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# 7M836

## *Animation & Rendering*

Global illumination, ray tracing

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# Local illumination models

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- What is missing in local (Phong) illumination model
  - (Shadows)
  - Real mirrors
  - Transparency
  - Area light sources
  - Indirect diffuse reflection

# Light paths

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- Light path notation
  - L: light source
  - D: diffuse reflection
  - S: specular reflection
  - E: eye point
- Local reflection models:  $L(D|S)E$
- Complete solution:  $L(D|S)^*E$

# Global illumination models

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- Illumination for complete scene
- All illumination, also indirect illumination
  
- Several approaches:
  - Ray tracing
  - Radiosity
  - ...

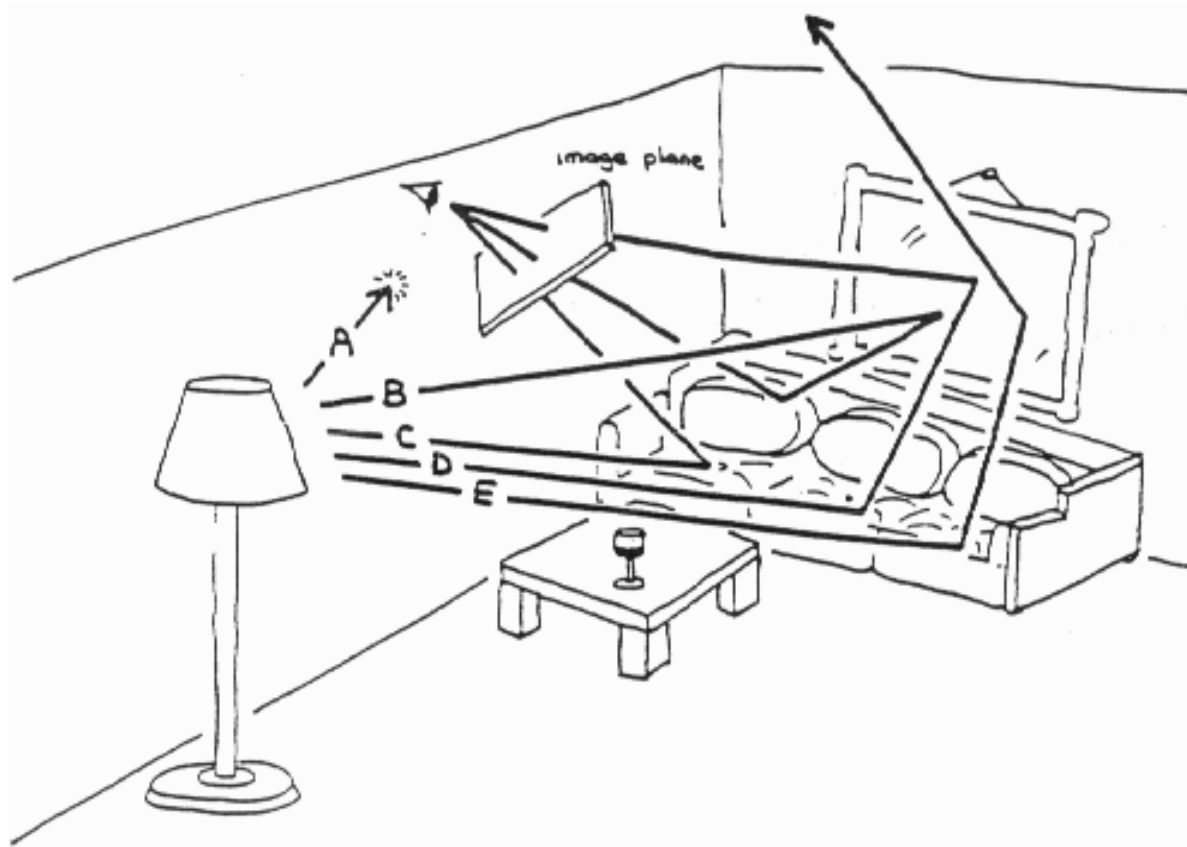
# Brute force solutions

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- Trace photons from light source into scene
- Follow paths of photons through reflections/transmissions
- At each reflection/transmission “energy” of photon is modified (part of energy absorbed by surface)
- Photons that go through image plane and reach the eye contribute to image.

# Trace photons

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# Trace photons

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- Light paths complete solution:  $L(D|S)^*E$
- Forward ray tracing
- Problem
  - Most photons will not contribute to image

# Backward ray tracing

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- Only trace light that arrives at viewpoint through viewing plane
- Trace light backwards from viewpoint until light source reached
- *(Backward) Ray tracing*

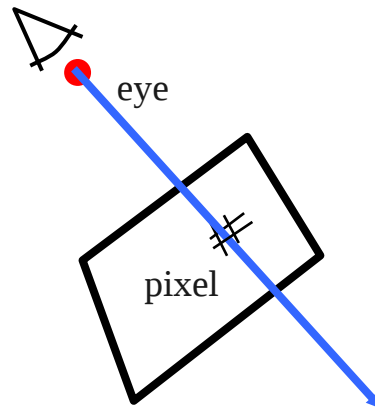


# Ray tracing – basic algorithm

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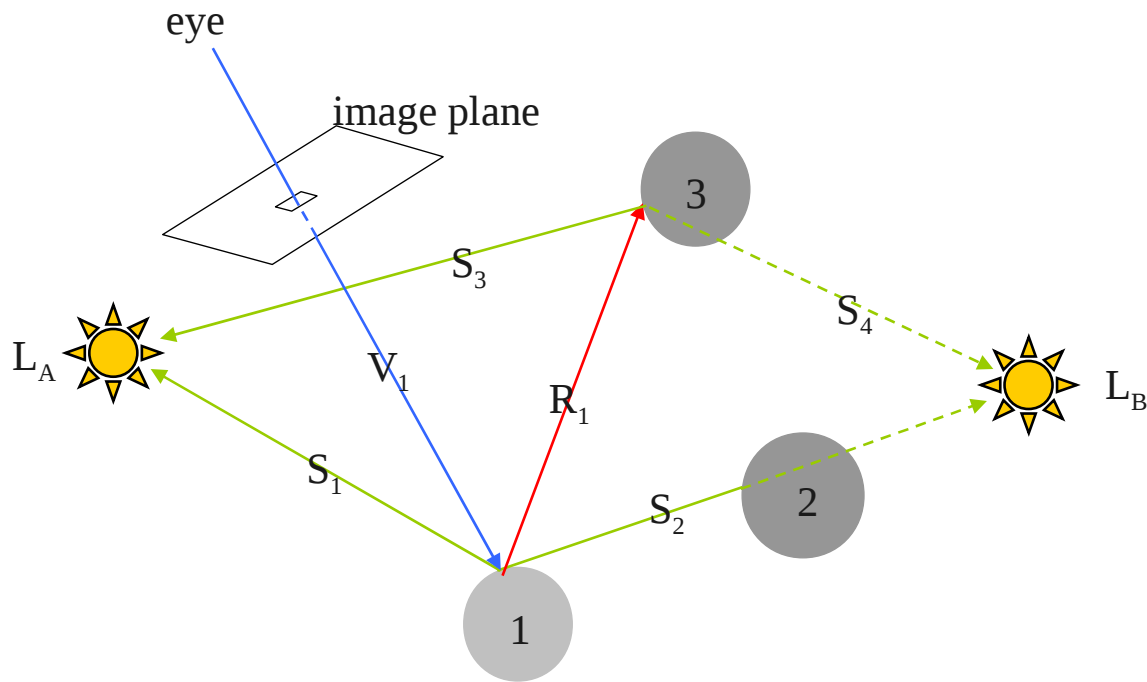
for all **pixels** in image plane

- create **ray** from eye point through pixel
- trace this ray on its path(s) through scene until it reaches light source(s) and collect illumination encountered during travel
- color of pixel = amount of collected light



# Ray tracing – example 1

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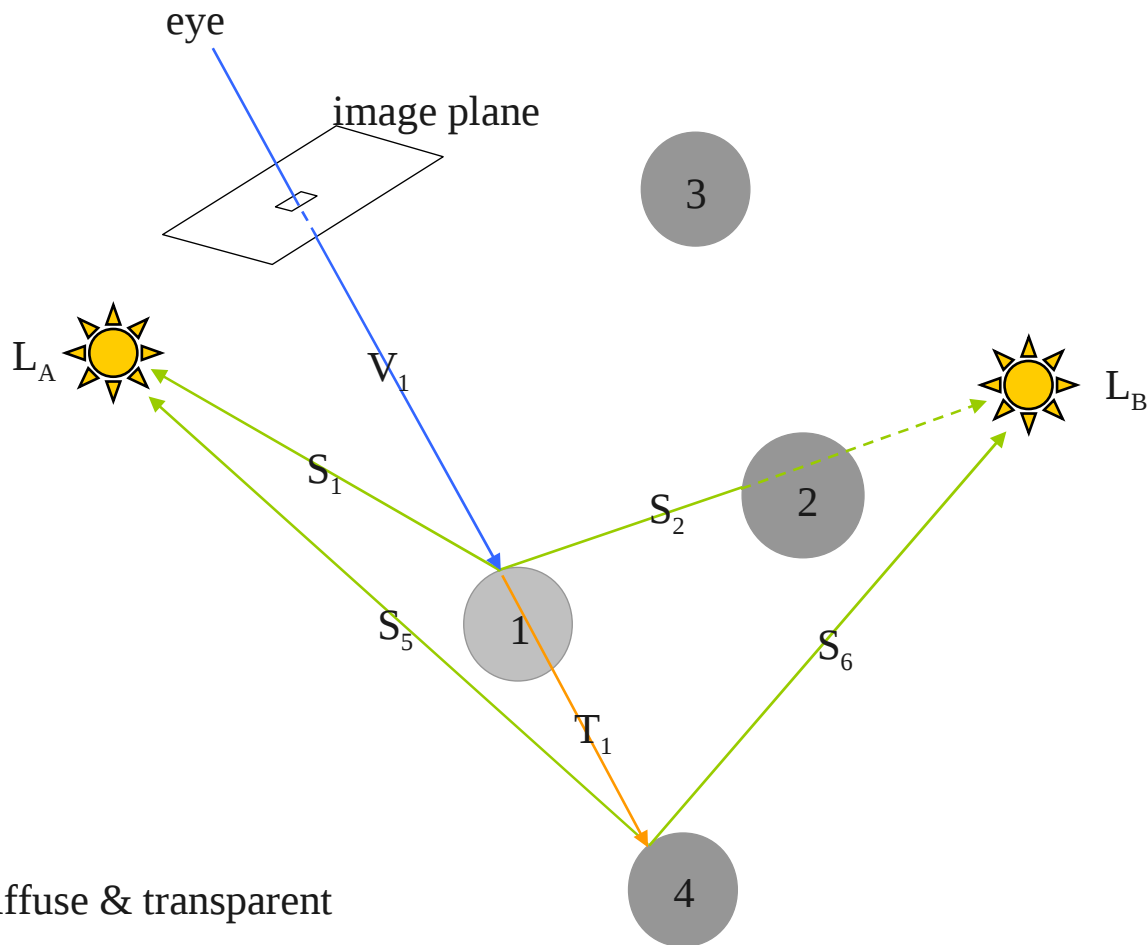


