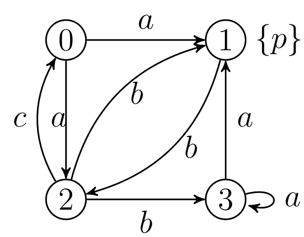
Modal μ-Calculus Exercises, January 12, 2011

1. Consider the following mixed Kripke Structure:



   Let ϕ be the following formula:

   $$\nu X.\ \mu Y.\mu Z.\Big(p \vee (\langle b\rangle Y \wedge [a]Z)\Big)$$

   (a) Transform the model checking problem to a BES.

   (b) Solve the BES, using Gauß Elimination, to determine the set of states of the Kripke Structure that satisfy ϕ.

   (c) Transform the BES you obtained into a Parity Game.

   (d) Solve the Parity Game using the recursive algorithm.

   (e) Give the definition of the set $\mathbb{M}_{\mathcal{G}}^{\top}$ for your Parity Game

   (f) Solve the Parity Game using the small progress measures algorithm.

2. Consider the LPE description of a lossy channel system, where actions r, s and l represent *receiving*, *sending* and *losing*, respectively, and the action τ represents some internal behaviour of the system.

   $$\begin{array}{rcl} P(b{:}Bool, c{:}Bool, n{:}Nat) & = & \sum_{m:Nat} \neg(b \vee c) \longrightarrow r(m) \cdot P(\mathsf{false}, \mathsf{true}, m) \\ & + & \neg b \wedge c \longrightarrow s(n) \cdot P(\mathsf{false}, \mathsf{false}, n) \\ & + & \neg b \wedge c \longrightarrow \tau \cdot P(\mathsf{true}, \mathsf{false}, n) \\ & + & b \wedge \neg c \longrightarrow l \cdot P(\mathsf{false}, \mathsf{true}, n) \end{array}$$

   Let ϕ be the first-order modal μ-calculus formula given below:

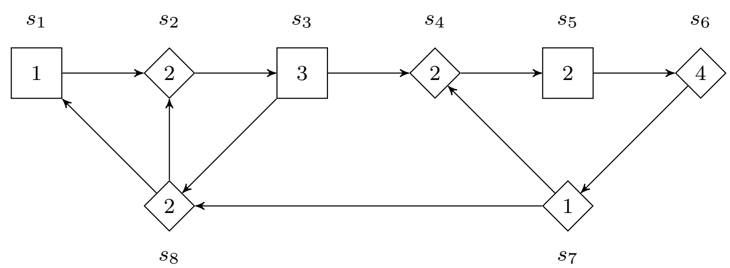
   $$\nu X.\ \mu Y.\ (([\neg(\tau \vee l)]X \wedge (\nu Z.\ \exists j{:}Nat.\langle r(j) \vee \tau \vee l\rangle Z)) \vee [\neg(\tau \vee l)]Y)$$

   (a) Compute the PBES that is the result of the transformation $\mathbf{E}(\phi)$ applied to P.

   (b) Solve the resulting PBES using symbolic approximation. Show all steps in all your computations.

(c) Solve the resulting PBES using instantiation. Hint: first eliminate redundant parameters of the given PBES, and use logic to rewrite the right-hand side of the PBES. Show all steps in all your computations.

3. Consider the following Parity Game $\mathcal{G}$, where the diamond vertices are owned by player *Even* and the square vertices are owned by player *Odd*. The priorities associated to each vertex are written inside the vertices.



(a) Give the set $\mathbb{M}^{\top}_{\mathcal{G}}$ (as used in the *Small Progress Measures* algorithm) for the Parity Game $\mathcal{G}$.

(b) Let μ_i be a game parity progress measure for $\mathcal{G}$, where μ_0 is the game parity progress measure $\lambda v \in V.(0, \ldots, 0)$, and for $j \geq 0$, we define:

$$
\begin{aligned}
\mu_{4j+1} &= \mathit{Lift}(\mu_{4j}, s_1) \\
\mu_{4j+2} &= \mathit{Lift}(\mu_{4j+1}, s_2) \\
\mu_{4j+3} &= \mathit{Lift}(\mu_{4j+2}, s_3) \\
\mu_{4j+4} &= \mathit{Lift}(\mu_{4j+3}, s_8)
\end{aligned}
$$

Calculate all game parity progress measures μ_i, $i \geq 0$. Show all intermediate steps in your calculations.

(c) Compute the set of vertices won by player *Even*. Use whichever transformation/algorithm you please (taken from the lecture notes). Mention which algorithm and approach you use and show the intermediate steps in all your computations.