

Constructing Franklin Magic Squares

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impossible number puzzles

Benjamin Franklin *discovered* squares with numbers 1, . . . , 64 arranged

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

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each ROW having sum $8 \cdot 32.5 = 260$.

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each ROW having sum $8 \cdot 32.5 = 260$,

each COLUMN having sum $8 \cdot 32.5 = 260$.

Benjamin Franklins square is NOT *magic*

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
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DIAGONALS have sums 262 and 258

Benjamin Franklins square is NOT *magic* but

52	61	4	13	20	29	36	45
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BENT DIAGONALS have sums 260.

Benjamin Franklins square is NOT *magic* but

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9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

BENT DIAGONALS have sums 260 (in both orientations).

Benjamin Franklin squares are more than magic:

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

SHIFTED BENT DIAGONALS have magic sum 260 (all types \vee , $>$, \wedge , $<$)

Franklin Magic squares further feature

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

HALF ROWS having fixed sum ($4 \cdot 32.5 = 130$)

Franklin Magic squares further feature

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

HALF ROWS having fixed sum ($4 \cdot 32.5 = 130$),

HALF COLUMNS having fixed sum ($4 \cdot 32.5 = 130$)

Franklin Magic squares finally have

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

2x2 SUBSQUARES all have sum $4 \cdot 32.5 = 130$
 even if wrapped-around

a general n by n Franklin Magic square M has

1. entries $1, 2, \dots, n^2$;
2. each row and each column has a fixed entry sum $n(1 + n^2)/2$;
3. each two by two sub-square $\begin{bmatrix} M_{i,j} & M_{i,j+1} \\ M_{i+1,j} & M_{i+1,j+1} \end{bmatrix}$ has sum $2(1 + n^2)$;
4. each half row starting in column 1 or $n/2 + 1$ has sum of entries equal to $n(1 + n^2)/4$, and similar for half columns starting in row 1 or $n/2 + 1$;
5. each shifted bent diagonal has entry sum equal to $n(1 + n^2)/2$.

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1. entries $1, 2, \dots, n^2$;
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4. each half row starting in column 1 or $n/2 + 1$ has sum of entries equal to $n(1 + n^2)/4$, and similar for half columns starting in row 1 or $n/2 + 1$;
5. each shifted bent diagonal has entry sum equal to $n(1 + n^2)/2$.

CHALLENGE:

find more Franklin Magic squares!

REQUIREMENT:

n is multiple of 4

a 4 by 4 Franklin Magic square would have entries $1, \dots, 16$

X	?	?	?
?	?	?	?
?	?	?	?
?	?	?	?

a 4 by 4 Franklin Magic square would have entries $1, \dots, 16$

X	$17 - X$?	?
$17 - X$?	?	?
?	?	?	?
?	?	?	?

half rows, half columns have sum 17

a 4 by 4 Franklin Magic square would have entries $1, \dots, 16$

X	$17 - X$?	?
$17 - X$	X	?	?
?	?	?	?
?	?	?	?

half rows, half columns have sum 17
2x2 subsquares have sum 34

a 4 by 4 Franklin Magic square would have entries $1, \dots, 16$

X	$17 - X$?	?
$17 - X$	X	?	?
?	?	?	?
?	?	?	?

half rows, half columns have sum 17

2x2 subsquares have sum 34

but entries should be distinct ! No such thing !

a 16 by 16 Franklin Magic square must have entries $1, \dots, 256$.

1 192 209 112 225 96 49 144	241 80 33 160 17 176 193 128
252 69 44 149 28 165 204 117	12 181 220 101 236 85 60 133
14 179 222 99 238 83 62 131	254 67 46 147 30 163 206 115
247 74 39 154 23 170 199 122	7 186 215 106 231 90 55 138
15 178 223 98 239 82 63 130	255 66 47 146 31 162 207 114
246 75 38 155 22 171 198 123	6 187 214 107 230 91 54 139
4 189 212 109 228 93 52 141	244 77 36 157 20 173 196 125
249 72 41 152 25 168 201 120	9 184 217 104 233 88 57 136
16 177 224 97 240 81 64 129	256 65 48 145 32 161 208 113
245 76 37 156 21 172 197 124	5 188 213 108 229 92 53 140
3 190 211 110 227 94 51 142	243 78 35 158 19 174 195 126
250 71 42 151 26 167 202 119	10 183 218 103 234 87 58 135
2 191 210 111 226 95 50 143	242 79 34 159 18 175 194 127
251 70 43 150 27 166 203 118	11 182 219 102 235 86 59 134
13 180 221 100 237 84 61 132	253 68 45 148 29 164 205 116
248 73 40 153 24 169 200 121	8 185 216 105 232 89 56 137