Object 1 diffuse & specular

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# Ray tracing – example 2

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Object 1 diffuse & transparent

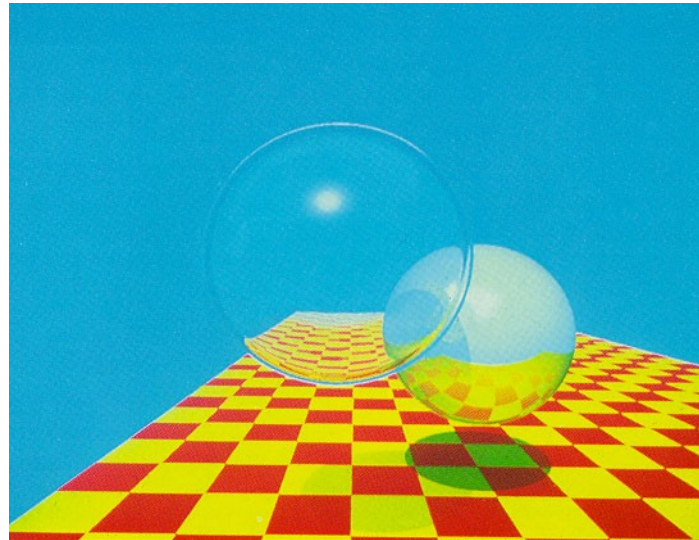
# Ray tracing

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1. Create **viewing ray**
2. Trace ray
3. At intersection point
  - a. Compute (local) illumination. Trace **shadow rays** to light sources to account for shadows.
  - b. If surface at intersection is specularly reflecting, trace **reflection ray** and compute its contribution:  
**continue at step 2**
  - c. If surface at intersection is transparent, trace **transparency ray** and compute its contribution:  
**continue at step 2**
  - d. Sum contribution of steps (a), (b) and (c)

# Early ray-tracing picture

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640x480, 74 minutes on VAX-11/780

An Improved Illumination Model for Shaded Display,  
Turner Whitted, Communications of the ACM,  
June 1980, volume 23, Number 6

# Example using ray tracing

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# Computations in ray-tracing

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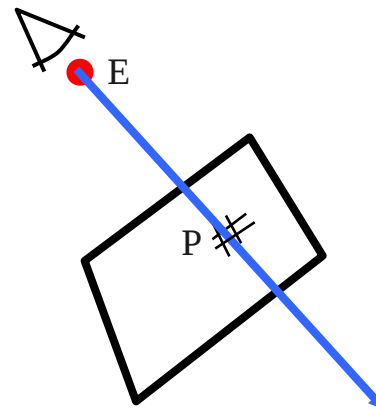
- Creation of viewing ray
- Intersection computation
  - Find first intersected object (ray-scene intersection)
  - Point of intersection
  - Normal (and local coordinates) of intersection point
- Creation of shadow rays
- Creation of reflection rays
- Creation of transparency rays
- Illumination

# Creation of viewing ray

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Ray is a half line that  
start in the eye point (E) and  
passes through a pixel (P) in the projection screen

$$\overbrace{\vec{R}(t) = \vec{E} + t \vec{V}}^{\text{viewing ray}} \quad \wedge \quad t > 0$$
$$\vec{V} = \frac{\vec{P} - \vec{E}}{\|\vec{P} - \vec{E}\|}$$

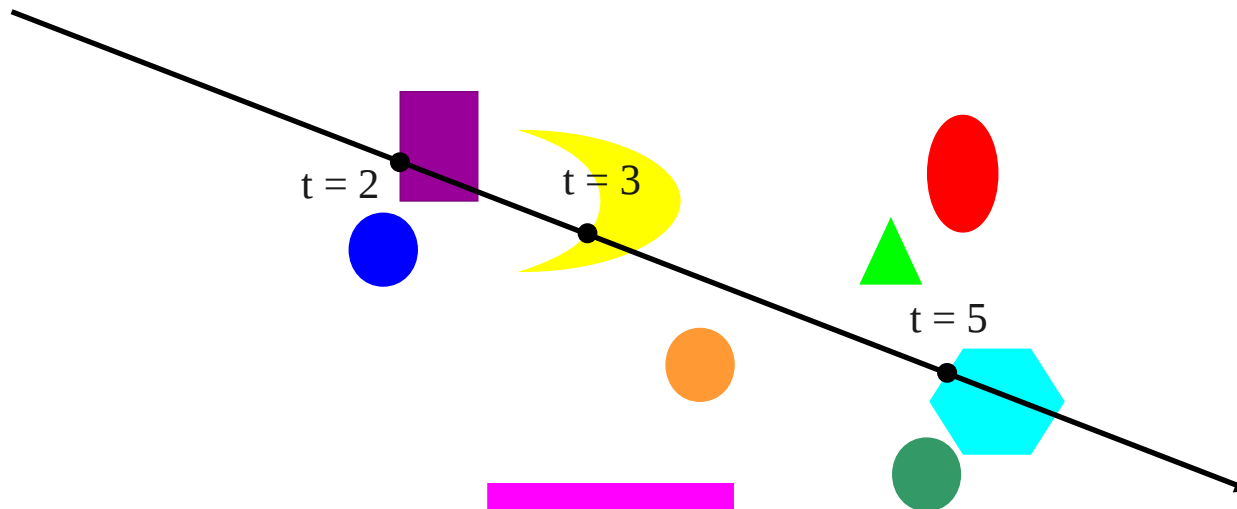




# Ray-scene intersection

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- Compute intersection of ray  $R(t)$  with all objects in scene
- Intersection with smallest positive intersection distance  $t$  gives intersected object



# Ray-object intersection

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- Ray equation

- $R_o$  : ray origin
- $R_d$  : ray direction
- $\|R_d\| = 1$

$$\overbrace{\vec{R}(t) = \vec{R}_o + t \vec{R}_d}^{\text{general ray}} \wedge t > 0$$

- Object description examples:

- Implicit surface  $S$  :
- Sphere
- Polygons
- ...

$$S = \{ \vec{P} \in \mathbb{R}^3 : f(\vec{P}) = 0 \}$$

# Ray-sphere intersection

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- Sphere with radius  $r$  and center  $C$ :

$$\|P - C\|^2 - r^2 = 0$$

- Substitution of  $R(t)$  for  $P$  gives

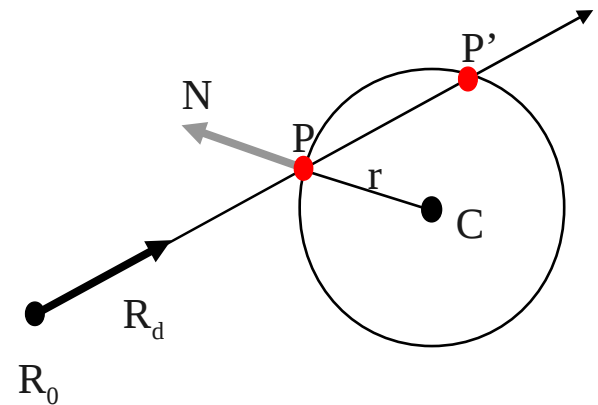
$$t^2 + 2bt + c = 0$$

- $b = R_d \cdot (R_o - C)$

- $c = \|R_o - C\|^2 - r^2$

- Solution for  $t$  gives intersection points

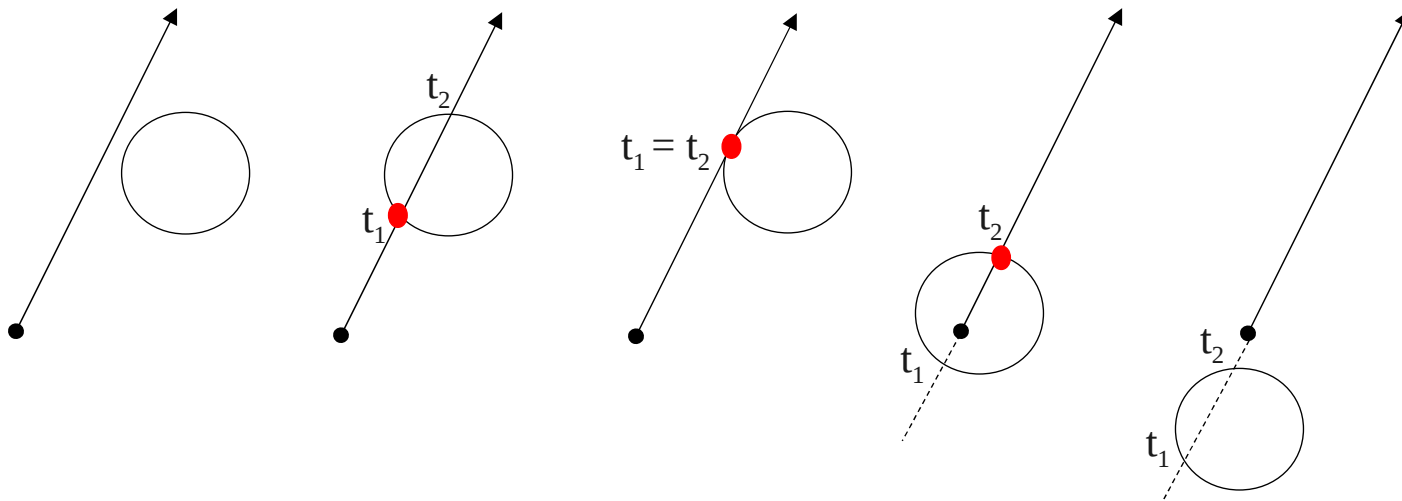
$$t_{1,2} = -b \pm \sqrt{b^2 - c}$$



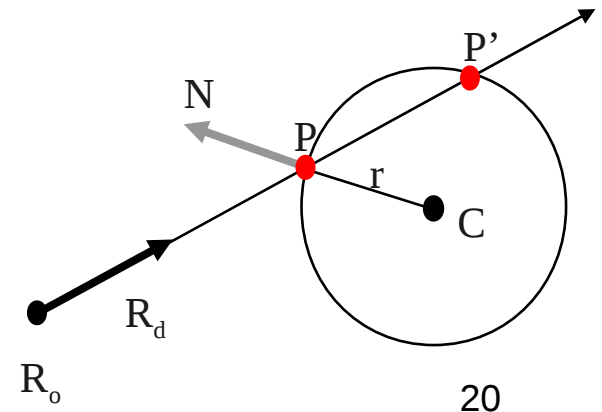
# Ray-sphere intersection

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- Solutions for  $t_1$  and  $t_2$ :



