## 2IV10/2IV60 Computer Graphics

### *Intermediate Examination, December 15 2014, 10:45 – 12:30*

This examination consist of **three** questions with in total 9 subquestion. Each subquestion weighs equally. In all cases: **EXPLAIN YOUR ANSWER. Use sketches where needed to clarify your answer. Read first all questions completely.** If an algorithm is asked, then a description in steps or pseudo-code is expected, which is clear enough to be easily transferred to real code. Aim at compactness and clarity. Use additional functions and procedures if desired. Give from each function and procedure a short description of input and output. The use of the book, copies of slides, notes and other material is not allowed.

**1** We use 2D transformations to transform a square, defined by $0 \le x \le 1$, and $0 \le y \le 1$ in local coordinates. New positions $X$ are derived from local positions $X'$ by $X=MX'$, where $M$ is a homogeneous transformation matrix. Suppose that we have a set of functions to define basic transformation matrices, *i.e.*,

- $T(x,y)$ gives a translation matrix over a vector $(x,y)$;
- $S(s_x, s_y)$ gives a scaling matrix with scale factors $s_x$ and $s_y$ for the axes; and
- $R(a)$ gives a counterclockwise rotation over $a$ radians around the origin.

a) Give a matrix $M$, expressed in the basic transformation functions, to obtain an axis-aligned rectangle with width $w$ and height $h$, centered around the point $(a, b)$.

We first center the square around the origin with $T(-1/2, -1/2)$; next we scale it using $S(w,h)$; and finally we move it to its new center with $T(a, b)$. Hence:
$M = T(a,b)S(w,h)T(-1/2, -1/2)$.
An alternative: scale the square first with $S(w,h)$; center the square ($T(-w/2, -h/2)$) and bring it to its new center with $T(a, b)$, giving $M = T(a,b)\ T(-w/2, -h/2)\ S(w,h)$. And, the last two translations can be combined: $M = T(a(-w/2, b-h/2)\ S(w,h)$.

b) Give a matrix $M^{-1}$ to transform a position $X$ into a local position $X'$ for the case of the previous question. Again, express $M^{-1}$ in terms of the basic transformation functions.

We need a matrix $M^{-1}$ such that $M^{-1}M = I$. To this end, we use the same transformations as for $M$, but in opposite order and using the inverses. This gives $M^{-1}=T(1/2, 1/2)S(1/w, 1/h)T(-a, -b)$.

c) Give a matrix $M$, expressed in the basic transformation functions, which transforms the initial square in a diamond shape (◊), centered around the origin: a parallelogram with vertices (1, 0), (0, 2), (–1, 0), and (0, –2).

The first step is to center the square around the origin with $T(-1/2, -1/2)$. Next, we rotate the square over 45 degrees with $R(\pi/4)$. This gives a diamond with width and height $\sqrt{2}$. To get the right size, we first give it a unit width and height with $S(1/\sqrt{2}, 1/\sqrt{2})$, and next scale in the $x$ direction with 2 and in the $y$ direction with 4. All in all this gives (combining the two scaling transformations):
$M = S(\sqrt{2}, 2\sqrt{2})R(\pi/4)\ T(-1/2, -1/2)$.

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**2** We consider projection on a plane. Suppose, a camera is located in the origin and pointing in the direction of the positive $z$-axis. The projection plane is a plane $z = d$. Answer the following questions.

*a)* Given a point $P=(P_x, P_y, P_z)$, calculate its projection $P'$ on the projection plane.

$P'$ is located on a line through the origin and $P$, and hence $P'=tP$, where $t$ has to be found. We know that $P'_z = tP_z = d$, and hence $t = d / P_z$. Therefore:

$P'=(d/P_z)P$, or $P'= ((d/P_z)P_x, (d/P_z)_y, d)$.

b) Given a line segment $PQ$ in a plane $z = a$ with length $s$. Does the length $s'$ of the projected line segment $P'Q'$ depend on the position and orientation of the line segment? Motivate your answer.

The projected positions $P'$ and $Q'$ are $(d/a)P$ and $(d/a)Q$, using the previous result and $P_z = P_z = a$. For the length $s'$ we find:

$s' = | P' - Q'| = | (d/a)P - (d/a)Q | = (d/a) | P - Q | = (d/a)\, s$ .

Hence, the length of the line segment is simply scaled with $d/a$ , and the position and the orientation have no influence.

c) Suppose that we want to display a wire frame of a cube. The cube is centered around the point (0, 0, $d$ ), and all its edges are aligned with the coordinate axes. After projection, we want the lengths of the projected edges of the front face to be twice as long as those of the back face. What should the length $r$ of the edges be?

The front face is located in the plane $z = d - r/2$ , the back face in the plane $z = d + r/2$. For the projected lengths we find $r_f = d / (d - r/2)$ and $r_b = d / (d + r/2)$. It is required that $r_f = 2\, r_b$. Substitution gives:

$d / (d - r/2) = 2\, d / (d + r/2)$ or

$d + r/2 = 2d - r$ or

$3r/2 = d$ or

$r = (2/3)d$.

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**3** We consider a surface patch defined by $z^2 = x^2 - y^2 + 4$ with $0 \le x \le 2$, $0 \le y \le 2$, and $z \ge 0$.

a) Give a parametric definition $P(u, v)$ with $0 \le u, v \le 1$ of this surface; and an implicit definition $f(x, y, z) = 0$ of the unbounded surface where this patch is part of.

We associate $u$ with $x$, taking scaling into account we get $x = 2u$. Similarly, $y = 2v$. Substitution in the right side of the given equation leads to $z^2=(2u)^2 - (2v)^2 + 4$, or $z = 2\sqrt{(u^2 - v^2 + 1)}$. Hence: $P(u,v)=(2u, 2v, 2\sqrt{(u^2 - v^2 + 1)})$.

For the implicit version, we move the right side of the equation to the left to get $f(x, y, z) = z^2 - x^2 + y^2 - 4 = 0$.

b) What is the shape of the boundary of the patch for $x = 0$ ?

If we substitute $x = 0$, we get $z^2 = -y^2 + 4$, or $z^2 + y^2 = 2^2$. This is the equation of a circle with radius 2, located in the YOZ-plane, centered around the $x$-axis. Because the patch is bounded, the boundary consists of only one quarter arc of this circle.

c) Give a unit length normal $N$ for an arbitrary point on this surface patch. Choose yourself if you assume the point to be given via coordinates ($x$, $y$, $z$) or parameter values ($u$, $v$).

Using the implicit definition is much easier here. We find:

$\nabla f = (\partial f/\partial x, \partial f/\partial y, \partial f/\partial z) = (-2x, 2y, 2z)$.

A unit length normal is given by :

$N(x,y,z) = \nabla f / |\nabla f| = (-x, y, z)/\sqrt{(x^2+y^2+z^2)}$.