Technische Universiteit **Eindhoven** University of Technology

Student name:

Student number:

Examination cover sheet

(to be completed by the examiner)

Course name: Algorithms for Model Checking | Course code: 2IMF35

Date: 10-04-2017

Start time: 13:30 | End time : 16:30

Number of pages: 2

Number of questions: 4

Maximum number of points/distribution of points over questions:100

Method of determining final grade: divide total of points by 10

Answering style: open questions

Exam inspection: With the lecturer

Other remarks:

Instructions for students and invigilators

Permitted examination aids (to be supplied by students):

- ☐ Notebook
- ☐ Calculator
- ☐ Graphic calculator
- ☑ Lecture notes/book
- ☐ One A4 sheet of annotations
- ☐ Dictionar(y)(ies). If yes, please specify:
- ☑ Other: Notes, sheets of annotations and other written material

Important:

- examinees are only permitted to visit the toilets under supervision
- it is not permitted to leave the examination room within 15 minutes of the start and within the final 15 minutes of the examination, unless stated otherwise
- examination scripts (fully completed examination paper, stating name, student number, etc.) must always be handed in
- the house rules must be observed during the examination
- the instructions of examiners and invigilators must be followed
- no pencil cases are permitted on desks
- examinees are not permitted to share examination aids or lend them to each other

During written examinations, the following actions will in any case be deemed to constitute fraud or attempted fraud:

- using another person's proof of identity/campus card (student identity card)
- having a mobile telephone or any other type of media-carrying device on your desk or in your clothes
- using, or attempting to use, unauthorized resources and aids, such as the internet, a mobile telephone, etc.
- using a clicker that does not belong to you
- having any paper at hand other than that provided by TU/e, unless stated otherwise
- visiting the toilet (or going outside) without permission or supervision

Examination Algorithms for Model Checking (2IMF35)

10 April, 2017, 13:30 – 16:30

Important notes:

- The exam consists of four questions.
- Weighting: 1: **20**, 2: **25**, 3: **35**, 4: **20**.
- Your grade is determined by dividing the points obtained by 10.
- Carefully read and answer the questions.
- The book, the course notes and other written material may be used during this examination. Laptops and other electronic equipment are not to be used.

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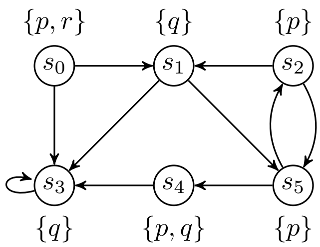
1. Consider the following μ-calculus formulae interpreted over mixed Kripke structures with action alphabet $\{a, b\}$.:

$$\begin{array}{ll} \mathsf{(A)} & \nu X.\ (\ \langle b\rangle X \wedge [a]\mathsf{false} \wedge \mu Y.\ (\langle b\rangle Y \vee \langle a\rangle \mathsf{true})\ ) \\ \mathsf{(B)} & (\mu X.(\ [a]X \vee \langle b\rangle \mathsf{true}\ )) \wedge (\nu Y.\ (\ [b]\nu Z.\ \langle a\rangle(Z \wedge Y)\ )) \\ \mathsf{(C)} & \nu X.\ \neg(\nu Y.\ (\ [a]\neg X \wedge [b]Y\ )) \end{array}$$

   (a) **(For 9pt).** Compute, for all three formulae A, B and C, their *nesting depth*, their *alternation depth* and their *dependent alternation depth*.

   (b) **(For 11pt).** Is there a mixed Kripke structure over action alphabet $\{a, b\}$ for which formula A holds for the initial state? If so, give such a mixed Kripke structure and prove using a transformation to Boolean equation systems and subsequent solving of the Boolean equation system using Gauß Elimination that this is the case. If not, prove that such a mixed Kripke structure cannot exist.

2. Consider the following Kripke Structure K over the set $AP = \{p, q, r\}$:



   (For 25pt). Determine the set of states in K where the CTL formula $\mathsf{E}\,[\,(\mathsf{E}\ \mathsf{G}\ p)\ \mathsf{U}\ (\mathsf{E}\ \mathsf{G}\ q)\,]$ holds using the **symbolic model checking algorithm** (lecture 2) for CTL. Use set notation to represent states instead of BDDs and include the relevant intermediate steps in your answer.

3. Consider the following four constraints for a parity game $G = (V, E, p, (V_\Diamond, V_\Box))$, consisting of 6 vertices:

    i. for every $1 \leq i \leq 6$, there is a vertex v_i such that $p(v_i) = i$; *i.e.*, all priorities are different and taken from the range $1 \ldots 6$,
    ii. G has at least 10 edges,
    iii. player $\Box$ **has** a strategy ρ that guarantees that priority 3 occurs infinitely often on **all** plays (i.e. plays starting in arbitrary vertices) consistent with ρ,
    iv. G has an $\Diamond$-dominion consisting of at least two vertices and an $\Box$-dominion consisting of at least two vertices.

    (a) **(For 5pt).** Give a parity game G that meets all of the above constraints.

    (b) **(For 5pt).** Define a strategy ρ for player $\Box$ and show that your game G of question (a) satisfies property (iii).

    (c) **(For 5pt).** Show your game G satisfies property (iv); i.e. state the two respective dominions and prove that these are dominions.

    (d) **(For 10pt).** Solve your game G using either the recursive algorithm or the small progress measures algorithm. In case you use the recursive algorithm, clearly indicate which subgames are solved in each recursive step of the algorithm. For the small progress measures algorithm clearly indicate the lifting strategy and the intermediate measures.

    (e) **(For 10pt).** Consider, in addition, the following constraint (v) on G:

    v. player $\Diamond$ **has** a strategy σ that guarantees that priority 4 occurs infinitely often on **all** plays (i.e. plays starting in arbitrary vertices) consistent with σ,

    Prove that player $\Box$ always wins the vertex with priority 4 in an arbitrary parity game that meets requirements (i)–(v), or give a counterexample by providing a parity game satisfying constraints (i)–(v) in which the vertex with priority 4 is won by player $\Diamond$.

4. Consider the following parameterised Boolean equation system:

$$
\begin{array}{ll}
(\nu X(n : Nat) & = (\mathsf{odd}(n) \wedge X(n+1)) \vee (\neg \mathsf{odd}(n) \wedge Y(n, n))) \\
(\mu Y(k : Nat, n : Nat) & = Y(k, n+1) \vee X(k+1))
\end{array}
$$

    Here, Nat represents the natural numbers, $\mathsf{odd}(n)$ yields true iff n is odd, and $+$ denotes addition.

    (a) **(For 10pt).** Compute the solution to $X(0)$ in the above parameterised Boolean equation system using Gauß Elimination and Symbolic approximation. Include the important steps in the computations in your answer.

    (b) **(For 10pt).** In case $X(0)$ has solution false, give a refutation graph, and, in case $X(0)$ has solution true, give a proof graph that explains this. In both cases, formally justify that this is a proof/refutation graph. That is, formally state the set of vertices, edges and labelling of the graph (or, if the graph is finite, draw the graph and clearly indicate the labelling of the graph) and show that it meets the properties of a refutation graph or a proof graph.

Good luck! $\Box$