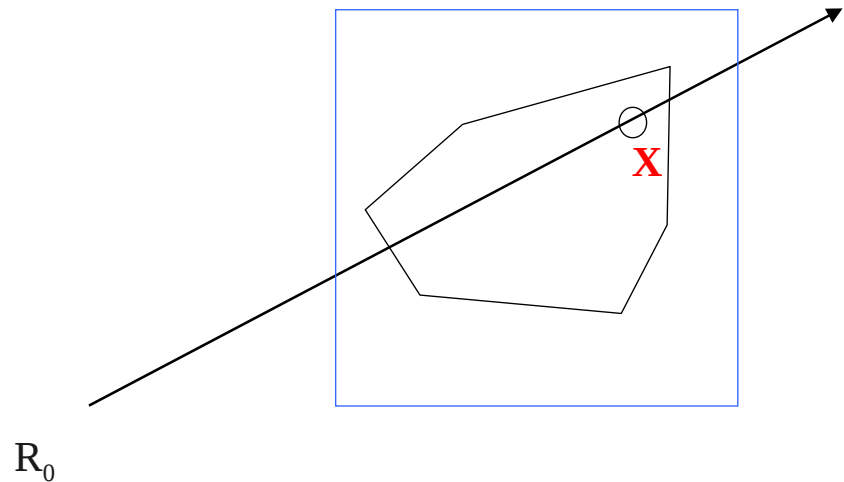
- Normal at intersection point P  
$$N = (P - C) / r$$



# Ray-polygon intersection

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- Two steps
  1. Intersect ray with plane of polygon.  
Compute intersection point **X**
  2. Determine if **X** is in the polygon
- Normal at intersection point is plane normal



# Ray-polygon intersection

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Intersection ray-plane:

Implicit equation of plane with normal  $N$  and point  $P$  using dot product:

$$\vec{N} \cdot (\vec{X} - \vec{P}) = 0$$

Substitution of  $X = R(t)$ :

$$t = \frac{\vec{N} \cdot (\vec{P} - \vec{R}_o)}{\vec{N} \cdot \vec{R}_d}$$

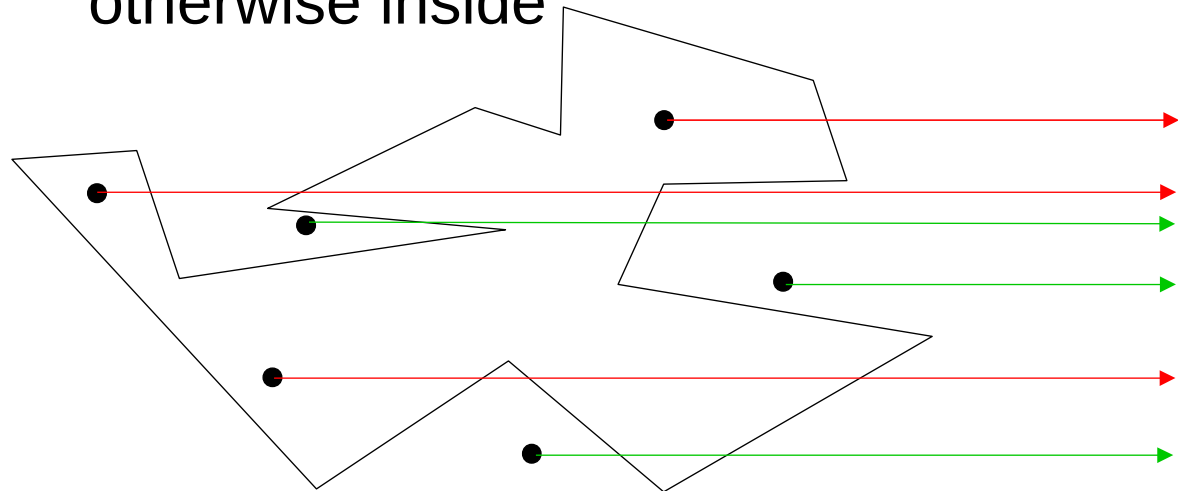
# Ray-polygon intersection

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Determine if intersection point  $P$  is in a polygon with a

**point in polygon test:**

- \* Draw half line  $L$  starting at point  $P$
- \* Count intersections of  $L$  with the edges of the polygon  
If count even, than point outside polygon,  
otherwise inside

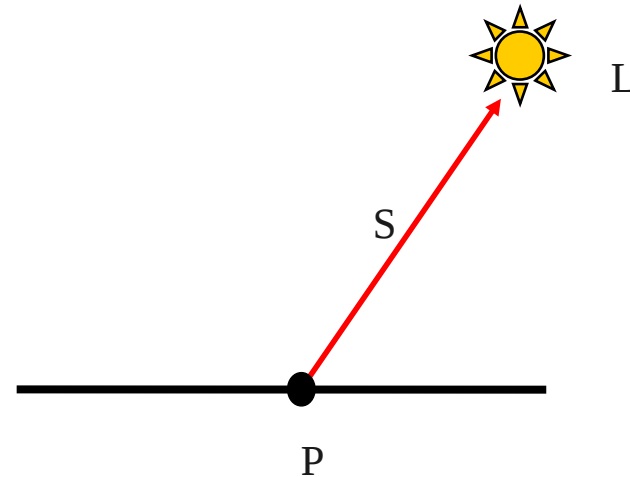


# Shadow ray

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- Determine if point is illuminated by light source

$$\overbrace{\vec{R}(t) = \vec{P} + t\vec{S} \quad \wedge \quad t > 0}^{\text{shadow ray}}$$
$$\vec{S} = \frac{\vec{L} - \vec{P}}{\|\vec{L} - \vec{P}\|}$$



- Point P is illuminated by light source L if there is no intersection of shadow ray with any object in the scene for  $0 < t < \|\vec{L} - \vec{P}\|$



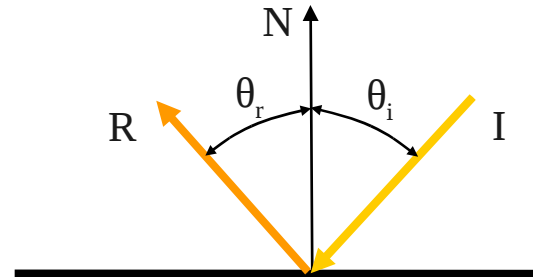
# Reflection ray

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- **Physical laws:**
  - R, N, and I are in same plane:  $\vec{R} = \alpha \vec{I} + \beta \vec{N}$
  - Angle of incidence = angle of reflection:  $\theta_r = \theta_i$

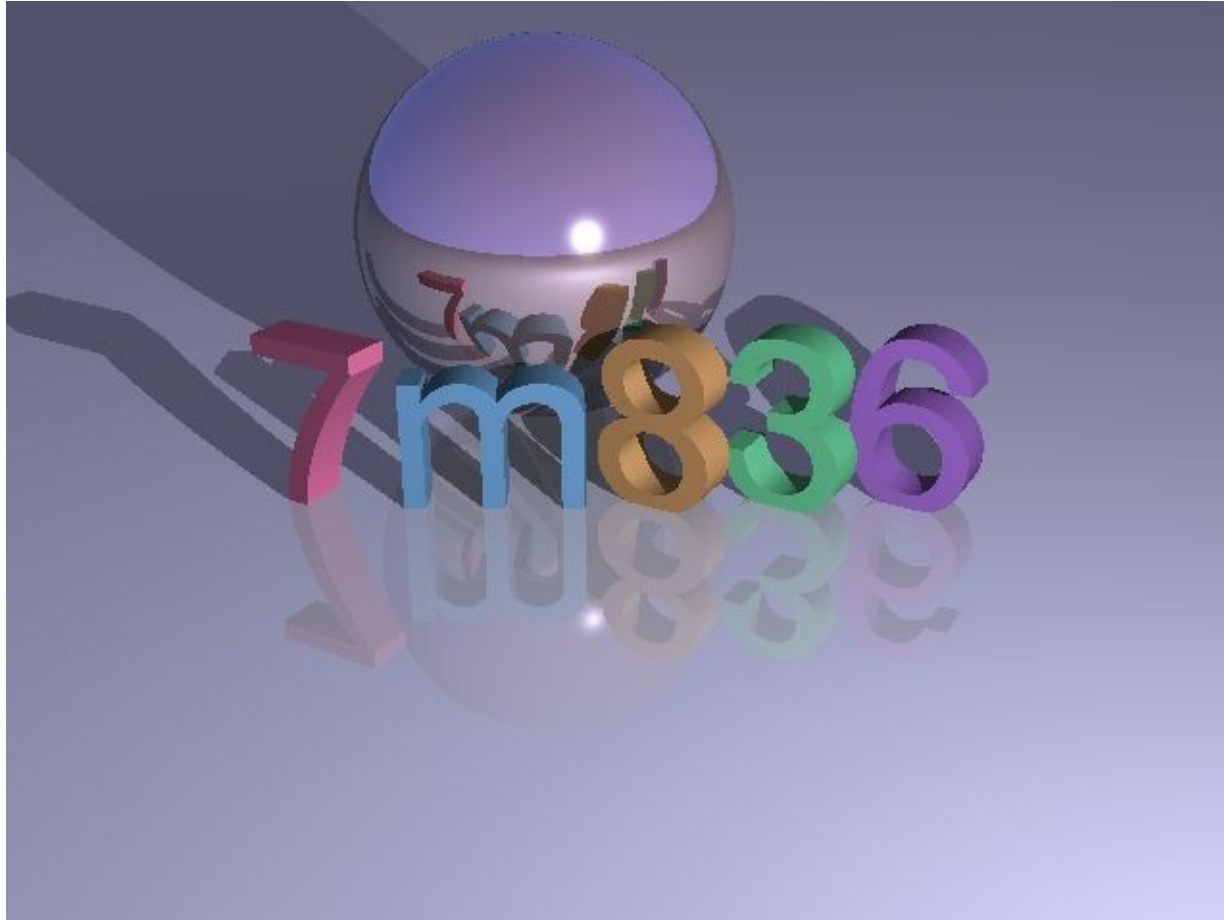
- **Reflection ray direction**

$$\vec{R} - \vec{I} = 2 \cos(\theta_i) \vec{N}$$
$$\vec{R} = \vec{I} + 2 \vec{N} \cdot \vec{I} \vec{N}$$



