Examination Algorithms for Model Checking (2IW55)

20 January, 2011, 14:00 – 17:00

Important notes:

- The exam consists of four questions.
- Weighting: 1: **30**, 2: **20**, 3: **20**, 4: **30**.
- Carefully read and answer the questions. The book, the course notes and other written material may be used during this examination.

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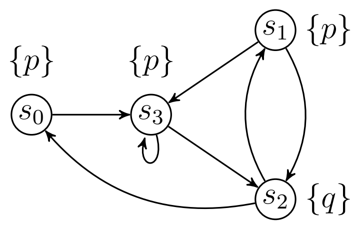
1. Consider the following Kripke Structure K:



Consider the following formulae, where p and q are atomic propositions:

($\mathbf{A}$) $\mathbf{A}\,[q\ \mathbf{R}\ p]$
($\mathbf{B}$) $\mathbf{AG}(\mathbf{AF}q)$

(a) Determine the set of states in K where ($\mathbf{A}$) holds using the **symbolic model checking algorithm** for CTL model checking. Use set notation to represent states instead of BDDs. Show all intermediate steps.

(b) Consider the fairness constraint $\mathcal{F} = \{\{s_2\}, \{s_3\}\}$. Determine the set of states in K where ($\mathbf{B}$) holds fairly under $\mathcal{F}$ using the **labelling algorithm** for fair CTL. Show the intermediate steps.

2. Consider the PBES $\mathcal{E}$ given below:

$$\Big(\nu Z(b{:}Bool, c{:}Bool, n{:}Nat) = (\neg b \wedge c \wedge Z(c,b,n)) \vee (b \wedge \neg c \wedge Z(\neg b, \neg c, n)) \vee \exists j{:}Nat.\exists m{:}Nat.(\neg(b \vee c) \wedge m = j \wedge Z(c, \neg b, m))\Big)$$

(a) Identify the set of "positive redundant" parameters in the PBES $\mathcal{E}$ and, if possible, simplify $\mathcal{E}$ as a result of your analysis. Show all necessary computations.

(b) Solve the PBES $\mathcal{E}$ using symbolic approximation. Show all intermediate steps.

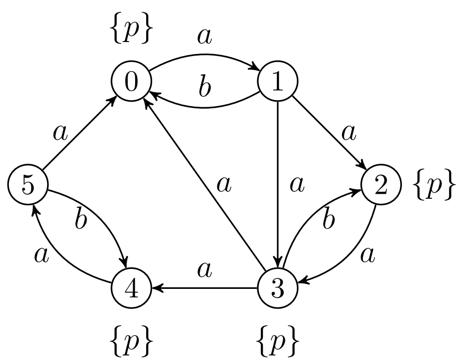
3. Let $\mathcal{G} = (V, E, p, (V_\Diamond, V_\Box))$ denote a parity game, in which $V_\Diamond$ is the set of vertices owned by player Even, and $V_\Box$ is the set of vertices owned by player Odd. Read the three statements below carefully, and, for each statement, give either a short proof or a counterexample.

   (a) For all parity games $\mathcal{G}$ with $V_\Box = \emptyset$, player Even wins all vertices in $\mathcal{G}$.

   (b) For all parity games $\mathcal{G}$ and all sets $U \subseteq V$, the following property holds: for all vertices $v, w \in V$ such that $v \to w$, if $v \in \mathit{Attr}_\Diamond(\mathcal{G}, U)$ then also $w \in \mathit{Attr}_\Diamond(\mathcal{G}, U)$.

   (c) Let $\mathcal{G}$ be an arbitrary parity game. Let $U \subseteq V$ such that for all $v \in U$, v has only a selfloop (*i.e.*, for all $w \in V$, if $v \to w$ then $w = v$), and there is an odd $k \in \mathbb{N}$, such that $k = p(v)$ for all $v \in U$ (*i.e.*, each vertex in U has the same odd priority). Then the Small Progress Measures algorithm requires lifting the initial progress measure $\lambda v.(0, \cdots, 0)$ for $\mathcal{G}$ at least $|U|^2$ times to become stable.

4. Consider the mixed Kripke Structure K, and the equation system $\mathcal{E}$.



$$
\begin{aligned}
&(\mu X_0 &&= Y_0)\\
&(\mu X_1 &&= Y_1)\\
&(\mu X_2 &&= Y_2)\\
&(\mu X_3 &&= Y_3)\\
&(\mu X_4 &&= Y_4)\\
&(\mu X_5 &&= Y_5)\\
&(\nu Y_0 &&= Y_2 \wedge Y_3)\\
&(\nu Y_1 &&= X_0)\\
&(\nu Y_2 &&= Y_0 \wedge Y_4)\\
&(\nu Y_3 &&= X_2 \vee (Y_1 \wedge Y_5))\\
&(\nu Y_4 &&= Y_0)\\
&(\nu Y_5 &&= X_4)
\end{aligned}
$$

   (a) Let the modal μ-calculus formula ϕ be defined as:

   $$\mu X.\nu Y.\Big(\langle b\rangle X \vee (p \wedge [a][a]Y)\Big)$$

   Prove or disprove that the BES $\mathcal{E}$ given above can be obtained from the translation of the model checking problem for $K \models \phi$ (up-to Boolean equivalence of the right-hand side expressions). Motivate your answer by means of a computation.

   (b) Give the Parity Game obtained by translating $\mathcal{E}$ and solve the game using either the recursive algorithm or the Small Progress Measures algorithm. Show all the intermediate steps.

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