CTL* Exercises, October 18, 2010

1. Consider the following formulae, where p, q are atomic propositions:

   (**A**) $\mathbf{A}(\mathbf{FG}(p \rightarrow q))$
   (**B**) $q \wedge \mathbf{A}(\mathbf{F}(q)) \wedge \neg(\mathbf{E}[(\neg q)\mathbf{R}(\neg p)])$

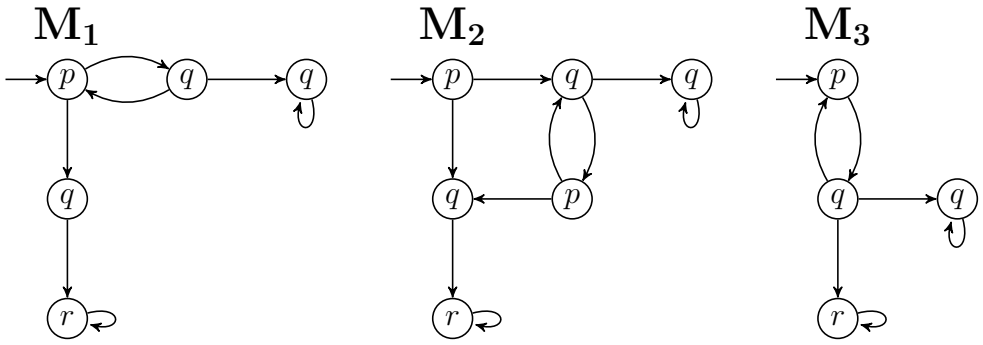
   Answer the following questions for **both** formulae **(A)** and **(B)** and provide a brief motivation for every answer that you give.

   (a) Is the formula in LTL? Is it in CTL? Is it in ACTL*?

   (b) Draw a Kripke Structure with a single initial state in which it holds.

   (c) Draw a Kripke Structure with a single initial state in which it does not hold, but in which it does hold fairly with an appropriate fairness constraint. Also provide this fairness constraint.

2. Consider the following three Kripke structures with initial states indicated by a "$\longrightarrow$" and where $\{p, q, r\}$ are the atomic propositions. Determine whether the following properties hold. If so, give the relation that supports your answer. If not, give a formula in CTL* that witnesses this fact.



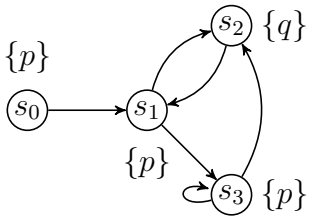
   (a) M_1 is strong bisimilar to M_2 (i.e. $M_1 \equiv M_2$)

   (b) M_2 is strong bisimilar to M_3 (i.e. $M_2 \equiv M_3$)

   (c) M_3 simulates M_2 (i.e. $M_2 \preccurlyeq M_3$)

   (d) M_2 simulates M_3 (i.e. $M_3 \preccurlyeq M_2$)

3. Consider the following Kripke Structure:



Consider the following formulae, where p and q are atomic propositions:

(**C**) $\mathbf{A}(\mathbf{F}(q))$
(**D**) $\mathbf{A}[q\ \mathbf{R}\ p]$
(**E**) $\mathbf{EF}(\mathbf{A}\ [q\ \mathbf{R}\ p])$

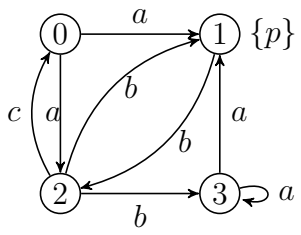
(a) Determine the set of states where (**C**) holds using the standard CTL model checking algorithm, based on graph algorithms. Show the intermediate steps.

(b) Consider the fairness constraint $\mathcal{F} = \{\{s_2\}, \{s_3\}\}$. Determine the set of states where (**C**) holds fairly under $\mathcal{F}$ using the labelling algorithm for fair CTL. Use explicit set notation to represents states instead of BDDs. Show the intermediate steps.

(c) Determine the set of states where (**D**) holds fairly (with $\mathcal{F}$ as defined above), using the symbolic model checking algorithm for CTL. Use explicit set notation to represent states instead of BDDs. Show the intermediate steps.

(d) Determine the set of states where (**E**) holds using the symbolic model checking algorithm for CTL model checking. Show the intermediate steps.

4. Consider the following mixed Kripke Structure:



Let ϕ be the following formula:

$$\nu X.\ \mu Y.\mu Z.\Big(p \vee (\langle b\rangle Y \wedge [a]Z)\Big)$$

Use both the naive μ-calculus model checking algorithm and the Emerson-Lei algorithm to determine the set of states satisfying ϕ. Show the intermediate approximations.

□