# Reflection

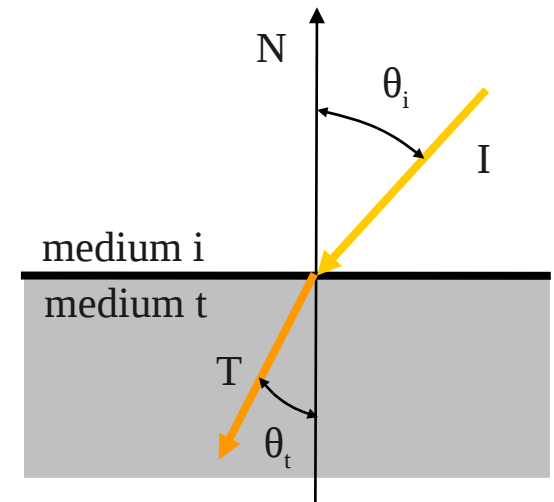
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# Transparency ray

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- **Snell's law:**  $\frac{\sin(\theta_1)}{\sin(\theta_2)} = \eta_{21} = \frac{\eta_2}{\eta_1}$ 
  - $\eta_i$  = index of refraction medium  $i$  with respect to vacuum
  - $\eta_{it}$  = index of refraction medium  $i$  with respect to medium  $t$

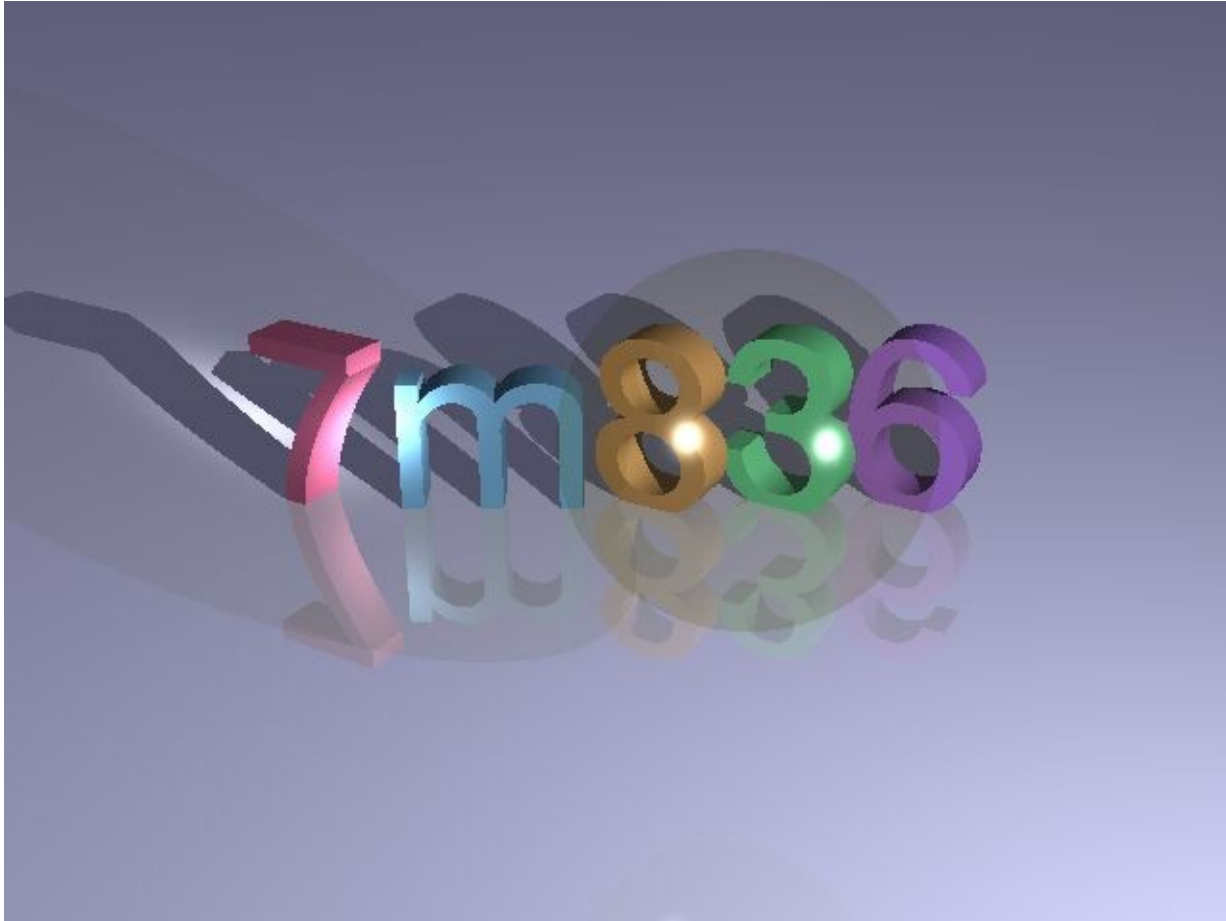


- Transparency ray direction

$$T = \eta_{it}I + (\eta_{it} \cos(\theta_i) - \sqrt{1 + \eta_{it}^2 (\cos^2(\theta_i) - 1)})N$$

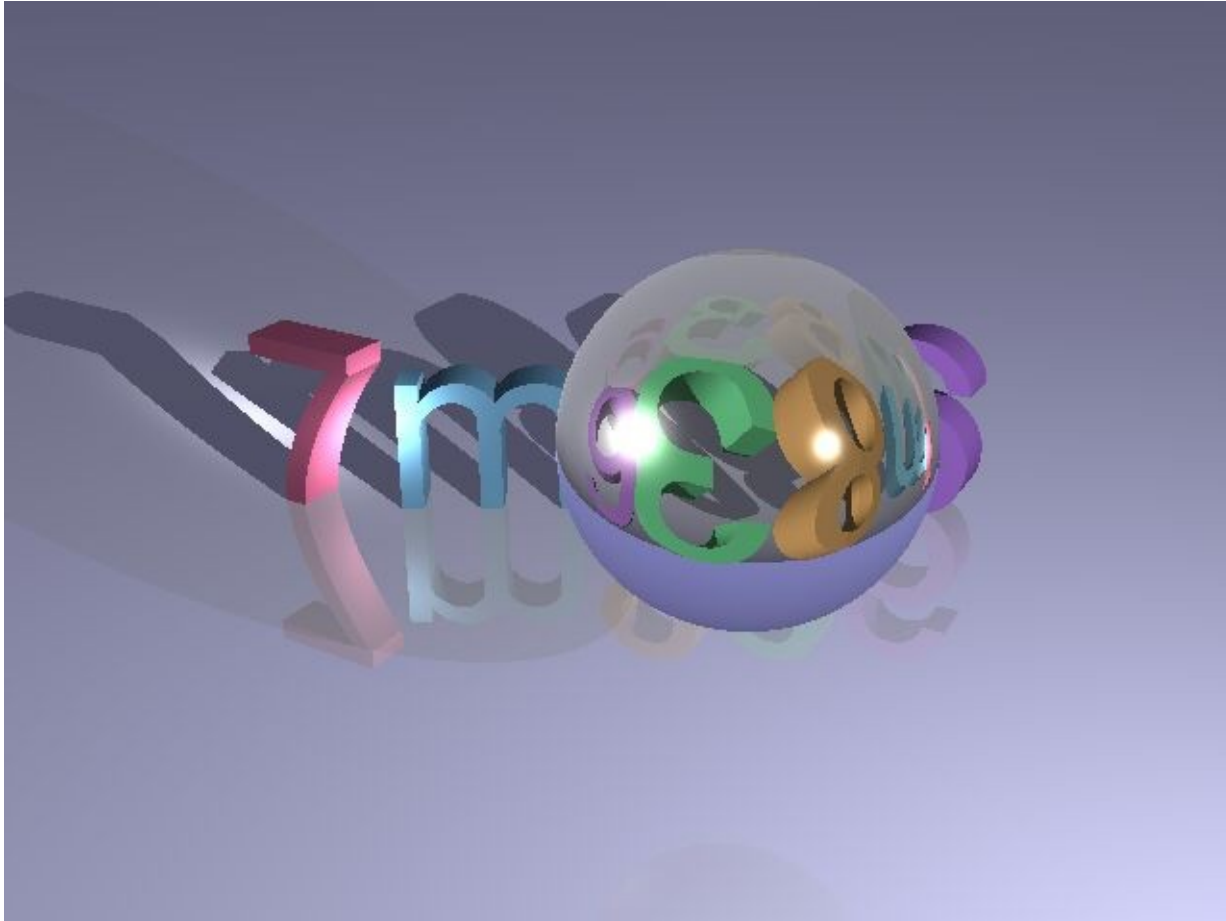
# Transparency

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# Transparency

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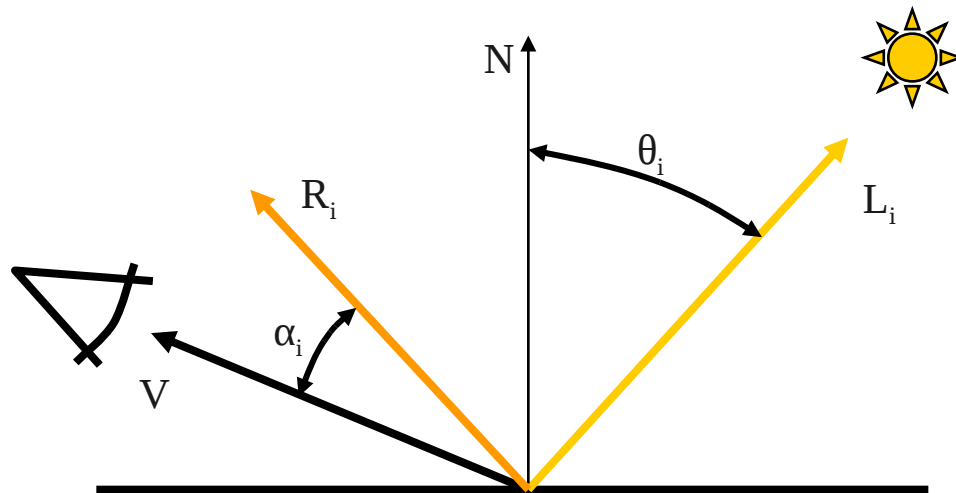