Some facts

- a 4 by 4 Franklin Magic square does not exist
- Franklin produced some 8 by 8 and some 16 by 16 Franklin squares (± 1750)
- web sites of magic squares display Franklin Magic squares of order 8 and 16, even 32
- constructing squares of order $8k$ is not so difficult
- March 22, 2007: HYPE in the Netherlands
high school students Hoekstra, Schulte, Alkema construct an ALMOST-FRANKLIN-MAGIC square of order 12 (Dec 14, 2006)

Question

do there exist Franklin Magic squares of order 12, 20, 28, ... ?

HSA Magic Square

1 142 11 136	8 138 5 139	12 135 2 141
120 27 110 33	113 31 116 30	109 34 119 28
121 22 131 16	128 18 125 19	132 15 122 21
48 99 38 105	41 103 44 102	37 106 47 100
73 70 83 64	80 66 77 67	84 63 74 69
60 87 50 93	53 91 56 90	49 94 59 88
85 58 95 52	92 54 89 55	96 51 86 57
72 75 62 81	65 79 68 78	61 82 71 76
97 46 107 40	104 42 101 43	108 39 98 45
24 123 14 129	17 127 20 126	13 130 23 124
25 118 35 112	32 114 29 115	36 111 26 117
144 3 134 9	137 7 140 6	133 10 143 4

HSA Magic Square

1 142	11 136	8 138	5 139	12 135	2 141
120 27	110 33	113 31	116 30	109 34	119 28
121 22	131 16	128 18	125 19	132 15	122 21
48 99	38 105	41 103	44 102	37 106	47 100
73 70	83 64	80 66	77 67	84 63	74 69
60 87	50 93	53 91	56 90	49 94	59 88
85 58	95 52	92 54	89 55	96 51	86 57
72 75	62 81	65 79	68 78	61 82	71 76
97 46	107 40	104 42	101 43	108 39	98 45
24 123	14 129	17 127	20 126	13 130	23 124
25 118	35 112	32 114	29 115	36 111	26 117
144 3	134 9	137 7	140 6	133 10	143 4

2 by 2 squares have sum $4 \cdot \frac{1}{2}(1 + 144) = 290$

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1 142 11 136	8 138 5 139	12 135 2 141
120 27 110 33	113 31 116 30	109 34 119 28
121 22 131 16	128 18 125 19	132 15 122 21
48 99 38 105	41 103 44 102	37 106 47 100
73 70 83 64	80 66 77 67	84 63 74 69
60 87 50 93	53 91 56 90	49 94 59 88
85 58 95 52	92 54 89 55	96 51 86 57
72 75 62 81	65 79 68 78	61 82 71 76
97 46 107 40	104 42 101 43	108 39 98 45
24 123 14 129	17 127 20 126	13 130 23 124
25 118 35 112	32 114 29 115	36 111 26 117
144 3 134 9	137 7 140 6	133 10 143 4

2 by 2 squares have sum $4 \cdot \frac{1}{2}(1 + 144) = 290$
 each **third** row or column has sum 290

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1 142 11 136	8 138 5 139	12 135 2 141
120 27 110 33	113 31 116 30	109 34 119 28
121 22 131 16	128 18 125 19	132 15 122 21
48 99 38 105	41 103 44 102	37 106 47 100
73 70 83 64	80 66 77 67	84 63 74 69
60 87 50 93	53 91 56 90	49 94 59 88
85 58 95 52	92 54 89 55	96 51 86 57
72 75 62 81	65 79 68 78	61 82 71 76
97 46 107 40	104 42 101 43	108 39 98 45
24 123 14 129	17 127 20 126	13 130 23 124
25 118 35 112	32 114 29 115	36 111 26 117
144 3 134 9	137 7 140 6	133 10 143 4

2 by 2 squares have sum $4 \cdot \frac{1}{2}(1 + 144) = 290$

each **third** row or column has sum 290

each bent-diagonal has sum $12 \cdot \frac{1}{2}(1 + 144) = 870$

Questions

- do there exist Franklin Magic squares of order 12, 20, 28, . . . ?
- can a brute-force search by computer find such square?
- or is there some sophisticated way to construct these squares?

Answers: Yes and No

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

RED equals BLUE equals GREEN.

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

RED equals BLUE equals GREEN.

