Mitered Fractal Tree Sculpture (late 1980s, bronze)
Mitered Fractal Tree Sculpture (late 1980s, wood)
Mitered Fractal Tree I in One-Month Art Exhibition
Mitered Fractal Tree II in Conference Art Exhibition
Earliest Theme: Closed 3D Paths with Miter Joints

‘Tinkering’
Bridges 2008

Lattice walking
Bridges 2008

Constant torsion
Bridges 2009

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Miter Joints with Square Cut Face

- Beam with $1 : \sqrt{2}$ rectangle as cross section
- Bevel at 45°
- Yields a square cut face
Tree Construction from $1 : \sqrt{2}$ Rectangular Beams: The Piece
Tree Construction from $1 : \sqrt{2}$ Rectangular Beams: Base
Tree Construction from $1 : \sqrt{2}$ Rectangular Beams: Growth 2

Make two copies of the piece and shrink them by a factor $\sqrt{2}$
Tree Construction from $1 : \sqrt{2}$ Rectangular Beams: Growth 3
Tree Construction from $1 : \sqrt{2}$ Rectangular Beams: Growth 4
Tree Construction from $1 : \sqrt{2}$ Rectangular Beams: Growth 5
Tree Construction from $1 : \sqrt{2}$ Rectangular Beams: Growth 6
Tree Construction from $1 : \sqrt{2}$ Rectangular Beams: Growth 7
Tree Construction from $1 : \sqrt{2}$ Rectangular Beams: Growth 8
Tree Construction from $1 : \sqrt{2}$ Rectangular Beams: Growth 9
Tree Construction from $1 : \sqrt{2}$ Rectangular Beams: Growth 10
Shorten the Piece Length: $c = 1$
Shorten the Piece Length further: $c = 1/2$
Extend the Piece Length: $c = 2$
Vary the Growth Pattern: Side – Side
Vary the Growth Pattern: Up – Up
Vary the Growth Pattern: Side – Up
Vary the Growth Pattern: Side – Front
Vary the Growth Pattern: Side-flip – Front
Vary the Growth Pattern: Back-flip – Front
Vary the Growth Pattern: Back – Up
Vary the Growth Pattern: Up – Front
Vary the Growth Pattern: Up-flip – Front
Tree Construction from Square Beams: The Piece
Tree Construction from Square Beams: Back – Front
Tree Construction from Square Beams: Rotated-Roof Piece

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Mitered Fractal Trees
Tree Construction from Square Beams: Back – Front
Tree Construction from Square Beams: Back-flip – Front-flip
Constraints for General Binary Mitered Fractal Trees

- The trunk has a **polygonal cross section**.

- Each subtree is a **scaled-down** copy of the whole tree, possibly reflected.

  All branches have a similar polygonal cross section as the trunk.

- The longitudinal edges of the beams properly meet up at the three-way joints.

- Sibling branches share the roof’s ridge, rather than a surface as in a **ternary miter joint** (cf. Bridges 2010)

- Three dimensional
Ternary Miter Joint versus Binary Tree Miter Joint
Variation Points

- **Cross section**: must be a strip (2-gon), triangle, or quadrangle

- **Cut face**: must be similar to roof panels
  
  Note: Symmetries of cut face determine growth options

- **Orientation (angles) of the roof panels**: could be asymmetric

We restrict ourselves to *rectangular* beams

Hence, cut face is *square*, or *rectangle*, or *rhombus*, or *parallelogram*
Tree Construction from 1: a Rectangular Beams: The Piece

Square cut faces: symmetric roof; roof angle at ridge makes squares

\[ a = 1 \]
\[ a = 1.1 \]
\[ a = 1.8 \]
\[ a = 2 \]
Tree Construction from 1 : 1.1 Rectangular Beams
Tree Construction from 1 : 1.8 Rectangular Beams
Tree Construction from $1 : \sqrt{2}$ Rectangular Beams: Variants

- cut
- ridge
- roof

square, horizontal, asymmetric
parallelogram, slanted, symmetric
rhombus, slanted, symmetric
rectangle, horizontal, incongruent
Squares as Cuts, Horizontal Ridge, Asymmetric Roof
Parallelograms as Cuts, Slanted Ridge, Symmetric Roof (2)
Rhombi as Cuts, Slanted Ridge, Symmetric Roof
Rectangles as Cuts, Horizontal Ridge, Asymmetric Scaling
Ternary Mitered Fractal Trees
Properties of Mitered Fractal Trees

- **Fractal dimension**: \( D = \frac{\log N}{\log f} \)

  where \( N \) = arity, and \( f \) = scale-down factor

  \( D > 3 \) implies self-intersection (for large number of generations)

- **Self-intersection** is hard to determine

- **Symmetries**: rotational, reflectional

- **Branch directions**: repetitive or not
Branch Directions

Analyze by *Turtle Geometry*: consider turn angle $\phi$ and roll angle $\psi$.

Branches on cone. Projected turn angle $\theta$ satisfies (Bridges 2011):

$$\cos(\theta/2) = \cos(\phi/2) \cos(\psi/2)$$

$\phi, \psi = 60^\circ, 19.4712 \ldots^\circ$

$\theta = 62.7994 \ldots^\circ$

$\phi, \psi = 60^\circ, 0$

$\theta = 60^\circ$
Leonardo’s Rule

Leonardo da Vinci writes in item 394 of his Notebook, Vol. 1:

“All the branches of a [natural] tree at every stage of its height when put together are equal in thickness to the trunk”

Eloy (2011) rephrased this as:

“the total cross section of branches is conserved across branching nodes”.

Eloy proposes the theory that this property evolved to help trees withstand wind-induced stresses.
Conclusion

- Explained the two earliest binary mitered fractal tree designs
- Explored various generalizations

To do:

1. General **quadrangle** as cross section
2. **Ternary** and higher
3. Grow trees with ‘deeper’ patterns, or randomly
4. Sibling branches that **share a cut surface** (cf. ternary miter joint)
Related Work

- Tom Verhoeff & Koos Verhoeff
  "The Mathematics of Mitering and Its Artful Application"
  *Bridges 2008*, Leeuwarden, Netherlands, pp.225–234

- Tom Verhoeff & Koos Verhoeff
  "Branching Miter Joints: Principles and Artwork"
  *Bridges 2010*, Pécs, Hungary, pp.27–34

- Tom Verhoeff & Koos Verhoeff
  "From Chain-link Fence to Space-spanning Helical Structures"
  *Bridges 2011*, Coimbra, Portugal, pp.73–80

Also see: [http://www.win.tue.nl/~wstomv/publications/](http://www.win.tue.nl/~wstomv/publications/)