Implicit surfaces
CSG
Lipschitz condition
scanline rendering

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**implicit surface** \( \{(x, y, z) \mid f(x, y, z) = 0\} \)

**implicit volume** \( \{(x, y, z) \mid f(x, y, z) \geq 0\} \)

**Example 1** sphere \( f(x, y, z) = x^2 + y^2 + z^2 - 41 = 0 \)

**Example 2** distance-induced function (skeleton)
\[
f(x, y, z) = r - d((x, y, z), S)
\]
Implicit, CSG, Lipschitz, Scan-conversion

primitives

point

torus

cylinder

disc

line

beam

triangle

quadrilateral
CSG functions

1. \((f \cup g)(p) := \max(f(p), g(p))\)

2. \((f \cap g)(p) := \min(f(p), g(p))\)

3. \((f \setminus g)(p) := \min(f(p), -g(p))\)
CSG functions with blending

1. \((f \cup g)(p) := \max(f(p), g(p)) + f_b(|g(p) - f(p)|)\)

2. \((f \cap g)(p) := \min(f(p), g(p)) - f_b(|g(p) - f(p)|)\)

3. \((f \setminus g)(p) := \min(f(p), -g(p)) - f_b(|g(p) + f(p)|)\)
A function $f$ has Lipschitz constant $\lambda$ iff for all $p$ and $q$ in the domain of $f$

$$|f(p) - f(q)| \leq \lambda||q - p||$$

Distance-induced functions have Lipschitz constant 1.

Let $f$ have LC $\lambda_f$ and $g$ have LC $\lambda_g$ then

- $f \circ g$ has Lipschitz constant $\lambda_f \lambda_g$

- $f + g$ has Lipschitz constant $\lambda_f + \lambda_g$

- $\lambda_f^{-1} f$ has Lipschitz constant 1
For a function $f$ with Lipschitz constant 1

$$|f(p)| \leq ||q - p||,$$

for all $q$ with $f(q) = 0$

Hence,

$$|f(p)| \leq \text{the distance of } p \text{ to the implicit surface}.$$

**Pruning property** An open ball with center $p$ and radius $|f(p)|$ is disjoint with the implicit surface.

**closed** The set of functions with Lipschitz constant 1 is closed under the following operators

- CSG
- blend (if $f_b$ is chosen properly)
Sphere tracing:
• implicit surfaces with **Lipschitz condition**

• rendering
  
  – ray tracing

  – polygonisation

  – **scan-conversion**

    • Davis, Nagel, Guber (1968) - quadratic surfaces

    • Sederberg (1989) - algebraic surfaces
**viewing volume** a cube $C$ (orthogonal projection)

**scanline** $y$

**scanplane** $P$ (plane containing $y$ ...)

**implicit surface** $S$

**intersection** $I := C \cap S \cap P$
Algorithm 1: scanline-rendering of surface $S$ in cube $C$

Find zero points of function in scanplane by sampling.
Algorithm 2: approximation of intersection $I$

Generate quadtree

- maximum depth
maximum depth of quadtree: 2,3,4,6,7,8
Generate quadtree using adaptivity

- Pruning property
+ minimal tree depth
+ 4 sign alterations
+ tolerance: $\alpha$
+ normal vector deviation: $\phi$
Approximation of straddling nodes with line segments.

- find zero point on each edge with alternating sign
- connect zero points with line segments
Solution: find cracks and refine tree
- faster function evaluations
  - lazy evaluation of CSG expression

- less function evaluations
  - use coherence between successive scan-lines
  - use coherence by using an octree
  - generate only ‘visible’ part of quadtree

- better support for unblended primitives

- perspective projection
CSG functions:

1. \((f \cup g)(p) := \max(f(p), g(p))\)

2. \((f \cap g)(p) := \min(f(p), g(p))\)

3. \((f \setminus g)(p) := \min(f(p), -g(p))\)
Implicit, CSG, Lipschitz, Scan-conversion

partial quadtree

screen
• directly scan-converting implicit surfaces

• orthogonal projection

• infinite surfaces with multiple parts

• fast prototyping
• polygonization using contours

• use time-coherence in the generation of octree

• ...
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![Image of a cube with a hole](image-url)
Implicit, CSG, Lipschitz, Scan-conversion

example

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