Algorithms for Model Checking (2IMF35)

Alternative assignment I

12 February 2020

Important notes:

- The assignment consists of five questions.
- Weighting: 1: **20**, 2: **20**, 3: **25**, 4: **20**, 5: **15**.
- Carefully read and answer the questions. Use formal reasoning whenever appropriate.
- Hand in your report by Friday 13 March (23.59 o'clock). **Only reports in PDF are accepted.**

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1. Consider the formula ϕ given by $\mathsf{A}\ \mathsf{F}\ \mathsf{A}\ [p\ \mathsf{U}\ q]$, in the context of the following mixed Kripke Structure, where $\{p, q, r\}$ is the set of atomic propositions and a is an action.

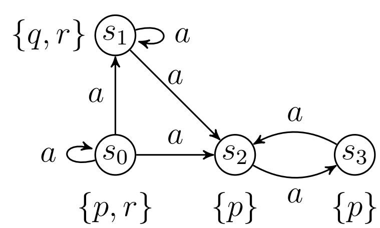

Figure 1: A mixed Kripke Structure with action alphabet $\{a\}$ and set of atomic propositions $\{p, q, r\}$.

   (a) Convert formula ϕ to an equivalent formula in the μ-calculus. Compute the nesting depth, the alternation depth and the dependent alternation depth of the formula you obtained. *(8 points)*

   (b) Solve the μ-calculus formula $\mu X.(\langle a\rangle X \vee \nu Y.\ (q \vee (p \wedge [a]Y)))$ using **both** the naive model checking algorithm for the μ-calculus, and the Emerson-Lei algorithm. Show the intermediate steps in your computations and discuss the differences (if any) in the behaviour of the two algorithms on this problem. *(12 points)*

2. Prove the correspondence $\mathsf{A}\ \mathsf{F}\ f = \mu Z.(f \cup \mathsf{A}\ \mathsf{X}\ Z)$. That is:

   (a) Prove that the transformer $\tau(Z) = f \cup \mathsf{A}\ \mathsf{X}\ Z$ is monotonic. *(5 points)*

   (b) Prove that $\mathsf{A}\ \mathsf{F}\ f$ is *a* fixpoint of τ; *(5 points)*

   (c) Prove that for any Z satisfying $\tau(Z) = Z$, we have $\mathsf{A}\ \mathsf{F}\ f \subseteq Z$. *(10 points)*

3. For each of the CTL^* formulae below, indicate whether (or not) it is (syntactically) a formula in LTL and/or CTL or neither. Determine for each formula in which states of the mixed Kripke Structure of Fig. 1 (ignoring the action labels) it holds. For CTL formulae, use the symbolic model checking algorithm of Lecture 2 to substantiate your answer; provide all steps in your computation in that case.

   (a) q, *(5 points)*

   (b) $\mathsf{E\ F\ G}\ p$, *(5 points)*

   (c) $\mathsf{A\ F\ G}\ p$, *(5 points)*

   (d) $\mathsf{A\ G\ E\ F}\ p$, *(5 points)*

   (e) $\mathsf{A}\ ((\mathsf{G}\ p) \vee (\mathsf{F}\ q))$ *(5 points)*

4. For each pair of CTL^* formulae below, *if possible*, give a Kripke Structure in which both are valid, a Kripke Structure in which both are not valid, and a Kripke Structure in which only one of them is valid.

   (a) $\mathsf{E\ F\ E\ G}\ p$ and $\mathsf{E\ G\ E\ F}\ p$ *(10 points)*

   (b) $\mathsf{A\ X\ E\ X}\ p$ and $\mathsf{A\ X\ X}\ p$ *(10 points)*

5. For a mixed Kripke structure ranging over the set of actions $\{a, b\}$, express the properties below using the modal μ-calculus, give a mixed Kripke structure in which the formula holds in the initial state and substantiate your answer using either the Emerson Lei algorithm or the naive algorithm.

   (a) There is an infinite path along which action a occurs infinitely often. *(5 points)*

   (b) Along all paths, every stretch of b actions is finite. *(5 points)*

   (c) Along all a-paths, after any even number of a's, a b is enabled. *(5 points)*

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