# Illumination

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- Phong illumination model for *local illumination*

$$I = I_e + k_a C_a I_a + \sum_i I_i \left( k_d C_d \cos(\theta_i) + k_s C_s \cos^n(\alpha_i) \right)$$



# Illumination for ray tracing

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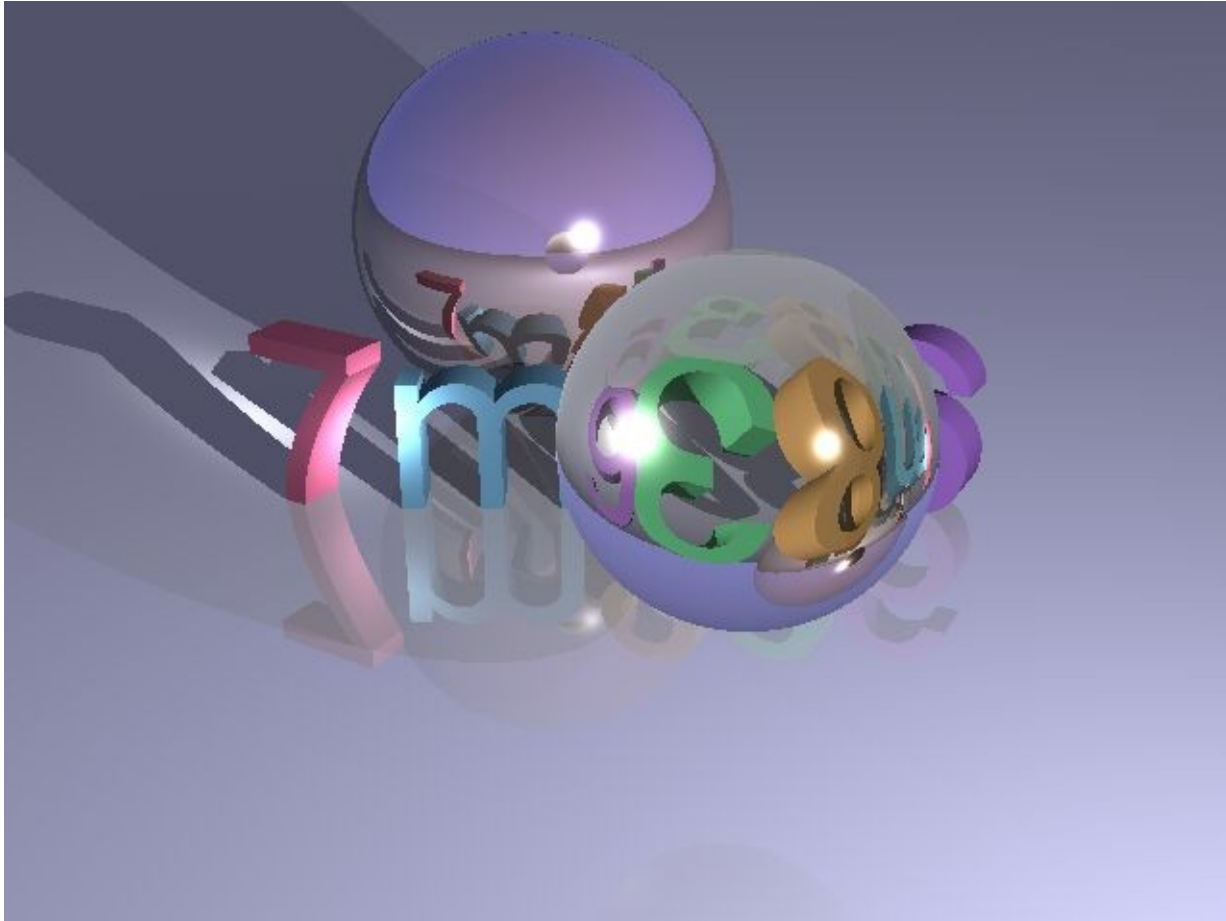
- Extension of local model with shadow information, mirroring and transparency

$$I = I_e + k_a C_a I_a + \sum_i S_i I_i \left( k_d C_d \cos(\theta_i) + k_s C_s \cos^n(\alpha_i) \right) + k_s C_s I_R + k_t C_t I_T$$

- $S_i$  = shadow factor (0, 1)
- $I_R$  = intensity of reflection ray
- $I_T$  = intensity of transmission ray

# Ray tracing

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# Ray tracing

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- Light paths ray tracing: LS\*E and LDS\*E
- Allows for transparency with refraction
- Easy shadow computation
- Easy to program
  
- Inefficient

# Ray tracing

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# Ray tracing - efficiency

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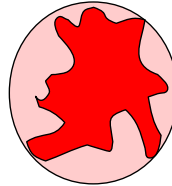
- Ray tracing is computationally expensive
- Need for efficient computation methods
- Efficiency ray tracing determined by
  - Number of rays
    - Number of pixels
    - Number of light sources
    - Number of specular and transparent objects
    - Recursion depth
  - Efficiency ray-object intersections
  - Number of intersections to be computed

# Ray tracing – bounding volume

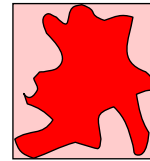
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- Reduce number of complex ray-object intersection computations by providing complex objects with a simple geometry around complex geometry

- Sphere



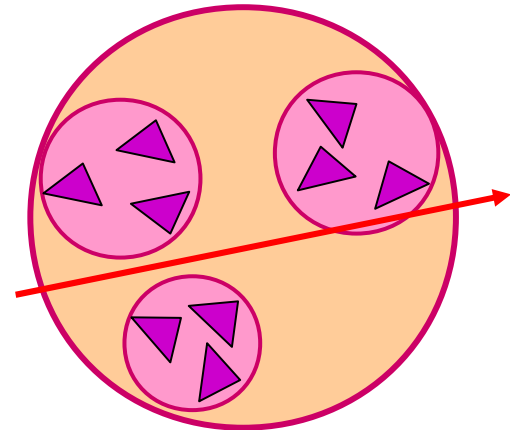
- Cube



- Do intersection with simple geometry

- Only if intersection found, do intersection with complex geometry

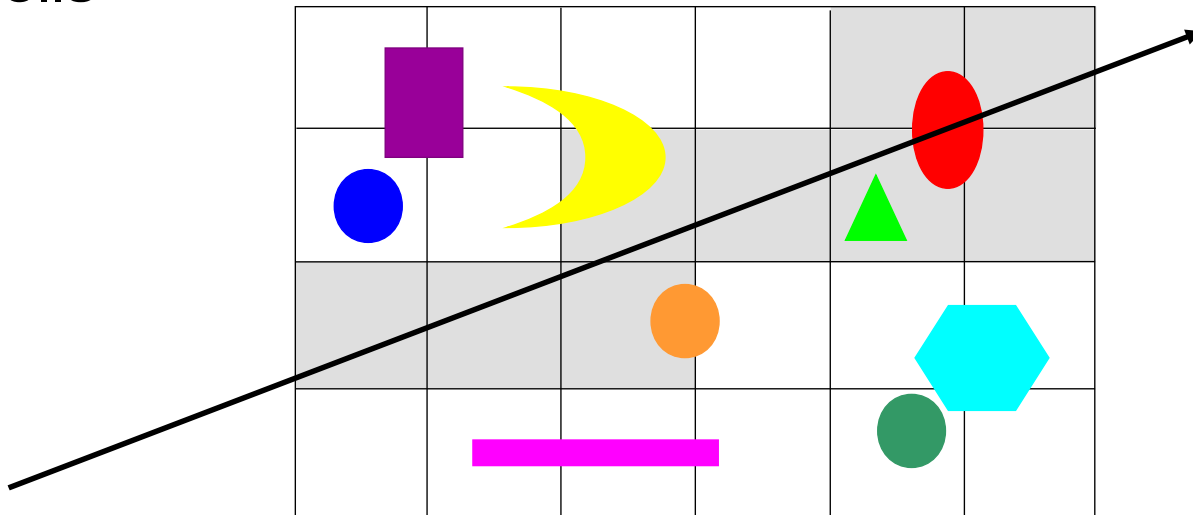
- Hierarchy of bounding volumes



# Ray tracing – space subdivision

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- Partition scene into small cells
- Store in each cell pointers to objects it contains
- Trace ray through cells and only compute intersections with objects in visited cell
- When intersection found, stop tracing rays through cells



