Examination Algorithms for Model Checking (2IW55)

6 April, 2011, 14:00 – 17:00

Important notes:

- The exam consists of four questions.
- Weighting: 1: **30**, 2: **25**, 3: **25**, 4: **20**.
- Carefully read and answer the questions. The book, the course notes and other written material may be used during this examination.

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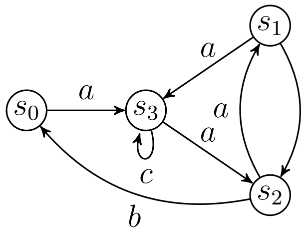
1. Consider the following Kripke Structure K:



Consider the formula ϕ: $\nu X.\mu Y.((\langle a\rangle X \wedge \nu Z.\langle c\rangle Z) \vee (\langle a\rangle Y \wedge \mu W.[c]W))$.

(a) Compute the *nesting depth*, the *alternation depth* and the *dependent-alternation depth* of ϕ.

(b) Compute the set of states of K that satisfy ϕ, using the **Emerson-Lei** algorithm. Show the intermediate steps in all your computations.

2. Consider the PBES $\mathcal{E}$ given below:

$$\begin{array}{ll}(\nu X(n{:}Nat) & = Y(n))\\ (\mu Y(n{:}Nat) & = (even(n) \wedge Y(n)) \vee (odd(n) \wedge X(n+1)))\end{array}$$

(a) Compute the solution to $X(n)$, for arbitrary $n \in Nat$ for the PBES $\mathcal{E}$ given above. Show all steps in your computations.

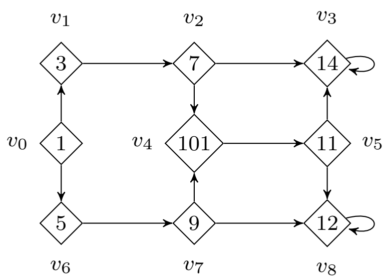
(b) Consider the process described by the LPE P given below.

$$\begin{array}{rrl}P(n{:}Nat) = & even(n) & \rightarrow stable \cdot P(n)\\ + & odd(n) & \rightarrow inc \cdot P(n+1)\\ + & \sum m{:}Nat.true & \rightarrow \tau \cdot P(m)\end{array}$$

Let ϕ be the first-order modal μ-calculus formula $\nu X.\mu Y.([stable]Y \wedge [inc]X)$

Verify whether the PBES $\mathcal{E}$ can be the result (up-to logical equivalence of right-hand side formulae) of the transformation E (ϕ) applied to P, by performing the transformation and indicating all (if any) differences between E (ϕ) and $\mathcal{E}$.

3. Solve the Parity Game, depicted below, using either the **Small Progress Measures** algorithm or the **Recursive** algorithm. State which of the two algorithms you choose to use, and **explicitly** mention which transformations you use on the Parity Game (if any) prior to solving the game, and **why** you choose to use them.



4. Let $M = (S, R, S_0, L)$ be an arbitrary Kripke Structure over a set AP of atomic propositions; S is the set of states, R is the transition relation, $\emptyset \neq S_0 \subseteq S$ is the set of initial states and $L{:}S \to 2^{AP}$ is the labelling. Read the statements below carefully, and, for each statement, give either a short proof or a counterexample.

   (a) Let M', M'' be Kripke Structures over AP such that:

   - $M \sqsubseteq M'$, i.e., M' simulates M, and
   - $M' \equiv M''$, i.e., M' and M'' are bisimilar.

   Let f be an ACTL^* formula such that $M'' \models f$. Then also $M \models f$.

   (b) Let state $s \in S$ be such that $L(s) = \emptyset$. Assume that for all $s', s'' \in S \setminus \{s\}$, we have $L(s') = L(s'')$ and $L(s') \neq \emptyset$. That is, all states in M, save state s, have the same non-empty labelling. Then, the relation B_0^* on S, defined as $B_0^* = \{(s', s'') \in S \times S \mid L(s') = L(s'')\}$ is always a bisimulation relation.

   (c) Let $s, s' \in S$ be two distinct, bisimilar states. That is, there is a bisimulation relation $B \subseteq S \times S$ such that $(s, s') \in B$. Then there is no fairness constraint $\mathcal{F}$ and no CTL formula f, such that $s \models_F f$ and $s' \not\models_F f$ using constraint $\mathcal{F}$.

□