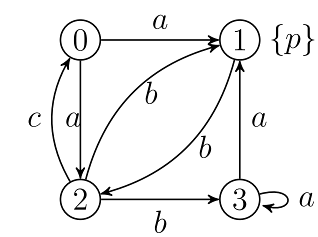
Modal μ-Calculus Exercises, December 15, 2009

1. Consider the following mixed Kripke Structure:



Let ϕ be the following formula:

$$\nu X.\ \mu Y.\mu Z.\Big(p \vee (\langle b \rangle Y \wedge [a]Z)\Big)$$

(a) Use the Emerson-Lei algorithm to determine the set of states satisfying ϕ. Show the intermediate approximations.

(b) Use a transformation to BES, and subsequently solve the BES, to determine the set of states satisfying ϕ.

(c) Transform the BES you obtained as an answer to the previous question into a Parity Game, and use the recursive algorithm for solving the resulting Parity Game.

2. Consider the LPE description of a lossy channel system, where actions r, s and l represent *receiving*, *sending* and *losing*, respectively, and the action τ represents some internal behaviour of the system.

$$\begin{array}{rcl} P(b{:}Bool, c{:}Bool, n{:}Nat) & = & \sum_{m:Nat} \neg(b \vee c) \longrightarrow r(m) \cdot P(\mathsf{false}, \mathsf{true}, m) \\ & + & \neg b \wedge c \longrightarrow s(n) \cdot P(\mathsf{false}, \mathsf{false}, n) \\ & + & \neg b \wedge c \longrightarrow \tau \cdot P(\mathsf{true}, \mathsf{false}, n) \\ & + & b \wedge \neg c \longrightarrow l \cdot P(\mathsf{false}, \mathsf{true}, n) \end{array}$$

Let ϕ be the first-order modal μ-calculus formula given below:

$$\nu X.\ \mu Y.\ ((([\neg(\tau \vee l)]X \wedge (\nu Z.\ \langle \forall j{:}Nat. \neg s(j) \rangle Z)) \vee [\neg(\tau \vee l)]Y)$$

(a) Compute the PBES that is the result of the transformation $\mathbf{E}(\phi)$ applied to P.

(b) Solve the resulting PBES (if possible). Eliminate redundant parameters of the given PBES, and use logic to rewrite the right-hand side of the PBES, if necessary. Show all steps in all your computations.