RED plus YELLOW is magic, hence YELLOW plus GREEN is magic.

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

YELLOW plus GREEN is magic; GREEN plus BLUE is magic

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

YELLOW plus GREEN is magic; GREEN plus BLUE is magic
 YELLOW equals BLUE !

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Bent-diagonal with magic sum

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Sum remains magic

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Sum remains magic

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Sum remains magic

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Sum remains magic

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Sum remains magic

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Sum remains magic

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

magic RED sum equals sum of THREE magic 2x2 squares

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

RED equals YELLOW (only used 2x2 magic subsquare property)

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

RED equals YELLOW (only used 2x2 magic subsquare property)
 RED plus YELLOW is magic ROW (870)

Searching for a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

RED has magic sum (435)

used 2x2 magic subsquare property, plus magic row sum

Let $P = \{1, 3, 5\}$, $Q = \{2, 4, 6\}$, $R = \{7, 9, 11\}$, $S = \{8, 10, 12\}$,

a Franklin Magic square M of order 12 is a square with

1. entries $1, 2, \dots, 144$;
2. each two by two sub-square $\begin{bmatrix} M_{i,j} & M_{i,j+1} \\ M_{i+1,j} & M_{i+1,j+1} \end{bmatrix}$ has sum 290;
3. in row (column) 1, entries in first half ($P + Q$) add up to 435;
4. in row (column) 1, entries in second half ($R + S$) add up to 435;
5. in row (column) 1, entries in first half odd positions plus entries in second half of even positions ($P + S$) add up to 435;

Note:

in row 1, entries in P and entries in R have the same sum

Let $P = \{1, 3, 5\}$, $Q = \{2, 4, 6\}$, $R = \{7, 9, 11\}$, $S = \{8, 10, 12\}$,

a Franklin Magic square M of order 12 remains Franklin Magic even

1. reflected along diagonal
2. reflected along horizontal axis of symmetry
3. reflected along vertical axis of symmetry
4. interchanging columns 1, 3, 5 (similar for Q, R, S);
5. interchanging columns in P with those in R (similar for Q and S);
6. applying above tricks to rows

Position of 1 in a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	1	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Position of 1 in a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	1	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Position of 1 in a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	1	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Position of 1 in a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
1	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Position of 1 in a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
I	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Position of 1 in a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
1	X	X	X	X	X	X	X	X	X	X	X

Position of 1 in a Franklin magic square of order 12

1	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

if there is a 12 by 12 FMS, there is one starting with 1

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	X	X	X	X	X	X	X	X	X	X	X
y_2	X	X	X	X	X	X	X	X	X	X	X
y_3	X	X	X	X	X	X	X	X	X	X	X
y_4	X	X	X	X	X	X	X	X	X	X	X
y_5	X	X	X	X	X	X	X	X	X	X	X
y_6	X	X	X	X	X	X	X	X	X	X	X
y_7	X	X	X	X	X	X	X	X	X	X	X
y_8	X	X	X	X	X	X	X	X	X	X	X
y_9	X	X	X	X	X	X	X	X	X	X	X
y_{10}	X	X	X	X	X	X	X	X	X	X	X
y_{11}	X	X	X	X	X	X	X	X	X	X	X

Fill in first row and column, rest follows from 2x2 squares

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	X	X	X	X	X	X	X	X	X	X	X
y_2	X	X	X	X	X	X	X	X	X	X	X
y_3	X	X	X	X	X	X	X	X	X	X	X
y_4	X	X	X	X	X	X	X	X	X	X	X
y_5	X	X	X	X	X	X	X	X	X	X	X
y_6	X	X	X	X	X	X	X	X	X	X	X
y_7	X	X	X	X	X	X	X	X	X	X	X
y_8	X	X	X	X	X	X	X	X	X	X	X
y_9	X	X	X	X	X	X	X	X	X	X	X
y_{10}	X	X	X	X	X	X	X	X	X	X	X
y_{11}	X	X	X	X	X	X	X	X	X	X	X

Fill in first row and column, rest follows from 2×2 squares

$$M_{22} = 290 - 1 - x_1 - y_1; M_{23} = 290 - x_1 - x_2 - M_{22}$$

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	X	X	X	X	X	X	X	X	X	X	X
y_2	X	X	X	X	X	X	X	X	X	X	X
y_3	X	X	X	X	X	X	X	X	X	X	X
y_4	X	X	X	X	X	X	X	X	X	X	X
y_5	X	X	X	X	X	X	X	X	X	X	X
y_6	X	X	X	X	X	X	X	X	X	X	X
y_7	X	X	X	X	X	X	X	X	X	X	X
y_8	X	X	X	X	X	X	X	X	X	X	X
y_9	X	X	X	X	X	X	X	X	X	X	X
y_{10}	X	X	X	X	X	X	X	X	X	X	X
y_{11}	X	X	X	X	X	X	X	X	X	X	X

First row: x_5 (in Q), x_{10} (in R), x_{11} (in S) follow from rules

Similar: y_5 , y_{10} , and y_{11} are fixed; 16 degrees of freedom.

