2IV60 Computer Graphics

Examination, January 25 2016, 9:00 – 12:00

This examination consist of **four** questions with in total 16 subquestions. Each subquestion weighs equally. In all cases: **EXPLAIN YOUR ANSWER. Use sketches where needed to clarify your answer. Read first all questions completely.** If a function is asked, then a description in steps or pseudo-code is expected, which is clear enough to be easily transferred to real code. Aim at compactness and clarity. The use of the book, copies of slides, notes and other material is not allowed.

1 We consider some basic concepts of computer graphics.

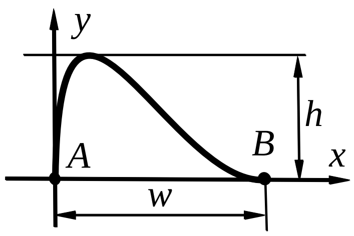
a) Explain what the view-up vector is and what the effect is when it is changed.

b) A basic model for specular reflection is $I_s = k_s I_L \, (V \cdot R)^n$. What do the parameters k_s , I_L , V, R, and n mean? Use a sketch to illustrate this.

c) Describe the z-buffer (or depth-buffer) algorithm for hidden surface removal.

d) Describe a method to test if a point P is inside or outside a polygon.

2 We want to draw a curve. The curve starts at point A = (0, 0) and ends at point B = (w, 0); at A the curve is tangent with the y-axis; at B the curve is tangent with the x-axis (see figure). The curve must be smooth everywhere. Answer the following questions, with formal definitions of the points *and* sketches:



a) Suppose we use one cubic Bézier segment for the curve, where a segment is given by

$$P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3 .$$

Draw a control polygon and give the possible positions of the control points (ignoring h), using new parameters where necessary.

b) What is the value of h, the maximum value of $P_y(t)$, $0 \le t \le 1$, given a set of (parametrized) control points?

c) Suppose we use two quadratic Bézier segments P and Q for the curve, given by

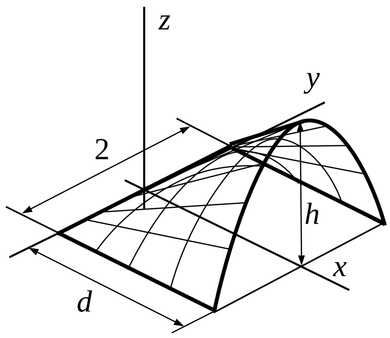
$P(t) = (1-t)^2 P_0 + 2t(1-t)P_1 + t^2 P_2$, and $Q(t) = (1-t)^2 Q_0 + 2t(1-t)Q_1 + t^2 Q_2$.

Draw a control polygon and give the possible positions of the control points, using new parameters where necessary.

d) Again, we use two quadratic Bézier segments P and Q, but now for arbitrary points A and B, and arbitrary tangent directions U and V at start and end. Give the possible positions of the control points, using new parameters where necessary.

See the reverse side for question 3 and 4.

3 We consider a surface $z = axy^2+bx$, with $a < 0$, $0 \le x \le d$, and $-1 \le y \le 1$, see the figure. Answer the following questions.



a) What values should a and b have such that for all corners z=0 and that the maximum value of z is equal to h?

b) Give a parametric description of the surface: define the surface as $P(u,v)$, $0 \le u,v \le 1$.

c) Given a point on the surface (by coordinates or by parameter value, choose yourself), give an expression for a normal vector N on the surface for that point.

d) We want to apply texture mapping. When viewed from the top (in the direction of the negative z- axis) we want to see a pattern of square tiles with width r. The base texture is given in a unit square $0 \le s, t \le 1$; furthermore, clamping (use of only the fractional value of texture parameters) is enabled for texture wrapping. Give the texture coordinates $s(u,v)$ and $t(u, v)$ to achieve this.

4 We consider the animation of a rotating object in 2D. Suppose that a round object (say, a wheel) is given in local coordinates, defined inside a circle with radius 1, centered around the origin. We want to make an animation of this wheel. The wheel should have a radius r, and roll over the x-axis: the line y=0. At time t = 0 the wheel touches the origin, the center has a constant velocity v. The wheel rotates such that there is no slip between the wheel and the x-axis.

a) Derive a formula for the rotation angle $\alpha(t)$ of the wheel as a function of time t.

b) Let A be the point on the wheel that touches the origin at t = 0.
What is the position $A(t) = (A_x(t), A_y(t))$ of this point as a function of time t?

Assume that a function *DrawWheel*(M) is available to draw the wheel, subject to a homogenous transformation M. Points X' in world coordinates are derived from local coordinates X by $X'=MX$, where M is a homogenous transformation matrix.

Furthermore, suppose that a standard set of functions to define transformation matrices is available:

- $T(d_x, d_y)$ gives a translation matrix over a vector (d_x, d_y);
- $S(s_x, s_y)$ gives a scaling matrix with scale factors s_x and s_y for the axes; and
- $R(a)$ gives a counterclockwise rotation over a radians around the origin.

c) Define a matrix $M_1(t)$ such that the wheel rolls along the x-axis according to the specification.

d) Define a matrix $M_2(t)$ such that the wheel rolls over a line $y = ax$ instead of over the x-axis. Matrix $M_1(t)$ can be reused.