# Ray tracing extensions

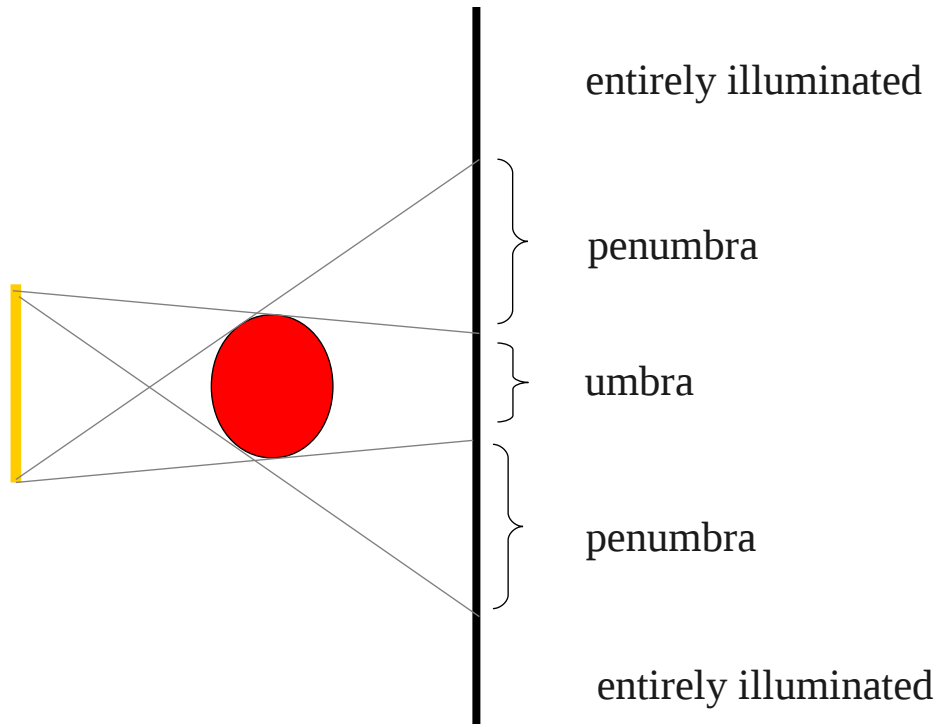
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- More effects are possible:
  - Area light sources, soft shadows
  - Depth of field
  - Motion blur
  - ...

# Area light sources

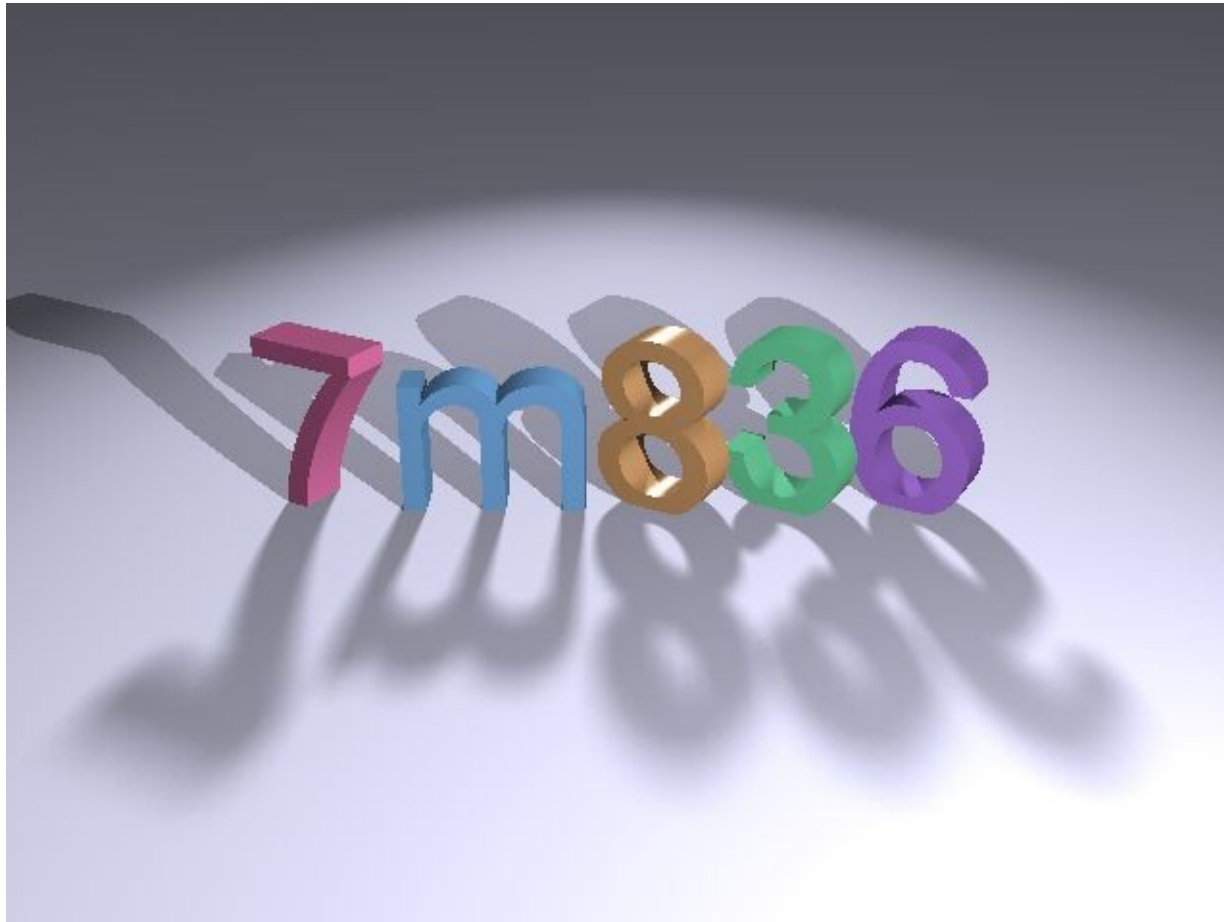
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- Area light sources generate soft shadows (penumbrae)



# Area light source

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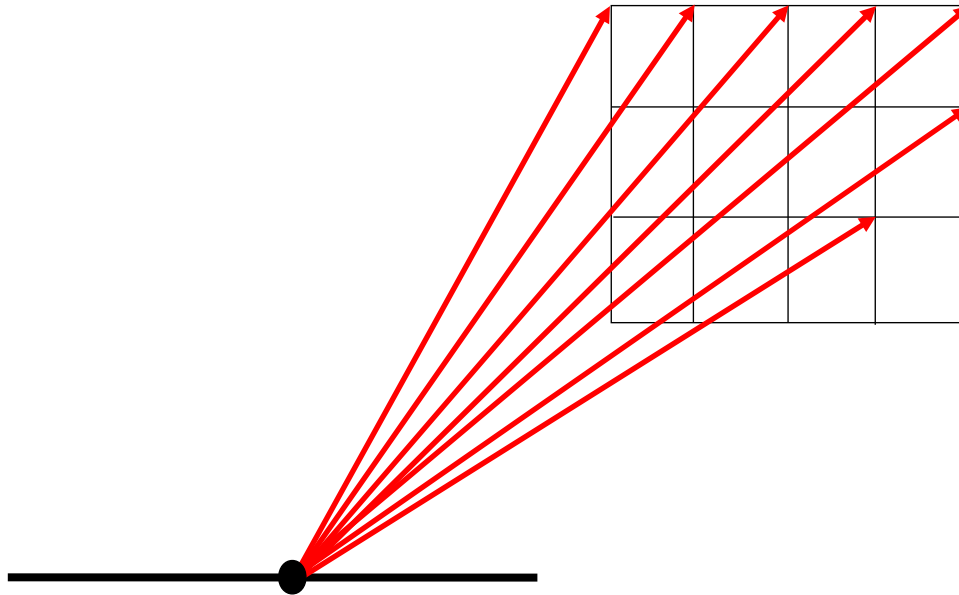




# Area light source

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- Subdivide area light source in number of point light sources
- Use regular grid of points



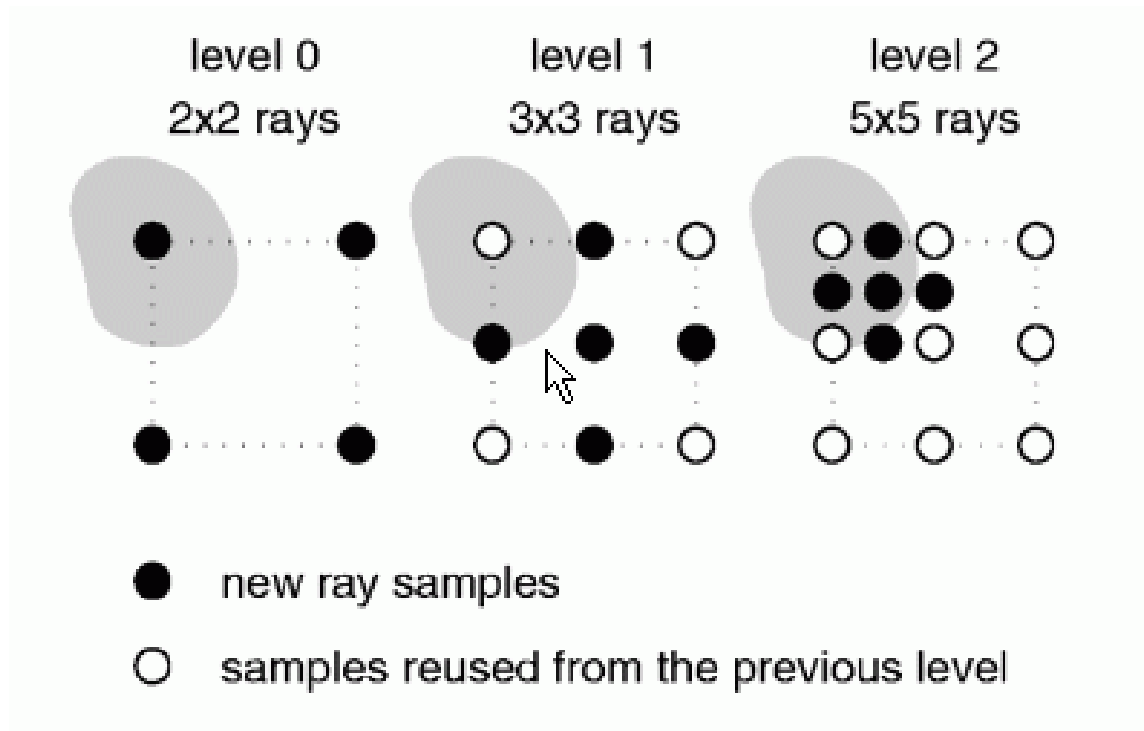
# Area light source

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- How many point light sources must be used to simulate area source?
  - Number depends on distance from point to light source
- Regular pattern of point sources generates shadow bands