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	X	X	X	X	X	X	X	X	X	X	X
y_2	X	X	X	X	X	X	X	X	X	X	X
y_3	X	X	X	X	X	X	X	X	X	X	X
y_4	X	X	X	X	X	X	X	X	X	X	X
y_5	X	X	X	X	X	X	X	X	X	X	X
y_6	X	X	X	X	X	X	X	X	X	X	X
y_7	X	X	X	X	X	X	X	X	X	X	X
y_8	X	X	X	X	X	X	X	X	X	X	X
y_9	X	X	X	X	X	X	X	X	X	X	X
y_{10}	X	X	X	X	X	X	X	X	X	X	X
y_{11}	X	X	X	X	X	X	X	X	X	X	X

rough number of possibilities: $144^{16} \approx 3.4 \cdot 10^{34}$.

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	X	X	X	X	X	X	X	X	X	X	X
y_2	X	X	X	X	X	X	X	X	X	X	X
y_3	X	X	X	X	X	X	X	X	X	X	X
y_4	X	X	X	X	X	X	X	X	X	X	X
y_5	X	X	X	X	X	X	X	X	X	X	X
y_6	X	X	X	X	X	X	X	X	X	X	X
y_7	X	X	X	X	X	X	X	X	X	X	X
y_8	X	X	X	X	X	X	X	X	X	X	X
y_9	X	X	X	X	X	X	X	X	X	X	X
y_{10}	X	X	X	X	X	X	X	X	X	X	X
y_{11}	X	X	X	X	X	X	X	X	X	X	X

number of possibilities: 1 473 501 105.

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	X	X	X	X	X	X	X	X	X	X	X
y_2	X	X	X	X	X	X	X	X	X	X	X
y_3	X	X	X	X	X	X	X	X	X	X	X
y_4	X	X	X	X	X	X	X	X	X	X	X
y_5	X	X	X	X	X	X	X	X	X	X	X
y_6	X	X	X	X	X	X	X	X	X	X	X
y_7	X	X	X	X	X	X	X	X	X	X	X
y_8	X	X	X	X	X	X	X	X	X	X	X
y_9	X	X	X	X	X	X	X	X	X	X	X
y_{10}	X	X	X	X	X	X	X	X	X	X	X
y_{11}	X	X	X	X	X	X	X	X	X	X	X

number of possibilities: 24 473 864 360.

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	X	X	X	X	X	X	X	X	X	X	X
y_2	X	X	X	X	X	X	X	X	X	X	X
y_3	X	X	X	X	X	X	X	X	X	X	X
y_4	X	X	X	X	X	X	X	X	X	X	X
y_5	X	X	X	X	X	X	X	X	X	X	X
y_6	X	X	X	X	X	X	X	X	X	X	X
y_7	X	X	X	X	X	X	X	X	X	X	X
y_8	X	X	X	X	X	X	X	X	X	X	X
y_9	X	X	X	X	X	X	X	X	X	X	X
y_{10}	X	X	X	X	X	X	X	X	X	X	X
y_{11}	X	X	X	X	X	X	X	X	X	X	X

number of possibilities: 121 404 978.

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	X	X	X	X	X	X	X	X	X	X	X
y_2	X	X	X	X	X	X	X	X	X	X	X
y_3	X	X	X	X	X	X	X	X	X	X	X
y_4	X	X	X	X	X	X	X	X	X	X	X
y_5	X	X	X	X	X	X	X	X	X	X	X
y_6	X	X	X	X	X	X	X	X	X	X	X
y_7	X	X	X	X	X	X	X	X	X	X	X
y_8	X	X	X	X	X	X	X	X	X	X	X
y_9	X	X	X	X	X	X	X	X	X	X	X
y_{10}	X	X	X	X	X	X	X	X	X	X	X
y_{11}	X	X	X	X	X	X	X	X	X	X	X

number of possibilities: 121 404 978, NONE of which extend to 12 by 12!
 165 hours of computing time (3.5 hours using 50 computers in parallel).