrow s(m)\cdot C(\text{true}, m) \\
& + & \neg b \longrightarrow l \cdot C(\text{true}, m)
\end{array}
$$

Action r stands for reading, s stands for sending and l stands for losing a message.

1. $\nu X.([\text{true}]X \wedge (\mu Y.[l]Y \wedge \forall m{:}M.[r(m)]Y \wedge \langle \text{true}\rangle \text{true}))$
2. $\nu X.\mu Y.\nu Z.(\forall m{:}M.[s(m)]X) \wedge ((\forall m{:}M.\ [s(m)]\text{false}) \vee ([l]Y \wedge \forall m{:}M.[r(m)]Y)) \wedge [l]Z \wedge \forall m{:}M.[r(m)]Z$

Questions:

- Translate both formulae to PBESs given process $C(\text{true}, m_0)$
- Use instantiation to compute BESs when $M = Bool$, and solve the BES ($m_0 = \text{true}$)
- Can you remove redundant parameters? If so, remove these redundant parameters and try instantiation to compute a BES when $M = Nat$ ($m_0 = 0$)

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