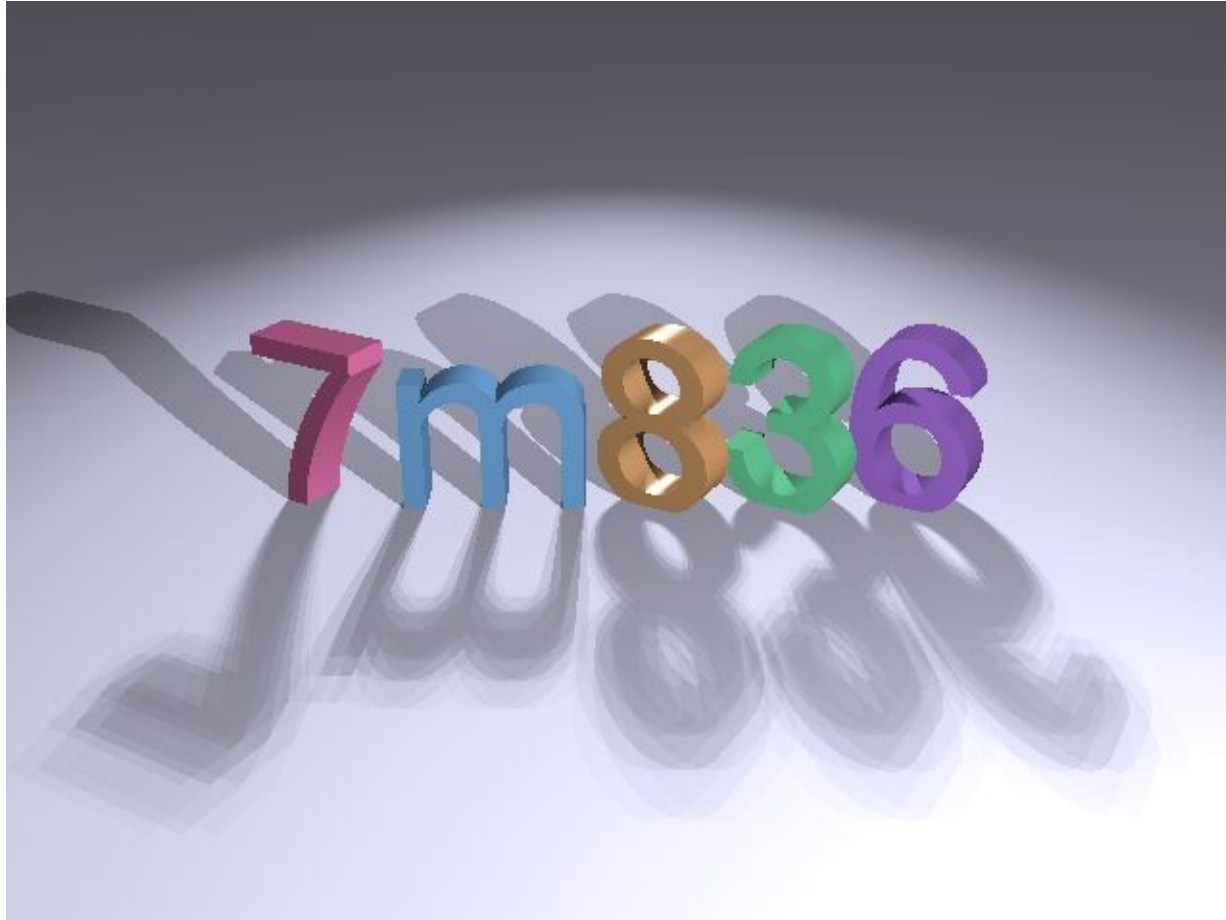
# Area source: adaptive subdivision

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# Shadow bands

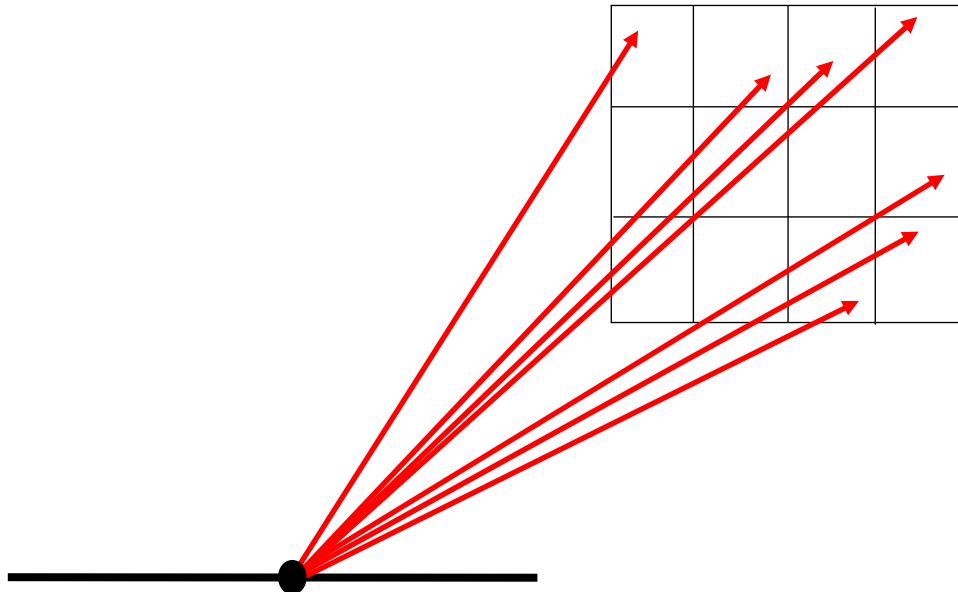
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# Area source: irregular pattern

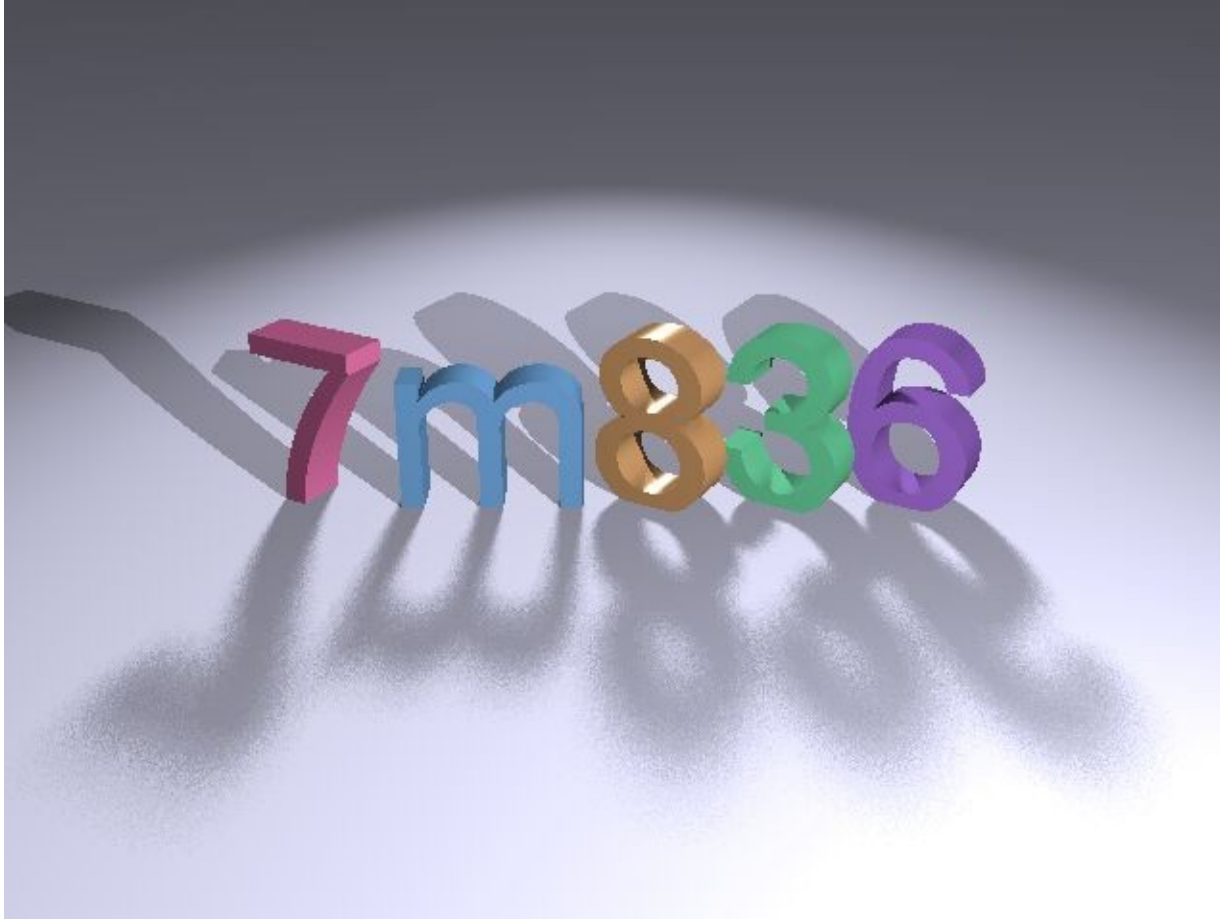
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- Trace shadow ray to random point on sub light source
- Degree of randomness often indicated with “jitter”
- Regular shadow patterns replaced by noise



# Area source: jittered subdivision

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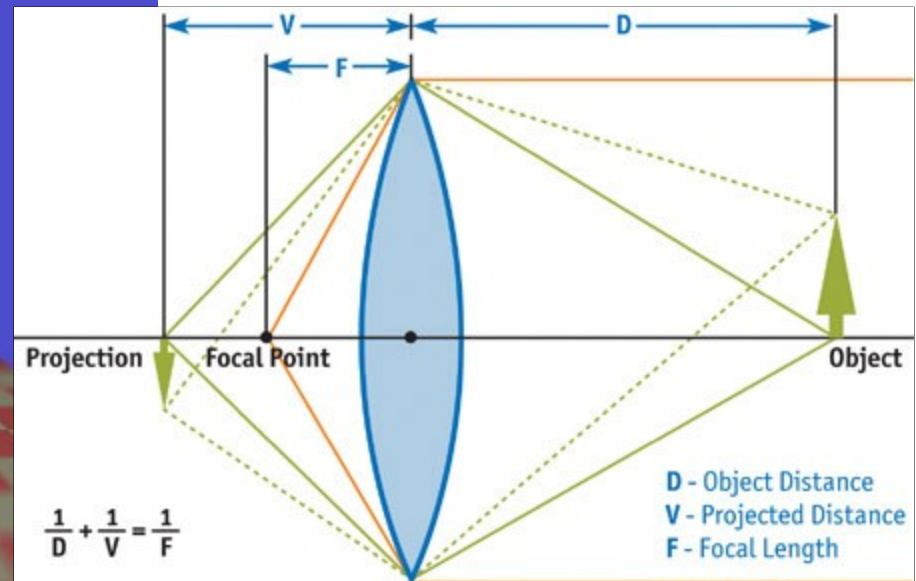
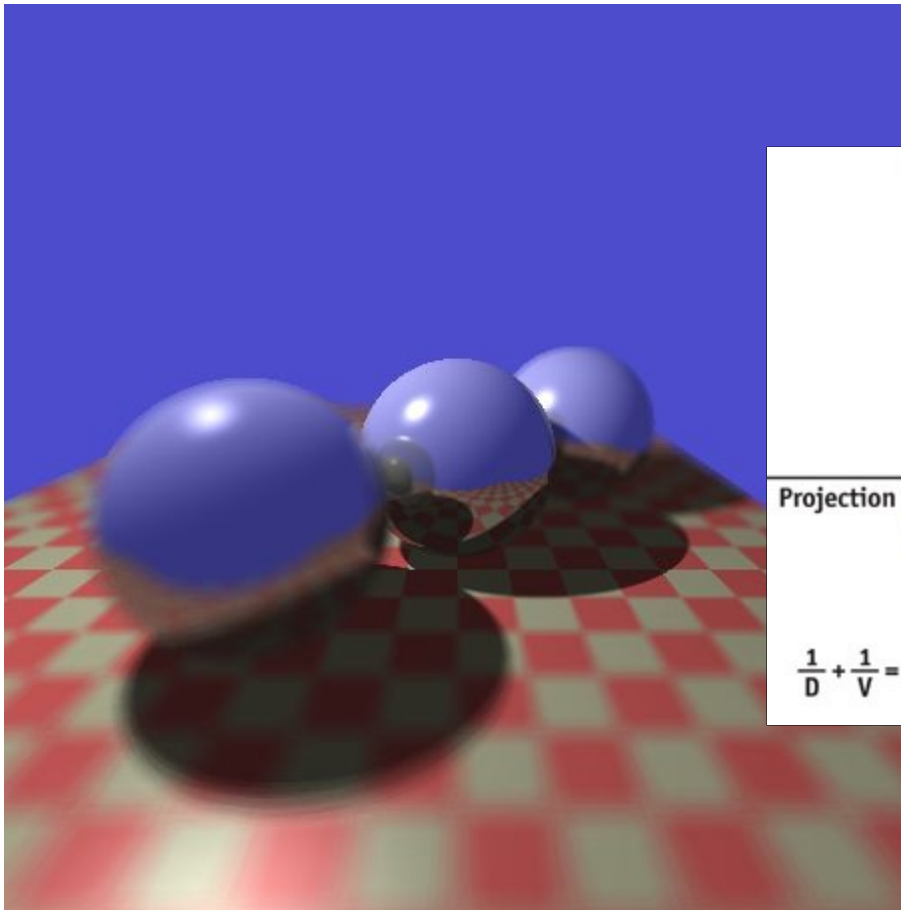


# Ray tracing: conclusion

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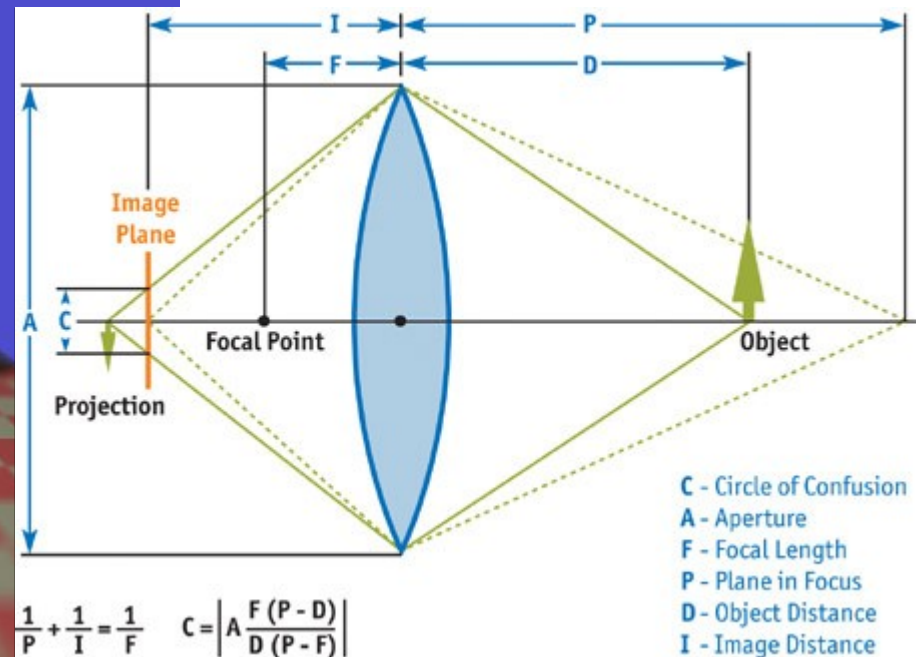
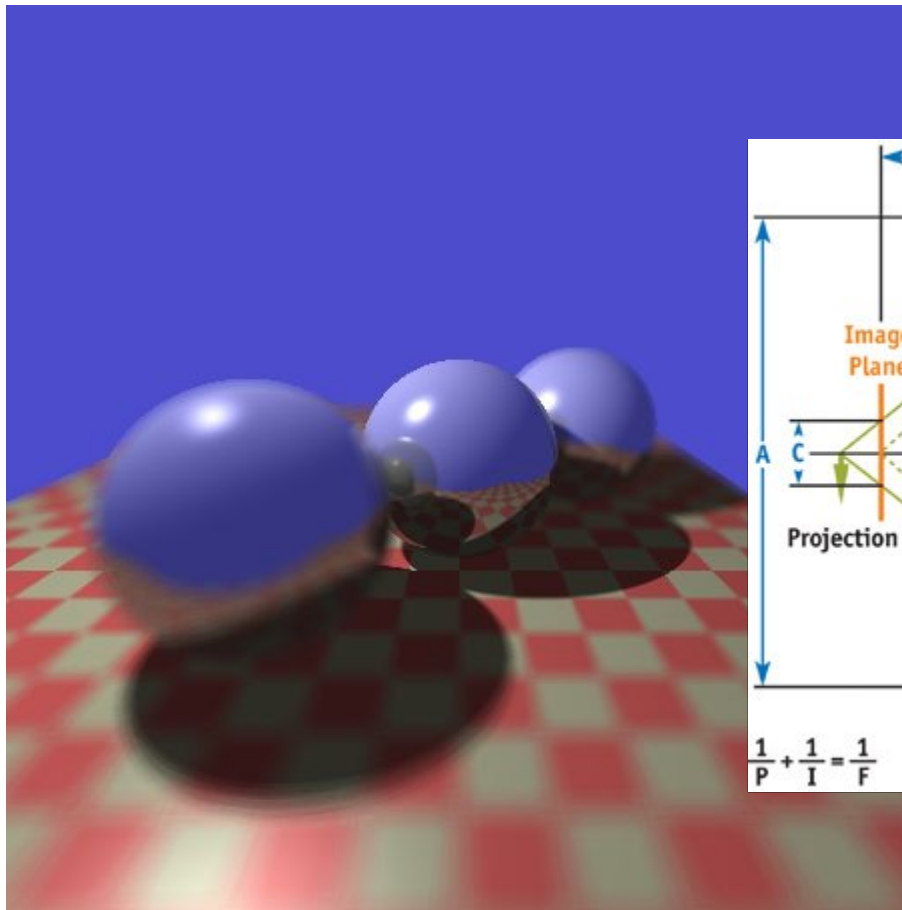
- Spectacular effects:
  - Shadows
  - Mirrors
  - Transparency, refraction
- Simple implementation
  
- Limitations
  - Expensive
  - Not all light paths possible, missing diffuse interreflection
  - Area light sources possible, but at high price
  
- => *Radiosity* method solves (parts of) limitations

# Depth of field





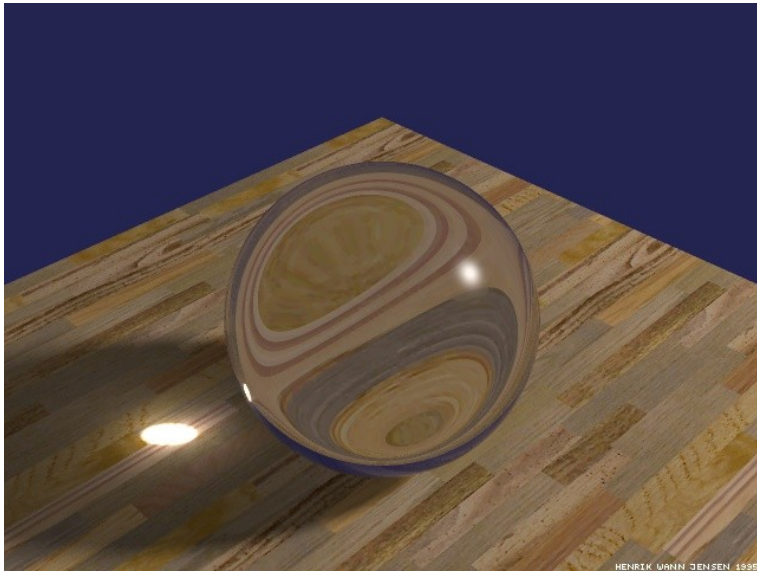
# Depth of field



Depth of field effect results from :  
Light does not converge to a single point on the image plane! 49

# Caustics

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# Two-pass ray tracing

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- Two-pass method
  - First pass: forward tracing (from lights into scene).
    - Limited to rays from light to reflective and transparent objects
    - When transparency ray hits surface, energy is stored at surface
  - Second pass: backward ray tracing
    - When local illumination applied, also check for stored intensity. Add this intensity to illumination
- Light paths ray tracing with caustics:  $LS^*E$  and  $LS^*DS^*E$